CONESCO, INC. consultants in engineering science

205 Sixth Street
Cambridge 42, Massachusetts

SHIP HULL VIBRATIONS - 5
ANALYSIS OF HULL STRUCTURES AS APPLIED TO SSB(N) 598 GEORGE WASHINGTON

by

F. E. Reed

Elizabeth Cuthill *

Contract Number:	NObs 86111
Sponsor:	Department of the Navy Bureau of Ships Research and Development Division Ships Silencing Branch Project S-F013-11-01, Task 1351

March 1963 CONESCO, INC. Report No. F-111-2

*Applied Mathematics Laboratory
David Taylor Model Basin

TABLE OF CONTENTS

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF FIGURES (continued)

LIST OF FIGURES (continued)

LIST OF FIGURES (continued)

LIST OF TABLES

ABSTRACT

A computer program for determining the natural frequency of a ship hull elastically connected to other elastic systems and to sprung masses and containing concentrated and hysteresis damping is developed. This program is applied to the mass-elastic system that represents the SSB(N) 598 GEORGE WASHINGTON in detail. The influence upon the hull response to propeller and hull excitation in the stern of variables such as the number of mass points used to represent the hull, the omission of the propulsion sub-system, the stiffness between the propulsion sub-system and the primary hull, the inclusion of the moment of inertia of the hull cross-sections, the treatment of water inertias, the treatment of sprung masses and the amount of hysteresis damping in the hull is shown.

SUMMARY

STATEMENT OF THE PROBLEM

A previous prediction[2] of the amplitude of longitudinal and vertical bending vibration excited by the propeller for the submarine GEORGE WASHINGTON, SSB(N) 598, gave results that agreed moderately well with tests in longitudinal vibration and did not agree for vertical bending vibration. This study has been undertaken to explore the factors that influence the structural response of the hull and eventually (in a subsequent report) to see whether it is possible to make a more accurate prediction of the amplitudes of propeller excited hull vibration.

WORK SUMMARY

1. A computing machine program that is sufficiently flexible to deal with parallel sub-systems (e.g., the propulsion system), sprung masses and concentrated and hysteresis damping in axial and bending vibration has been developed.

2. The mass-elastic characteristics of the SSB(N) 598 GEORGE WASHINGTON have been determined in considerable detail.

3. The effects of various approximations such as omitting the propulsion sub-system, treating sprung masses as rigidly connected, neglecting rotary inertia, representing the hull with a limited number of mass points, spacing the mass points uniformly rather than to conform to hull mass and stiffness variations, have been investigated

4. The effects of various amounts of hysteresis damping in the hull have been investigated.

CONCLUSIONS

1. In predicting the vibration response of a ship in axial and bending modes, it is necessary to include the propulsion system (propellers and shafting) as an elastic sub-system in the calculations if reliable results are to be obtained.

2. Hull damping not only reduces the vibration amplitude in the vicinity of the excitation but also attenuates the relutive amplitude at locations remote from the point of excitation.

3. In order to obtain reliable predictions of vibration response of a ship it is necessary to define the hull mass and stiffnesses accurately. A less accurate definition is required if only hull frequencies are desired. Because of the attenuation caused by damping, the accurate definition of the hull is particularly important in the regions near the excitation.

4. On the submarines the effect of including rotary inertia of the hull cross sections upon the response of the ship are so small as to be negligible.

5. When a careful effort has been expended to define the hull mass and stiffnesses accurately, it is not advisable to use only a few mass points to represent the hull. The computation program can accept many points at a negligible extra cost. The greater number of points will result in better accuracy and a better definition of the hull motions will be obtained.

6. The hull response is sensitive to the stiffness of the connections between the propulsion system and the hull, and therefore careful attention should be paid to these stiffnesses. By controlling these stiffnesses to locate the resonant frequencies between the propulsion sub-system and the hull, it is possible to influence the hull vibration amplitudes at low speeds as compared with those at high speeds.

7. Although the total weight of the sprung masses is small compared to the weight of the hull, including them as sprung weights rather than as solidly mounted weights has an

important effect upon the hull response.

RECOMMENDATIONS

If reliable estimates of the response of a ship hull to vibratory excitation are to be obtained, it is necessary to determine the mass and elastic characteristics of the hull in considerable detail. Particular attention should be paid to the propulsion sub-system and its connection to the hull. The local springing of hull weights must be included.

I INTRODUCTION

In an early report[1] CONESCO outlined a program for predicting the amplitude of propeller excited hull vibration and presented ways of estimating the excitation arising from the propellers working in irregular wakes. A subsequent report[2] applied this program to a prediction of the longitudinal and vertical bending propeller excited vibrations on the USS GEORGE WASHINGTON, SSB(N) 598. For this purpose the excitation was computed by the methods of the earlier report but the structural response of the hull was determined from existing calculations[3]. It was considered that if the ship resonant frequencies as measured on trials and the amplitudes of vibration on the flanks of the resonances agreed with those that were predicted, that the method of prediction would be validated. The amplitude of vibration at the resonances would depend upon more detailed considerations of damping in the calculations and more experimental comparison between the damped calculations and trial results than could be made at the time.

When this comparison was made it was found that there was a moderate support for the calculation method as applied of longitudinal vibration of the hull since the predicted amplitudes of vibration on the flanks agree moderately well with measured values and the calculated and measured natural frequencies agreed within about 10%. For the bending calculation there appeared to be no agreement. However, the representation of the hull in the axial response calculations took account of the elasticity of the connection between the propeller and the hull where the bending calculation did not.

In an effort to improve the accuracy of the computation process and to study the effects of damping on the hull vibration amplitude, a thorough study of the structural response of the hull has been undertaken. To accomplish this it has been necessary to develop a new program for computing hull response in bending which is sufficiently adaptable to include not only the mass and elasticity of the elements in the propulsion system, but also the elastically supported elements throughout the ship. A procedure for computing the response of a ship that is represented in the most general way of a beam having

distributed mass and stiffness was also considered but because of complexity in the programming could not be completed and is therefore not reported at this time.

The results of these studies, involving (1) a more precise representation of the hull mass and structure (2) the inclusion of elastic sub-systems and (3) an investigation of the effects of simplifications and assumptions made in the calculations, are presented in this unclassified report. Because the actual vibration of the ship falls in the realm of classified information, the comparison of calculated and experimental amplitudes of vibration will be presented in a subsequent classified report.

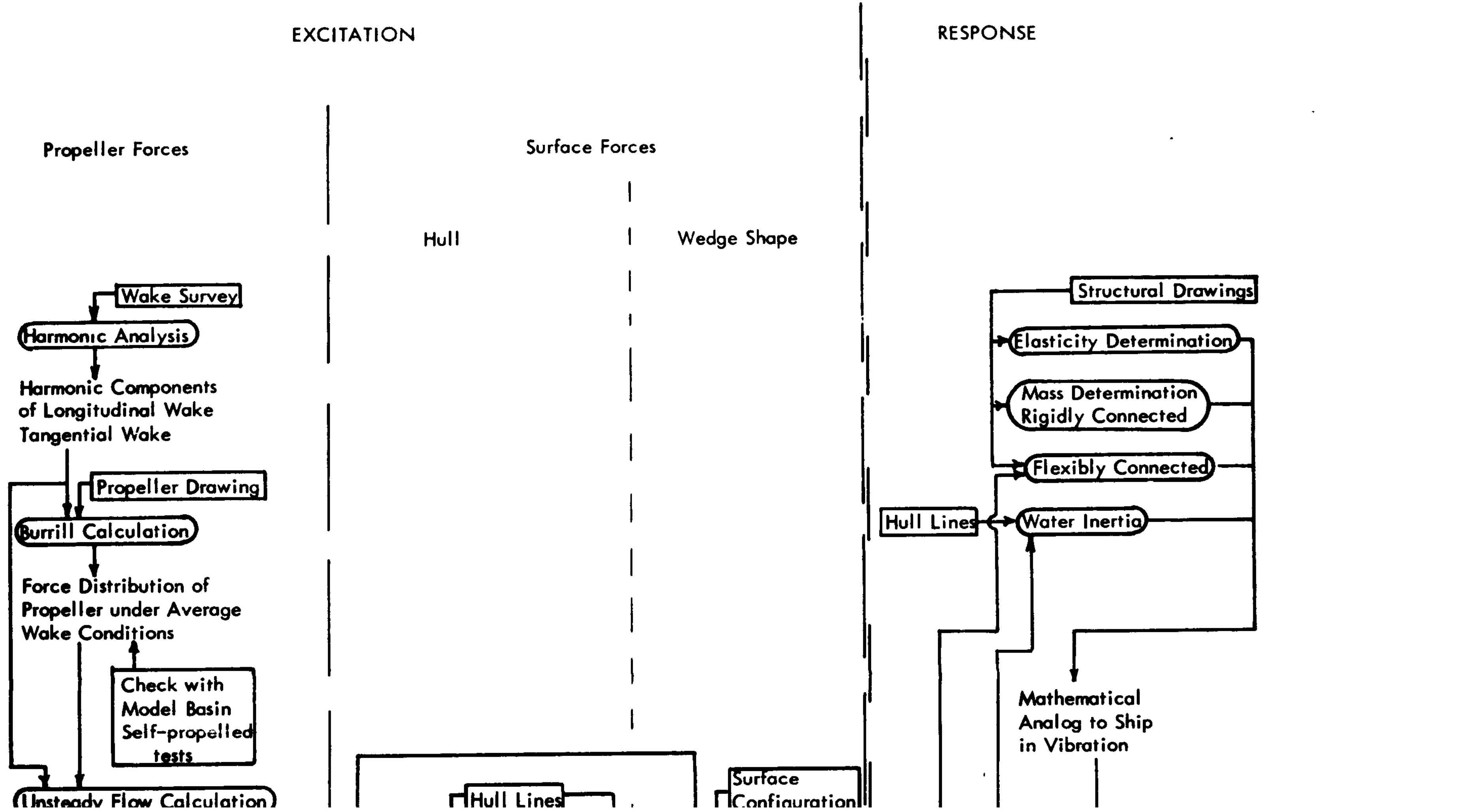
II METHOD OF SOLUTION

The method of computing the amplitudes of the propeller excited hull vibration falls into two categories: (1) the calculation of the exciting forces and (2) the calculation of the response of the hull to these exciting forces. This is essentially the standard procedure followed in the calculation of the forced vibrations of any system but in the case of ships involves many difficulties. The flow of work in the prediction process is diagrammed in Figure 1. The prediction of the forces has been discussed in previous reports.[2, 4] This report is concerned solely with the response of the hull to these forces.

III DEFINITION OF THE MASS AND ELASTIC PROPERTIES OF THE SHIP

It should be pointed out that since the ship under consideration is a submarine, the definition of the mass and stiffness can be made quite exact. The requirement of adequate transverse strength results in heavy structural members which carry their loads without buckling or local resonances. Furthermore, the shape is readily adaptable to theoretical predictions of the inertias of entrained water. It should not be assumed however, that even the submarine can be defined precisely. The flexibilities and water inertias associated with the control surfaces are inadequately handled in the present study and there are questions about the stiffness of the superstructure and the mass of the free flooding water

SEQUENCE OF CALCULATIONS FOR ESTIMATING
THE AMPLITUDE OF PROPELLER EXCITED HULL VIBRATION



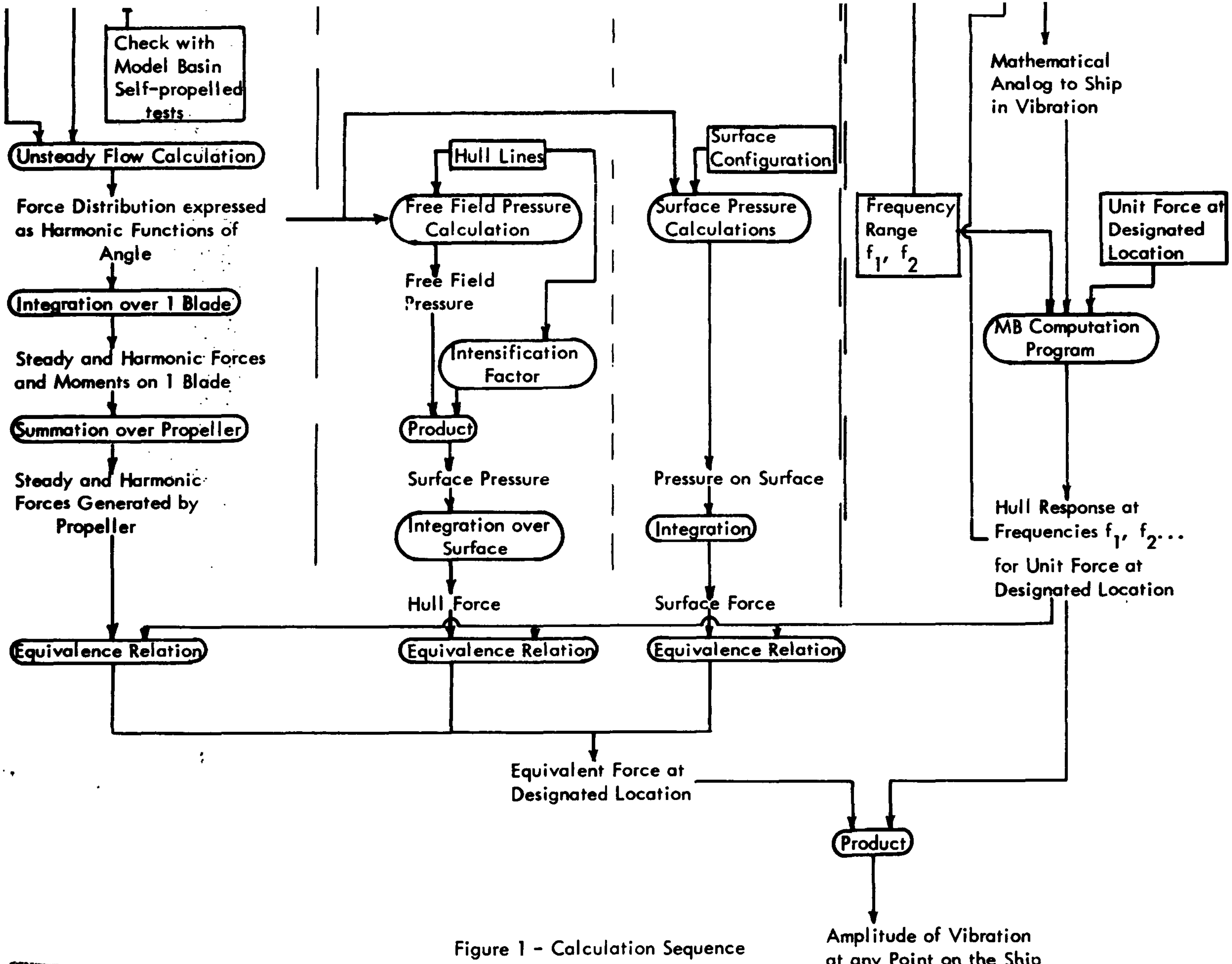


Figure 1 - Calculation Sequence

2

between the superstructure and the hull.

In this study the mass distribution and the stiffness distribution were determined to a rather high degree of precision and then plotted. From these plotted results a variety of hull representations were developed which seemed to fit the distributions of mass and stiffness well. A small scale plot of stiffnesses is given in Figure 2 and of weight distribution in Figure 3. The weight moments of inertia of the hull are presented in Figure 4. (Apparently E. B division feels that the entrained water contributes to moment of inertia. We feel that the contribution, as to the axial mass, is negligible). The weight and stiffness of the propulsion system in longitudinal vibration is given in Figure 5 and in vertical vibration in Figure 6. The characteristics of the sprung mounted weights and their mountings are given in Table I. A discussion of the methods by which these quantities were determined and the factors that entered into their determination are given in Appendices A-H.

IV CALCULATION PROCEDURE

For the longitudinal vibration it was found that an existing Taylor Model Basin Code was adequate for handling all of the variables that it was desired to impose upon the problem For the analysis of the bending, all of the existing codes were quite inadequate. CONESCO originally proposed some additions by which the existing codes might be made workable but the addition of these additional complications to an already inefficient code was not considered desirable. Dr. Cuthill (one of the authors of this report) therefore prepared an entirely new code which is adaptable to a wide variety of hull representations. This code can deal not only with the bending but also is sufficiently general to cover the longitudinal vibration and with small changes can deal with coupled longitudinal and vertical bending or coupled torsion and transverse bending. Because it was developed after the hull had been defined, the present calculations do not fully exploit the potentials of the new code (specifically in the treatment of control surfaces). A presentation of the basis for this code and the way it is used is given in Appendix I. This general bending

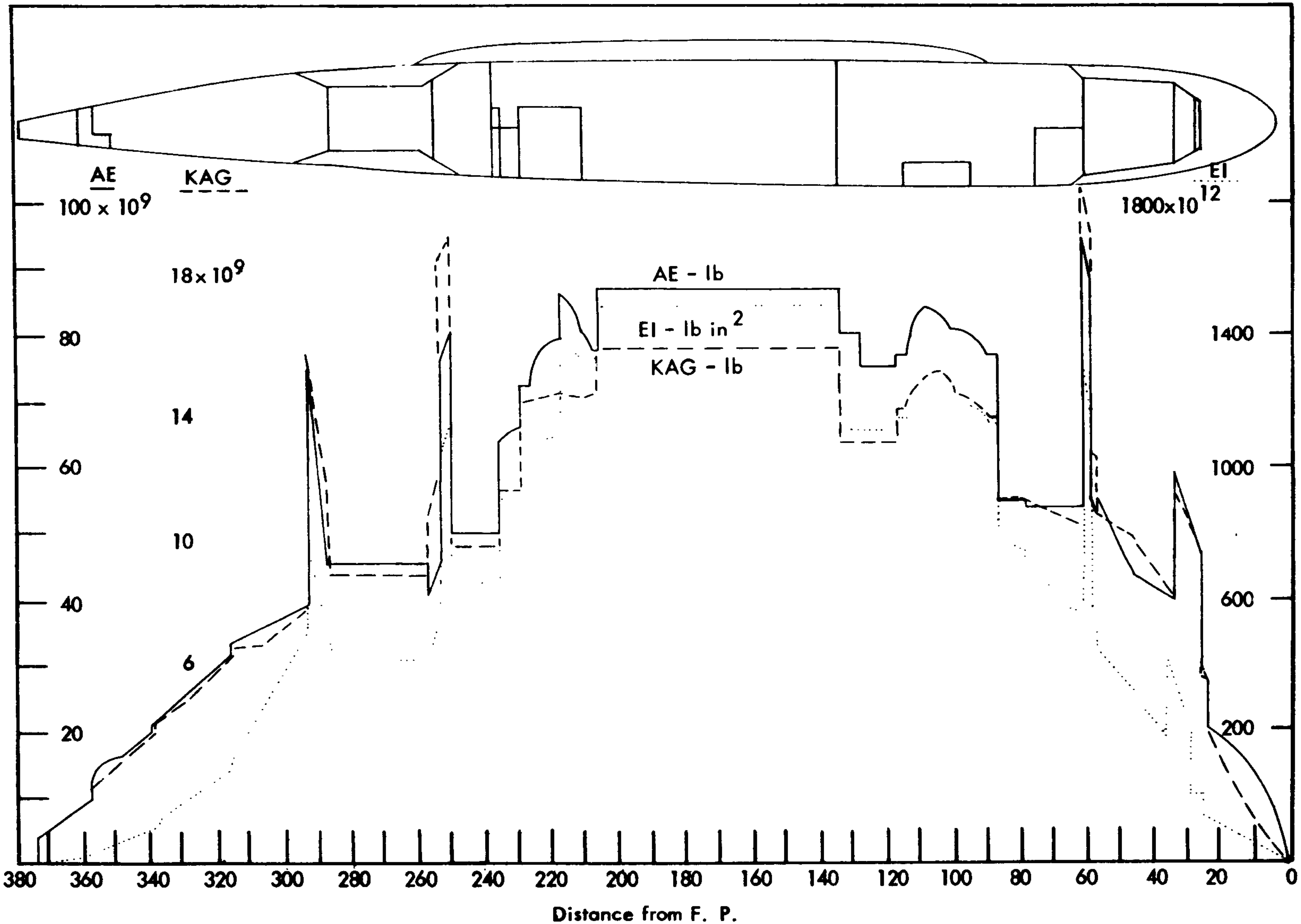
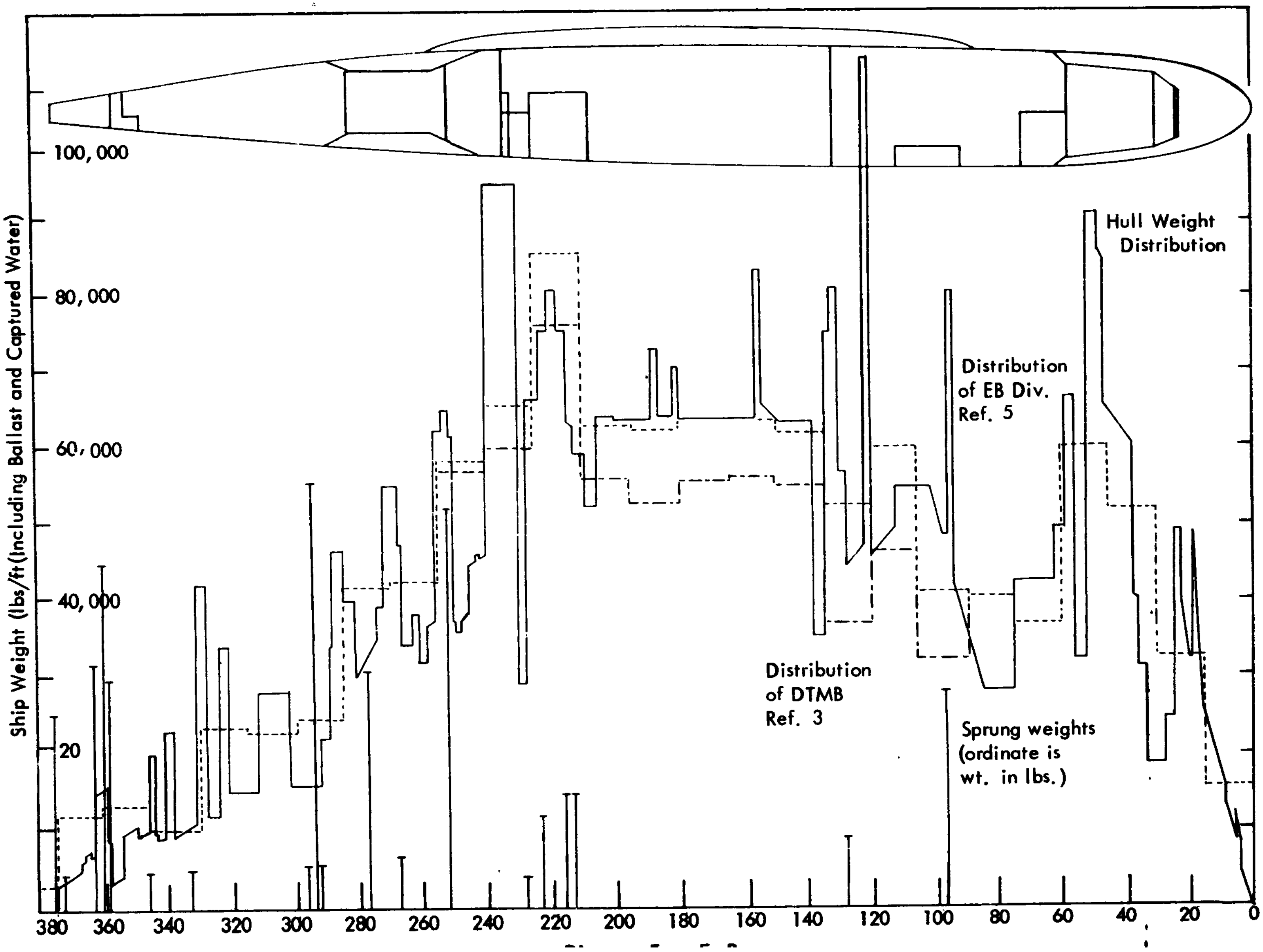


Figure 2 - SSB(N) 598 - Axial, Shear and Bending Stiffness



100,000
80,000
60,000
40,000
20
Ship Weight (lbs/ft(Including Ballast and Captured Water)
Hull Weight Distribution
Distribution of EB Div. Ref. 5
Distribution of DTMB Ref. 3
Sprung weights (ordinate is wt. in lbs.)
380
360
340
320
300
280
260
240
220
200
180
160
140
120
100
80
60
40
20
0

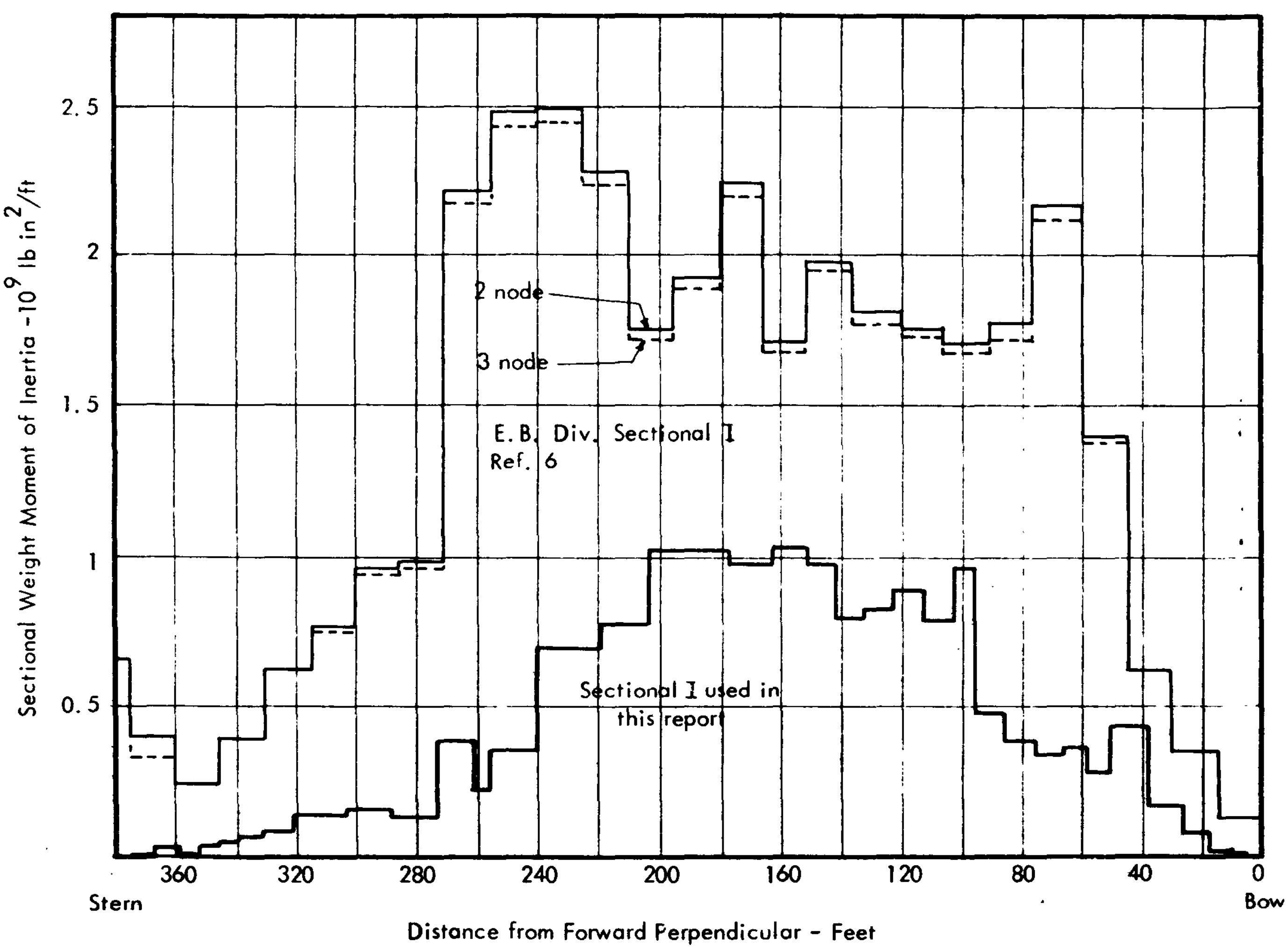


Figure 4 - Sectional Weight Moments of Inertia - SSB(N) 598

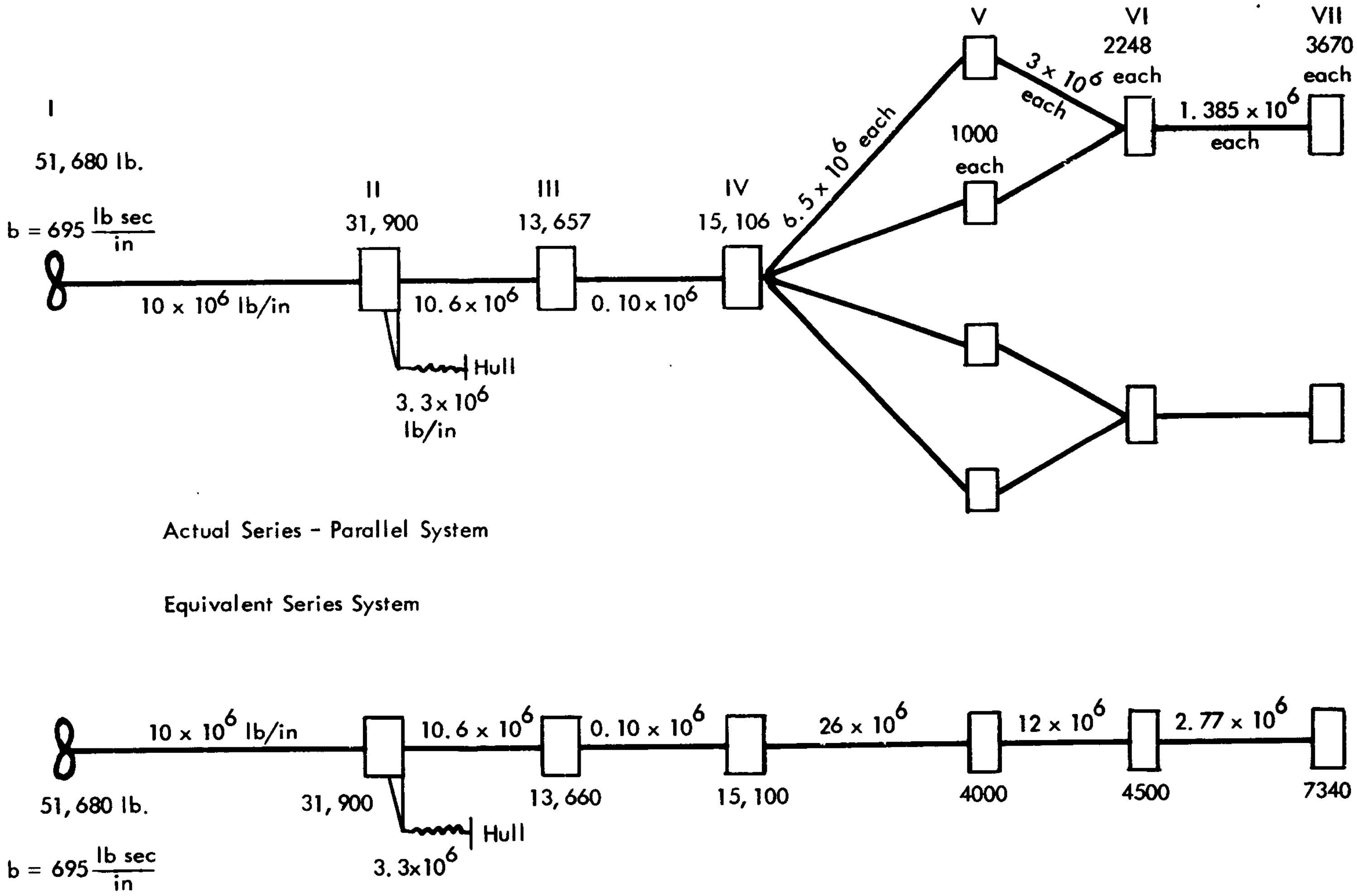


Figure 5-SSB(N) 598 Weight and Stiffness of Propulsion System in Longitudinal Vibration

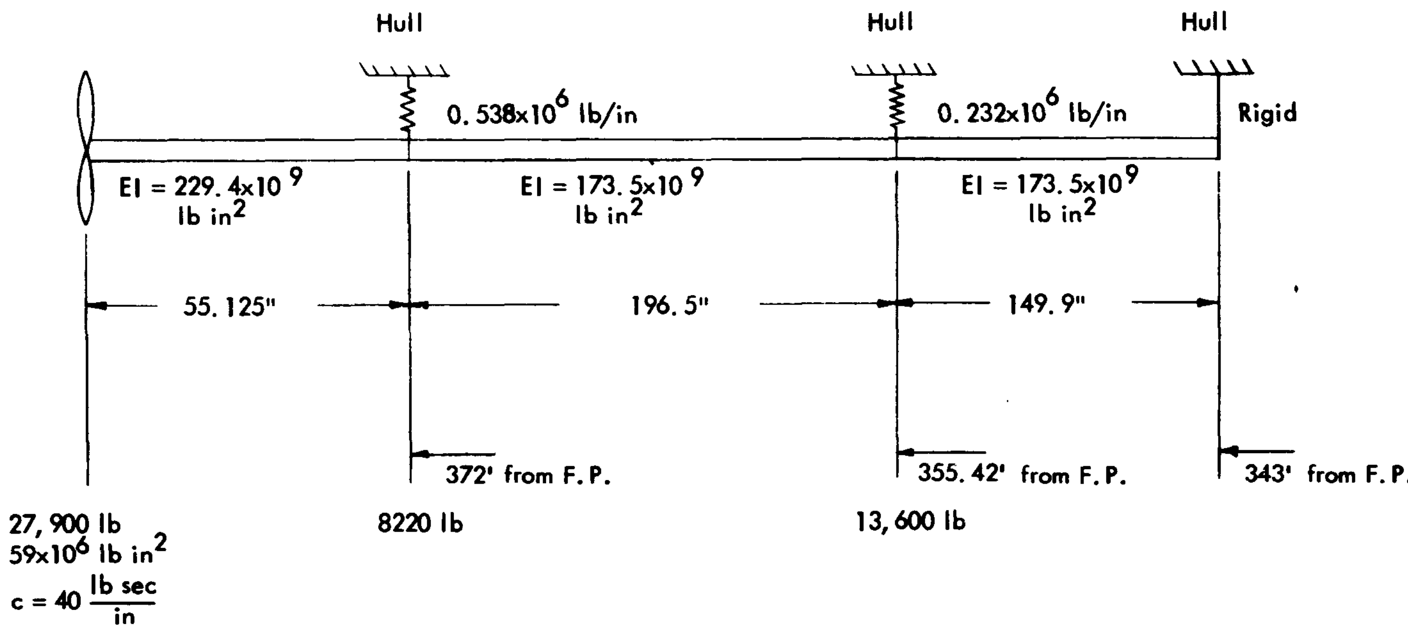


Figure 6 - Mass and Stiffness of Propulsion System in Vertical Vibration

TABLE I - SSB(N) 598 GEORGE WASHINGTON SPRUNG WEIGHTS

Flexibly-Mounted Element	Distance from F.P. (feet)	Weight (lbs)	Longitudinal		Vertical	
			Stiffness lb/in	Damping* lb sec/in	Stiffness lb/in	Damping* lb sec/in
Each of two Fair-water diving planes	98	Long. 12,771 Vert. 7,750 37,300	Rigid		Rigid 122,200	60n
Gyro. Hyd. System	125	6,900	12,400	24.6	31,440	29.3
L.P. Blower	128	3,145	5,720	11.5	13,600	12.6
O2 Gen. Plant	213	15,200	11,600	27.8	50,000	60.7
Air Cond. Set	217	15,200	11,200	19.3	51,600	41.6
400 Cycle M.G. sets	224	12,261	8,100	15.2	38,700	33.3
H.P. Air Comp.	228	3,450	6,160	25.25	14,400	14.5
Main Coolant Pumps	251.5	51,893	307,000	192	Rigid	
Trim and Drain Pumps	268	5,300	9,000	15.9	16,000	16.4
300 KW M.G. Sets	276	32,920	20,640	107	46,000	104
H.P. Air Comp.	292	3,991	7,150	14.2	16,700	16.7
Air Cond. Sets each of 2	294	27,470	36,120	81.5	80,500	79.1
H.P. Air Comp.	298	3,991	7,150	14.2	16,700	16.7
Hydraulic Plant	346	4,300	6,400	13.6	16,800	19.8
Each of 2 stabilizers + Stern Planes	360	Long. 36,495 Vert. 68,100 125,000	Rigid		Rigid 1.820×10^6	110n

* Damping is located between mass and foundation except for fairwater and stern planes which are to ground. Fairwater and stern plane damping constant is proportional to ship speed. This proportionality is represented by n, the frequency of oscillation, cps, for the case of a 5-bladed propeller.

response code carries the AML problem number designation 840-277.

V DAMPING

Damping on a complex structure like a ship is difficult to define – in fact, it can only be determined by experience. However, there are certain types of damping that can be calculated by theory or from the results of experiments external to the ship. Among these are the damping of the propeller in longitudinal, torsional and transverse motion, the aerodynamic damping of control surfaces and the damping in the rubber mounts used under flexibly mounted machinery. Although these are significant sources of damping, the major damping is in the hull. By experiments and theory it is known that this damping is in the hull itself not in the water and is thus related to the slope of the deflection curve, not the absolute amplitude, i.e., the relative amplitudes between adjacent points. In the damping of structures it has been found that one of the most reasonable ways to deal with this type of damping is to tie it with the stress in the structure. If it is assumed that a certain percentage of the stress energy in a volume of stressed material is dissipated because of damping, then it is possible to incorporate the effects of this damping in the calculations by the simple expedient of making the modulus of elasticity a complex term. It should be understood that the actual damping in a ship is much larger than that due to the hysteresis in the material. Most of the damping probably comes from sliding between elements of structure and cargo. It is tied to stress only because this procedure offers a good basis for calculations, and is not incompatible with experience. The actual value of the damping constants can only be determined by many comparisons between carefully calculated amplitudes of vibration and those obtained on trials. The damping constants will probably vary from ship to ship. It will be noted in the subsequent results that the damping not only reduces the amplitudes of vibration in the vicinity of the exciting force but also reduces the transmission of the vibration into parts of the structure remote from the excitation.

VI REPRESENTATION OF THE HULL IN THE CALCULATIONS

Because the hull was so carefully defined and because a computer program had been developed that could deal with a wide variety of hull representations, the stage was well set for a broad exploration of many factors that were expected to influence the hull response.

The various calculations that were made on the hull of the SSB(N) 598 are given in Table II. Supporting data giving the constants for longitudinal vibration are given in Figures 6, 7 and 8. For the bending vibration calculations, the arrangement of the stations are given in Figures 9-14 in conjunction with Table III. The location of the stations is given in Table IV. The specific values of mass and stiffnesses in bending vibration are given in the computer input sheets in Appendix J.

In both the longitudinal and the bending vibrations the first computation was a repeat of the earlier one made in Reference 3 (Cases 1 and 12). In addition, a repeat of the calculations with a 10% hysteresis damping was made in order to compare with later calculations (Cases 1a and 12a). The second calculation was of a hull represented by the same number and location of stations as in Reference 3, but with each station assigned the mass and stiffness determined by the more detailed analysis of the hull. (Cases 2 and 13). The comparison between these calculations gives an indication of the possible increase in accuracy that could be gained by the laborious definition of the hull. In longitudinal vibration an exploration was made of the number of masses used to represent the ship (Cases 2, 3 and 4); of the effects of hull damping (Cases 4, 5, 6 and 7) and of the effects of changes in the difficult-to-compute stiffness between the propulsion system and the hull (Cases 4, 8 and 9). The final two runs (Cases 10 and 11) were made for use in computing the hull vibration and indicate the difference in hull response for forces on the propeller and on the hull.

For the bending vibrations there were additional factors to be investigated and so the range of calculations was larger. The difference in hull response as the propeller and

Text continues on page 34

TABLE II - SCHEDULE OF HULL VIBRATION CALCULATIONS

Case No.	Mode	Hull Definition	Excitation	Water Inertia *	Damping (coef. of imaginary term is hysteresis damping)	Remarks
1	Longi-tudinal	DTMB Report No. 1464 Table 2	Unit Axial F. Prop. Sys. Sta. 0	Inc. in table	No damping	Check Case
2	Longi-tudinal	"	Unit Axial F. Hull Sta. 0	"	"	Check Case
3	Longi-tudinal	24 uniformly spaced masses Figure 6	Unit Axial F. Propeller	Included in Figure 6	Propeller, sprung masses, $\frac{\Delta x}{AE}$ divided by (1+0.1j)	
4	Longi-tudinal	16 concentr. masses Figure 7	Unit Axial F. Propeller	Included in Figure 7	Propeller, sprung masses, $\frac{\Delta x}{AE}$ divided by (1+0.1j)	
5	Longi-tudinal	35 concentr. masses Figure 8	"	Included in Figure 8	Propeller, and sprung masses only	
6	Longi-tudinal	"	"	"	Propeller, sprung masses, $\frac{\Delta x}{AE}$ divided by (1+0.04j)	

TABLE II - SCHEDULE OF HULL VIBRATION CALCULATIONS (continued)

Case No.	Mode	Hull Definition	Excitation	Water Inertia*	Damping (coef. of imaginary term is hysteresis damping)	Remarks
7	Longi-tudinal	35 concentr. masses Figure 8	Unit Axial Force at Propeller	Included in Figure 8	Propeller, sprung masses $\frac{\Delta x}{AE}$ divided by $(1+0.1j)$	
8	Longi-tudinal	"	"	"	Propeller, sprung masses $\frac{\Delta x}{AE}$ divided by $(1+0.25j)$	
9	Longi-tudinal	35 concentr. masses. Fig. 8 increase shaft-hull stiffness from 3.3×10^6 to 6.6×10^6	"	"	Propeller, sprung masses $\frac{\Delta x}{AE}$ divided by $(1+0.10j)$	
10	Longi-tudinal	35 concentr. masses. Fig. 8 decrease shaft-hull stiffness from 3.3×10^6 to 1.0×10^6	Unit Axial force at propeller	Included in Fig 8	Propeller, sprung masses $\frac{\Delta x}{AE}$ divided by $(1+0.10j)$	
11	Longi-tudinal	35 concentr. masses, Fig. 8	Unit Axial force at Sta. 36	"	"	

TABLE II - SCHEDULE OF HULL VIBRATION CALCULATIONS (continued)

Case No.	Mode	Hull Definition	Excitation	Water Inertia *	Damping (coef. of imaginary term is hysteresis damping)	Remarks
12	Vertical Bending	DTMB Report No. 1464 Table 1-Subm. Fig. 9 Appendix J1-3	Unit vertical force at Station 0	Included in mass	No damping	Check Case
12A	Vertical Bending	"	"	"	$\frac{1}{EI}$ and $\frac{1}{KAG}$ divided by $(1+0.10j)$	12 with 10% hysteresis
13	Vertical Bending	24 uniformly spaced masses, hull only Fig. 10 Appendix J4-11	Unit vertical force at Stern	$J=0.87$ for $0<n<3.5$ cps $J=0.82$ for 3.5 cps $<n<25.7$ cps	Sprung masses, $\frac{1}{EI}$ and $\frac{1}{KAG}$ divided by $(1+0.10j)$	
14	Vertical Bending	24 uniformly spaced masses + propulsion sys. Fig. 11 Appendix J12-16	"	"	Propeller, sprung masses, $\frac{1}{EI}$ and $\frac{1}{KAG}$ divided by $(1+0.10j)$	
15	Vertical Bending	" Figure 11 Appendix J17-20	Unit vertical force at Propeller	"	"	

TABLE II - SCHEDULE OF HULL VIBRATION CALCULATIONS (continued)

Case No.	Mode	Hull Definition	Excitation	Water Inertia*	Damping (coef. of imaginary term is hysteresis damping)	Remarks
16	Vertical Bending	16 conc. masses + propulsion sys. Figure 12 Appendix J21-27	Unit vertical force at Propeller	$J = 0.87$ for $0<n<3.5$ cps $J = 0.82$ for 3.5 cps $<n<25.7$ cps	Propeller, sprung masses, $\frac{1}{EI}$ and $\frac{1}{KAG}$ divided by $(1+0.10j)$	
17	Vertical Bending	35 conc. masses + propulsion sys. Figure 13 Table III - Appendix J28-36	Unit vertical force at Propeller	$J = 0.87$ for $0<n<3.5$ cps $J = 0.82$ for 3.5 cps $<n<25.7$ cps	Propeller, sprung masses, $\frac{1}{EI}$ and $\frac{1}{KAG}$ divided by $(1+0.10j)$	
18	Vertical Bending	" J37-38	"	J determined for sinusoidal mode shape	"	
19	Vertical Bending	" Fig. 13 Table III Appendix J39-41	"	"	Propeller and sprung masses only	
20	Vertical Bending	35 conc. masses + propulsion sys. Fig. 13 Table III Appendix J42-44	Unit vertical force at Propeller	J determined for sinusoidal mode shape	Propeller, sprung masses, $\frac{1}{EI}$ and $\frac{1}{KAG}$ divided by $(1+0.04j)$	

TABLE II - SCHEDULE OF HULL VIBRATION CALCULATION (continued)

Case No.	Mode	Hull Definition	Excitation	Water Inertia*	Damping (coef. of imaginary term is hysteresis damping)	Remarks
21	Vertical Bending	35 conc. masses + propulsion sys. Fig. 13 Table III Appendix J45-47	Unit vertical force at Propeller	J determined for sinusoidal mode shape	Propeller, sprung masses $\frac{1}{EI}$ and $\frac{1}{KAG}$ divided by $(1+0.25j)$	
22	Vertical Bending	35 conc. masses + propulsion sys. hull stiffness conn. at B increased from 0.538×10^6 to 1.2×10^6 lb/in at C from 0.232×10^6 to 0.7×10^6 lb/in Fig. 13 Table III Appendix J48-50	Unit vertical force at Propeller	J determined for sinusoidal mode shape	Propeller, sprung masses $\frac{1}{EI}$ and $\frac{1}{KAG}$ divided by $(1+0.10j)$	
23	Vertical Bending	35 conc. masses + propulsion sys. hull stiffness of conn. at B decreased from 0.538×10^6 to 0.20×10^6 lb/in and at C from 0.232×10^6 to 0.10×10^6 lb/in Fig. 13 Table III Appendix J51-52	"	"	"	

TABLE II - SCHEDULE OF HULL VIBRATION CALCULATIONS (continued)

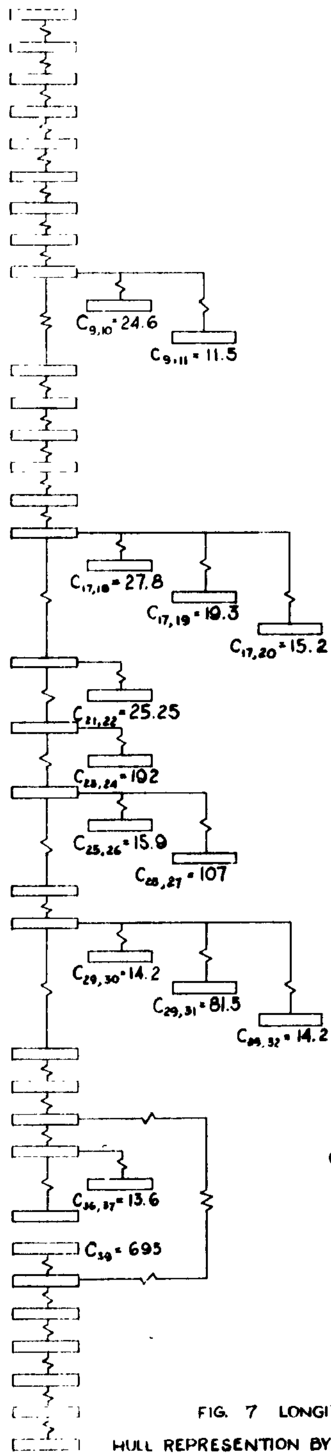
Case No.	Mode	Hull Definition	Excitation	Water Inertia *	Damping (coef. of imaginary term is hysteresis damping)	Remarks
24	Vertical Bending	35 conc. masses + propulsion sys. Fig. 13 Table III Appendix J53-55	Unit moment about horizontal axis at Propeller	J determined for sinusoidal mode shape	Propeller, sprung masses $\frac{1}{EI}$ and $\frac{1}{KAG}$ divided by $(1+0.10j)$	
25	Vertical Bending	35 conc. masses + propulsion sys. Fig. 13 Table III Appendix J56-57	Unit vertical force at Station 54	J determined for sinusoidal mode shape	Propeller, sprung masses $\frac{1}{EI}$ and $\frac{1}{KAG}$ divided by $(1+0.10j)$	
26	Vertical Bending	" Fig. 13 Table III Appendix J58-59	Unit vertical force at Station 51	"	"	
27	Vertical Bending	35 conc. masses + propulsion sys. hold all sprung masses rigidly Fig. 14 Appendix J60-63	Unit vertical force at Propeller	J determined for sinusoidal mode shape	Propeller $\frac{1}{EI}$ and $\frac{1}{KAG}$ divided by $(1+0.1j)$	

TABLE II - SCHEDULE OF HULL VIBRATION CALCULATIONS (continued)

Case No.	Mode	Hull Definition	Excitation	Water Inertia*	Damping (coef. of imaginary term is hysteresis damping)	Remarks
28	Longi-tudinal	Distributed + conc. weights + propulsion sys.	Unit axial force at Propeller	J determined for sinusoidal mode shape	Propeller, sprung masses, $\frac{1}{EI}$ and $\frac{1}{KAG}$ divided by $(1+0.1j)$	This was not run
29	Vertical Bending	"	Unit vertical force at Propeller	"	"	This was not run
30	Vertical Bending	35 conc. masses including rotary inertia + propul-sion sys. + sprung weights Fig. 13 Table III Appendix J 64-70	Unit vertical force at Propeller	J determined for sinusoidal mode shape	Propeller, sprung masses, $\frac{1}{EI}$ and $\frac{1}{KAG}$ divided by $(1+0.1j)$	

* The sectional water inertia values are reduced to account for bending vibration modes by the value J. See Appendix C.

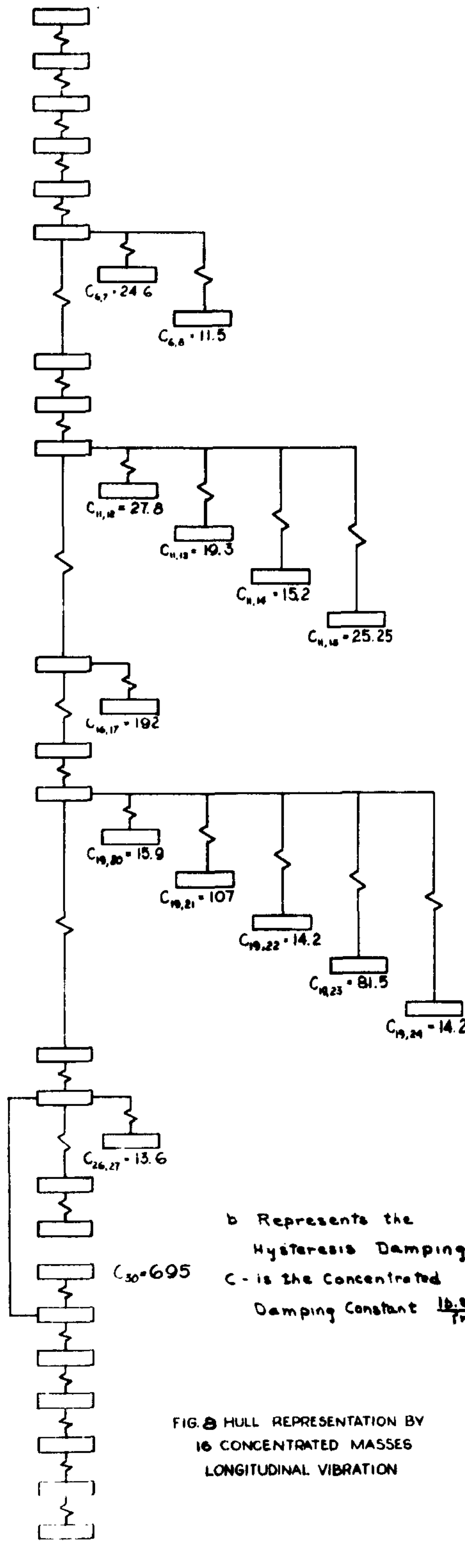
STIFFNESS $k = \frac{LBS.}{IN}$	WEIGHT LBS.	STATION NO.
	201,580	1
$10^{-6}\times(1+bi)\,94.34$		
	520,900	2
$10^{-6}\times(1+bi)\,210.08$		
	661,350	3
$10^{-6}\times(1+bi)\,252.53$		
	922,550	4
$10^{-6}\times(1+bi)\,313.48$		
	654,575	5
$10^{-6}\times(1+bi)\,295.86$		
	449,000	6
$10^{-6}\times(1+bi)\,389.11$		
	785,925	7
$10^{-6}\times(1+bi)\,456.62$		
	765,675	8
$10^{-6}\times(1+bi)\,420.17$		
	973,370	9
12400		
	6,900	10
5720		
	3,145	11
$10^{-6}\times(1+bi)\,458.72$		
	893,700	12
$10^{-6}\times(1+bi)\,480.77$		
	992,600	13
$10^{-6}\times(1+bi)\,480.77$		
	945,000	14
$10^{-6}\times(1+bi)\,480.77$		
	978,000	15
$10^{-6}\times(1+bi)\,480.77$		
	921,800	16
$10^{-6}\times(1+bi)\,456.62$		
	1,072,725	17
11600		
	15,200	18
11200		
	15,200	19
8100		
	12,261	20
$10^{-6}\times(1+bi)\,401.61$		
	1,157,040	21
6160		
	3,450	22
$10^{-6}\times(1+bi)\,288.18$		
	727,900	23
307000		
	51,893	24
$10^{-6}\times(1+bi)\,271.74$		
	631,500	25
9000		
	5,300	26
20640		
	32,920	27
$10^{-6}\times(1+bi)\,251.26$		
	559,900	28
$10^{-6}\times(1+bi)\,281.69$		
	301,450	29
7150		
	3,991	30
36120		
	54,940	31
7150		
	3,991	32
$10^{-6}\times(1+bi)\,209.21$		
	354,675	33
$10^{-6}\times(1+bi)\,176.367$		
	280,850	34
$10^{-6}\times(1+bi)\,138.696$		
	215,050	35
$10^{-6}\times(1+bi)\,94.16$		
	142,275	36
6400		
	4,300	37
$10^{-6}\times(1+bi)\,52.91$		
	124,230	38
$10^{-6}\times(1+bi)\,3.3$		
	51,680	39
$10^{-6}\times(1+bi)\,10$		
	31,900	40
$10^{-6}\times(1+bi)\,10.6$		
	13,660	41
$10^{-6}\times(1+bi)\,.10$		
	15,100	42
$10^{-6}\times(1+bi)\,26$		
	4,000	43
$10^{-6}\times(1+bi)\,12$		
	4,500	44
$10^{-6}\times(1+bi)\,2.77$		
	7,340	45



b Represents the Hysteresis Damping
C is the Concentrated Damping Constant $\frac{lb\,sec}{in}$

FIG. 7 LONGITUDINAL VIBRATION HULL REPRESENTION BY 25 UNIFORMLY SPACED CONCENTRATED MASSES

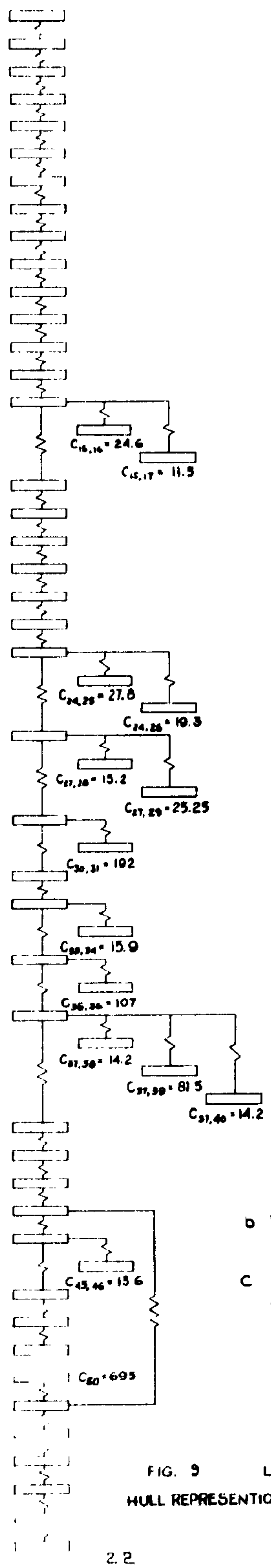
Weight (lbs.)	Stiffness K = lbs/in.	Sta. No
8145		1
	$(1+bi)61.27\times10^{-6}$	
584,813		2
	$(1+bi)70.82\times10^{-6}$	
1,580,247		3
	$(1+bi)185.87\times10^{-6}$	
1,028,411		4
	$(1+bi)174.22\times10^{-6}$	
1,327,915		5
	$(1+bi)224.72\times10^{-6}$	
1,642,546		6
	12400	
6,900		7
	5720	
3145		8
	$(1+bi)249.38\times10^{-6}$	
2,357,950		9
	$(1+bi)240.38\times10^{-6}$	
916,000		10
	$(1+bi)283.29\times10^{-6}$	
3,312,928		11
	11600	
15,200		12
	11200	
15,200		13
	8100	
12,261		14
	6160	
3,450		15
	$(1+bi)204.08\times10^{-6}$	
790,880		16
	307,000	
51,893		17
	$(1+bi)404.86\times10^{-6}$	
286,905		18
	$(1+bi)162.60\times10^{-6}$	
1,425,200		19
	9000	
5,300		20
	20640	
32,920		21
	7150	
3,991		22
	36120	
54,940		23
	7150	
3,991		24
	$(1+bi)56.883\times10^{-6}$	
669,413		25
	$(1+bi)91.575\times10^{-6}$	
201,169		26
	6400	
4,300		27
	$(1+bi)88.889\times10^{-6}$	
89,941		28
	$(1+bi)41.841\times10^{-6}$	
33,730		29
	$(1+bi)3.3\times10^{-6}$	
51,680		30
	$(1+bi)10\times10^{-6}$	
31,900		31
	$(1+bi)10.6\times10^{-6}$	
13,660		32
	$(1+bi).1\times10^{-6}$	
15,100		33
	$(1+bi)26\times10^{-6}$	
4,000		34
	$(1+bi)12\times10^{-6}$	
4,500		35
	$(1+bi)2.77\times10^{-6}$	
7,340		36



b Represents the Hysteresis Damping
C - is the Concentrated Damping Constant $\frac{\text{lb. sec.}}{\text{in.}}$

FIG. 8 HULL REPRESENTATION BY 16 CONCENTRATED MASSES LONGITUDINAL VIBRATION

STIFFNESS k - LBS/IN	WEIGHT LBS	STA. NO.
98.9×10^{-6}	8,145	1
166.6×10^{-6}	70,394	2
161.55×10^{-6}	158,570	3
253.81×10^{-6}	355,899	4
250.6×10^{-6}	293,623	5
408.16×10^{-6}	861,205	6
719.42×10^{-6}	425,419	7
649.35×10^{-6}	341,547	8
396.83×10^{-6}	387,288	9
505.05×10^{-6}	299,576	10
806.45×10^{-6}	317,196	11
757.58×10^{-6}	492,448	12
653.59×10^{-6}	518,271	13
625.0×10^{-6}	583,650	14
12400	534,144	15
5720	6,900	16
729.93×10^{-6}	3,145	17
515.46×10^{-6}	524,752	18
450.45×10^{-6}	599,000	19
598.80×10^{-6}	813,450	20
537.63×10^{-6}	945,000	21
363.64×10^{-6}	916,000	22
465.12×10^{-6}	786,000	23
11600	1,010,291	24
11200	15,200	25
404.85×10^{-6}	15,200	26
8100	1,516,637	27
6160	12,261	28
465.12×10^{-6}	3,450	29
307000	790,880	30
404.85×10^{-6}	51,893	31
523.56×10^{-6}	286,905	32
9000	484,457	33
235.85×10^{-6}	5,300	34
20640	562,527	35
276.24×10^{-6}	32,920	36
7150	378,235	37
36120	3,991	38
7150	54,940	39
152.67×10^{-6}	3,991	40
134.95×10^{-6}	325,238	41
235.85×10^{-6}	226,996	42
216.45×10^{-6}	117,179	43
485.44×10^{-6}	101,375	44
6400	55,308	45
174.83×10^{-6}	4,300	46
180.83×10^{-6}	44,486	47
418.41×10^{-6}	89,941	48
3.3×10^{-6}	35,730	49
10×10^{-6}	51,680	50
10.6×10^{-6}	31,900	51
$.10 \times 10^{-6}$	13,660	52
26×10^{-6}	15,100	53
12×10^{-6}	4,000	54
2.77×10^{-6}	4,500	55
	7,340	56

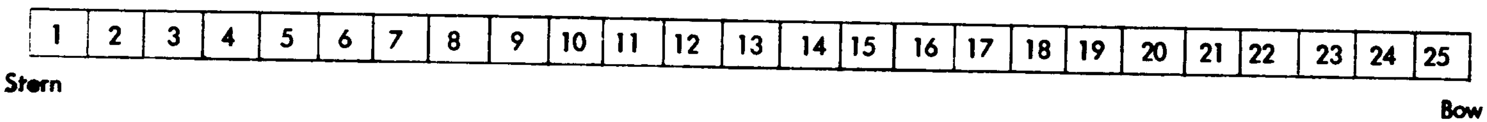


b Represents the Hysteresis Damping

C is the Concentrated Damping Constant $\frac{\text{lb. sec.}}{\text{in}}$

FIG. 9 LONGITUDINAL VIBRATION

HULL REPRESENTION BY 56 CONCENTRATED MASSES

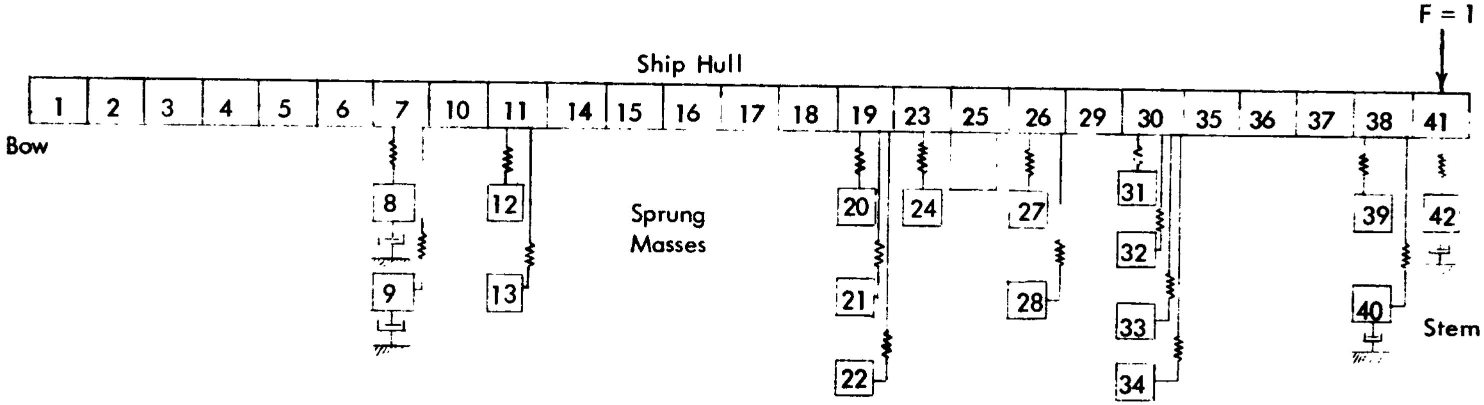


SSB(N) 598

Vertical Bending Vibration Calculations

Case 12 Ship Representation	25 Uniformly Spaced Masses Including Water Inertia Mass and Stiffness as Given in DTMB Report 1464 Excitation at the Stern (Sta. 1) No Damping
Case 12A	Identical but with 10% Hysteresis Damping

Figure 10 - Ship Representation for Calculation

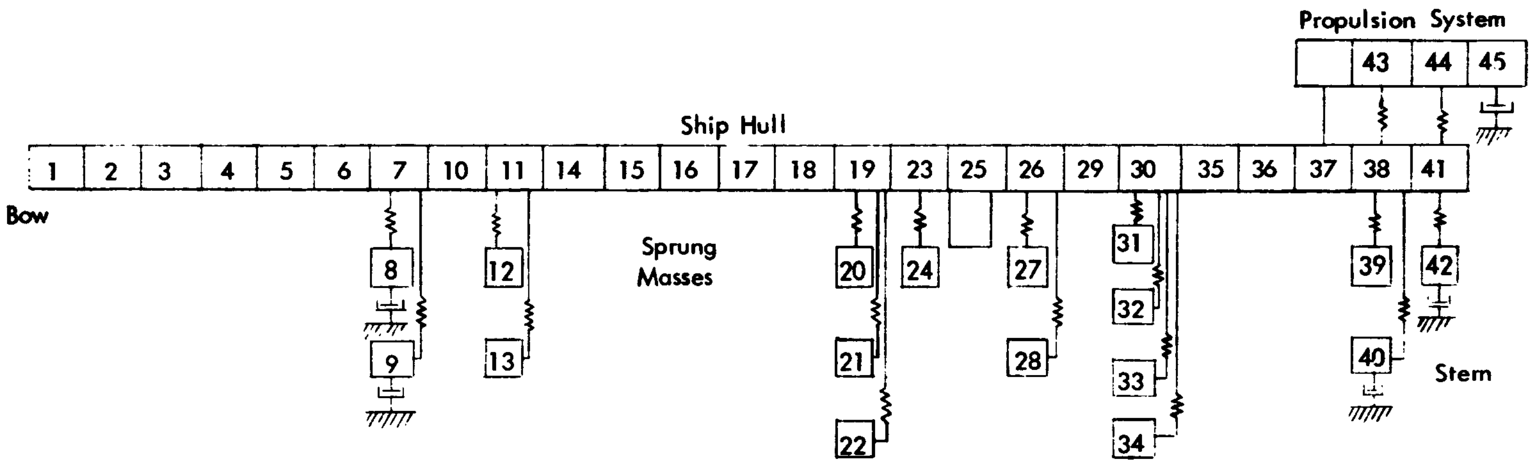


SSB(N) 598

Vertical Bending Vibration Calculations

Case 13	24 Uniformly Spaced Masses
Ship	Damped Sprung Masses
Representation	Water Inertia $J = 0.87$ for $0 < n < 3.5$ cps
	$= 0.82$ for $3.5 < n$ cps
	Excitation at Stern (Sta. 41)
	10% Hysteresis Damping

Figure 11 - Ship Representation for Calculation

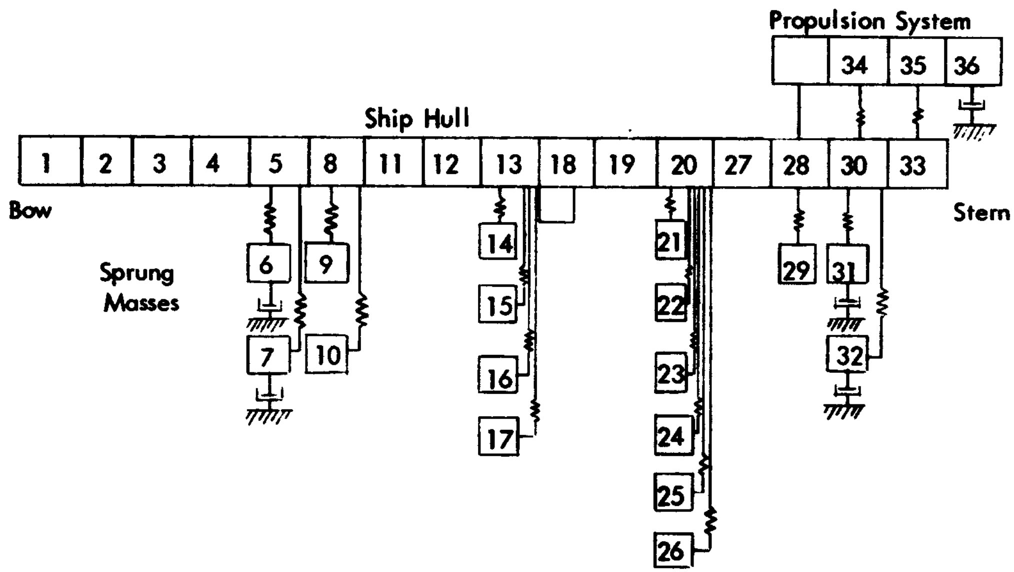


SSB(N) 598

Vertical Bending Vibration Calculations

Case 14 Ship Representation	24 Uniformly Spaced Masses Damped Sprung Masses, Propulsion System Water Inertia $J = 0.82$ for $0 < n < 3.5$ cps; $= 0.87$ for $3.5 < n$ cps Excitation at Stern (Sta. 41) 10% Hysteresis Damping
Case 15 Ship Representation	Identical with Case 14 except Excitation is at the Propeller (Sta. 45)

Figure 12 - Ship Representation for Calculation

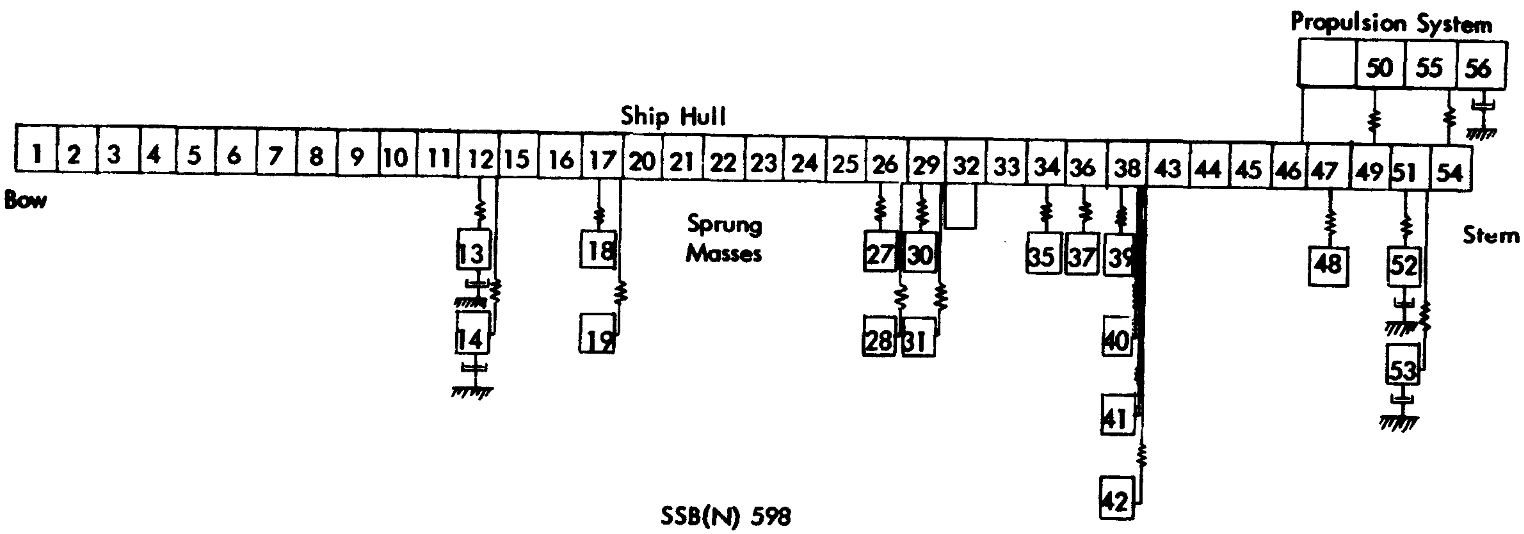


SSB(N) 598

Vertical Bending Vibration Calculation

Case 16	16 Concentrated Masses
Ship	Damped Sprung Masses; Propulsion System
Representation	Water Inertia, $J = 0.87$ for $0 < n < 3.5$ cps, $= 0.82$ for $3.5 < n$ cps
	Excitation at the Propeller (Sta. 36)
	10% Hysteresis Damping

Figure 13 - Ship Representation for Calculation



SSB(N) 598

Vertical Bending Vibration Calculations

Cases 17-26 and 30 Ship Representation

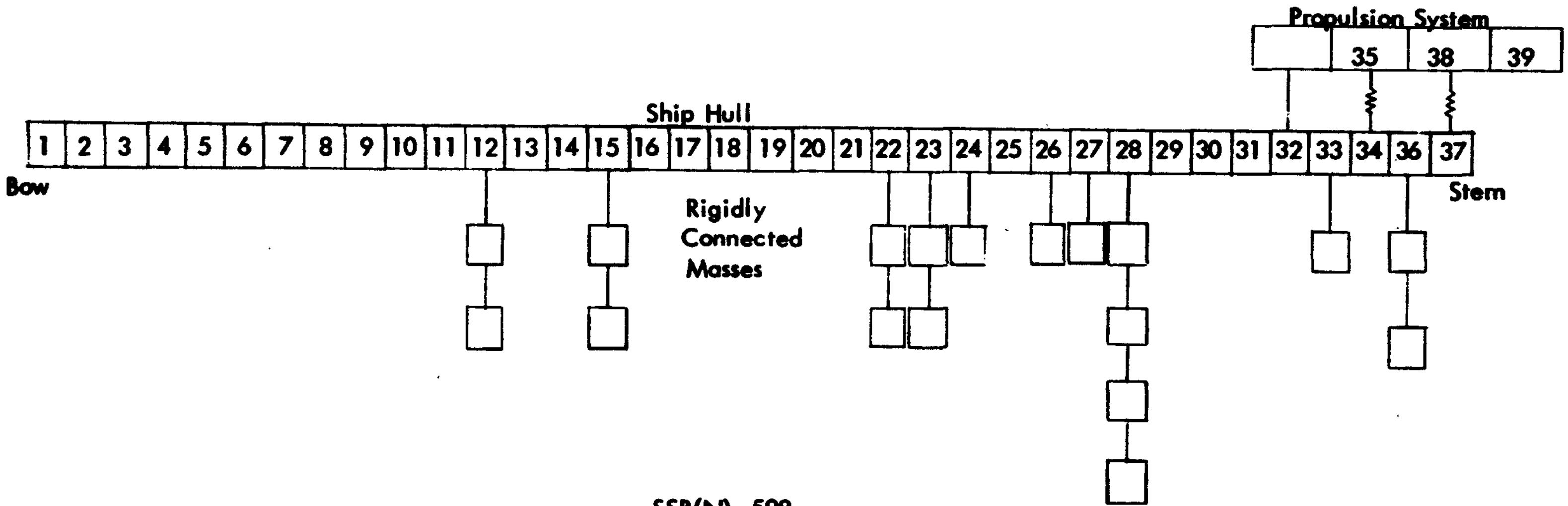
35 Concentrated Masses
Damped Sprung Masses - Propulsion System
Water Inertia - Representation Varies
Damping - Representation Varies
Excitation - Various Locations

See Following Table

Figure 14 - Ship Representation for Calculation

TABLE III - 35 MASS REPRESENTATION - VARIATIONS OF VERTICAL BENDING VIBRATION CALCULATIONS

Case No.	Water Inertia	Hull Damping	Excitation	Other
17	$J = 0.87$ for $0 < n < 3.5$ $= 0.82$ for $3.5 < n$	10% Hysteresis	Vertical force at Propeller (Sta. 56)	
18 Standard	J determined for sinusoidal mode shape	10% Hysteresis	Vertical force at Propeller (Sta. 56)	
19	Standard	No hull damping	Standard	
20	Standard	4% Hysteresis	Standard	
21	Standard	25% Hysteresis	Standard	
22	Standard	Standard	Standard	Stiffened connections between propulsion system and hull
23	Standard	Standard	Standard	Softened connections between propulsion system and hull
24	Standard	Standard	Moment about the horizontal axis at the propeller	
25	Standard	Standard	Vertical force at stern (Sta. 54)	
26	Standard	Standard	Vertical force forward of the stern (Sta. 51)	
30	Standard	Standard	Standard	Include rotary inertia of the hull cross-section



SSB(N) 598

Vertical Bending Vibration Calculations

Case 27	35 Concentrated Masses
Ship	Sprung Masses Assumed Rigidly Connected
Representation	Water Inertia J Based on Sinusoidal Mode Shape

Excitation at Propeller (Sta. 39)
10% Hysteresis Damping

Figure 15 - Ship Representation for Calculation

TABLE IV - LOCATION OF CALCULATION STATIONS IN FEET FROM FORWARD PERPENDICULAR

(s) indicates sprung mass

Case No. / Sta. No.	1, 2	3	4	5, 6, 7, 8, 9, 10, 11	12	13	14, 15	16	17→26 30	27
1	Prop.	7.5	1.5	1.5	367.5	7.5	7.5	1.5	1.5	1.5
2	Prop. shaft	22.5	12	6	352.5	22.5	22.5	12	6	6
3	Prop. shaft	37.5	45	12	337.5	37.5	37.5	45	12	12
4	Thrust brg.	52.5	69	21	322.5	52.5	52.5	69	21	21
5	Pwr. plant	67.5	98	30	307.5	67.5	67.5	98	30	30
6	367.5	82.5	127	45	292.5	82.5	82.5	98	45	45
7	352.5	97.5	127(s)	55	277.5	97.5	97.5	98(s)	55	55
8	337.5	112.5	127(s)	62	262.5	97.5(s)	97.5(s)	127	62	62
9	322.5	127.5	155	69	247.5	97.5(s)	97.5(s)	127(s)	69	69
10	307.5	127.5(s)	185	80	232.5	112.5	112.5	127(s)	80	80
11	292.5	127.5(s)	210	90	217.5	127.5	127.5	155	90	90
12	277.5	142.5	210(s)	98	202.5	127.5(s)	127.5(s)	185	98	98
13	262.5	157.5	210(s)	107	187.5	127.5(s)	127.5(s)	210	98(s)	107
14	247.5	172.5	210(s)	117	172.5	142.5	142.5	210(s)	98(s)	117
15	232.5	187.5	210(s)	127	157.5	157.5	157.5	210(s)	107	127
16	217.5	202.5	250	127(s)	142.5	172.5	172.5	210(s)	117	136
17	202.5	217.5	250(s)	127(s)	127.5	187.5	187.5	210(s)	127	145

TABLE IV - LOCATION OF CALCULATION STATIONS IN FEET FROM (continued) FORWARD PERPENDICULAR

(s) indicates sprung mass

Case No. / Sta. No.	1, 2	3	4	5, 6, 7, 8, 9, 10, 11	12	13	14, 15	16	17→26 30	27
18	187.5	217.5(s)	259	136	112.5	202.5	202.5	250	127(s)	155
19	172.5	217.5(s)	280	145	97.5	217.5	217.5	259	127(s)	169
20	157.5	217.5(s)	280(s)	155	82.5	217.5(s)	217.5(s)	280	136	185
21	142.5	232.5	280(s)	169	67.5	217.5(s)	217.5(s)	280(s)	145	197
22	127.5	232.5(s)	280(s)	185	52.5	217.5(s)	217.5(s)	280(s)	155	210
23	112.5	247.5	280(s)	197	37.5	232.5	232.5	280(s)	169	228
24	97.5	247.5(s)	280(s)	210	22.5	232.5(s)	232.5(s)	280(s)	185	250
25	82.5	262.5	326	210(s)	7.5	247.5	247.5	280(s)	197	259
26	67.5	262.5(s)	348	210(s)		262.5	262.5	280(s)	210	264
27	52.5	262.5(s)	348(s)	228		262.5(s)	262.5(s)	326	210(s)	280
28	37.5	277.5	360	228(s)		262.5(s)	262.5(s)	348	210(s)	295
29	22.5	292.5	372	228(s)		277.5	277.5	348(s)	228	315
30	7.5	292.5(s)	Prop.	250		292.5	292.5	360	228(s)	326
31		292.5(s)	Thrust brg.	250(s)		292.5(s)	292.5(s)	360(s)	228(s)	335
32		292.5(s)	L.S.Gr.	259		292.5(s)	292.5(s)	360(s)	250	343
33		307.5	L.S.pinion	264		292.5(s)	292.5(s)	372	259	348
34		322.5	H.S.Gr.	264(s)		292.5(s)	292.5(s)	Shaft Seal	264	355.42

TABLE IV - LOCATION OF CALCULATION STATIONS IN FEET FROM (continued) FORWARD PERPENDICULAR

(s) indicates sprung mass

Case No. / Sta. No.	1, 2	3	4	5, 6, 7, 8, 9, 10, 11	12	13	14, 15	16	17-26 30	27
35		337.5	H. S. pinion	280		307.5	307.5	Stern Brg.	264(s)	Shaft Seal
36		352.5	Turb.	280(s)		322.5	322.5	Prop.	280	360
37		352.5(s)		295		337.5	337.5		280(s)	372
38		367.5		295(s)		352.5	352.5		295	Stern Brg.
39		Prop.		295(s)		352.5(s)	352.5(s)		295(s)	Prop.
40		Thrust Brg.		295(s)		352.5(s)	352.5(s)		295(s)	
41		Low speed Gr. Hub.		315		367.5	367.5		295(s)	
42		Low speed Pinions		326		367.5(s)	367.5(s)		295(s)	
43		H. S. gear		335			Shaft Seal		315	
44		H. S. Pinions		343			Stern Brg.		326	
45		Turb.		348			Prop.		335	
46				348(s)					343	
47				355.42					348	

TABLE IV - LOCATION OF CALCULATION STATIONS IN FEET FROM FORWARD PERPENDICULAR (continued)

(s) indicates sprung mass

Case No. / Sta. No.	1, 2	3	4	5, 6, 7, 8, 9, 10, 11	12	13	14, 15	16	17-26 30	27
48				360					348(s)	
49				372					355. 42	
50				Prop.					Shaft seal	
51				Thrust Brg.					360	
52				L. S. Gr. Hub.					360(s)	
53				L. S. Pinions					360(s)	
54				H. S. Gear					372	
55				H. S. Pinions					Stern Brg.	
56				Turb.					Prop.	

its connection to the hull is brought into consideration is shown by a comparison between Cases 13 and 14 where the excitation remains on the hull and with Case 15 where the excitation is transferred to the propeller. The influence of the number of masses used to define the ship is shown by a comparison of Cases 15, 16 and 17. A comparison between runs 17 and 18 indicates the effects upon the hull response of different assumptions for the reduction of the sectional water inertia to allow for the modal pattern of the hull vibration (See Appendix C). The effects of hull damping are determined through Cases 18, 19, 20 and 21. The effects of changes in the stiffness of the connections between the propulsion system and the hull are shown by Cases 18, 22 and 23. Case 24 is included to give a base for estimating the hull response to a moment at the propeller and Cases 25 and 26 to forces acting on the hull. Case 27 is included for comparison with Case 18 to show the influences of the sprung masses (whose gross weight is very small) upon the hull response and Case 30 by comparison with Case 18 shows the importance of including the rotary inertia of the hull cross sections in making calculations.

VII RESULTS OF CALCULATIONS

The results of the calculations, which are in the form of deflections at each station for a range of frequencies from 1 to 25 cps, are best shown by curves of hull motion per pound of excitation force at selected positions and phase angle relative to the excitation. These curves are presented in Figures 16 through 60. From a study of these curves it is possible to determine the answers to some of the questions raised in the previous section of this report.

1. Longitudinal Vibration:

In comparing the results of the several longitudinal vibration calculations there are several lessons to be noted but there does not appear to be any trend that could not be expected. Case 7(Figures 21 and 22) is the standard to which other cases are compared.

In comparing Case 1 with Case 7, the first noticeable difference is the larger

Text continues on Page 46

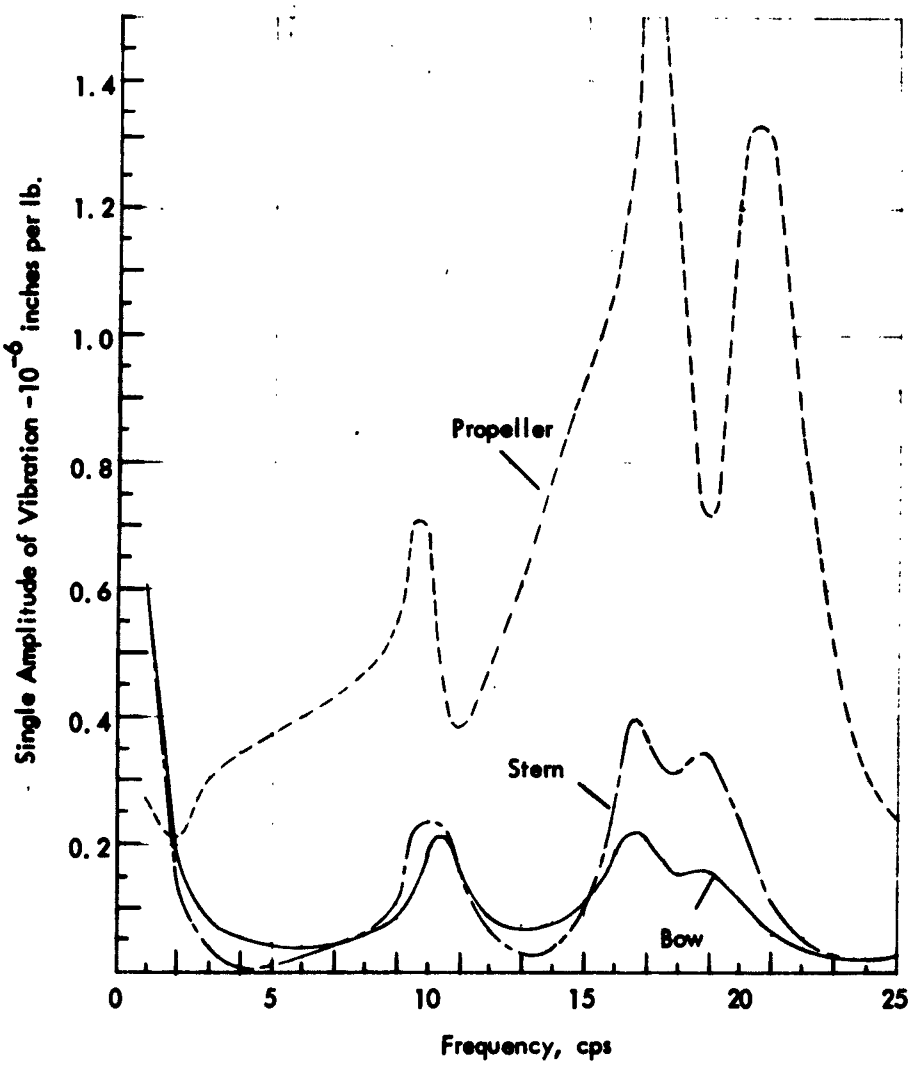


Figure 16 – Response Curves in Longitudinal Vibration

Case 1a. Excitation at Propeller. 10% Hysteresis Damping
Mass and Stiffness taken from DTMB Report 1464

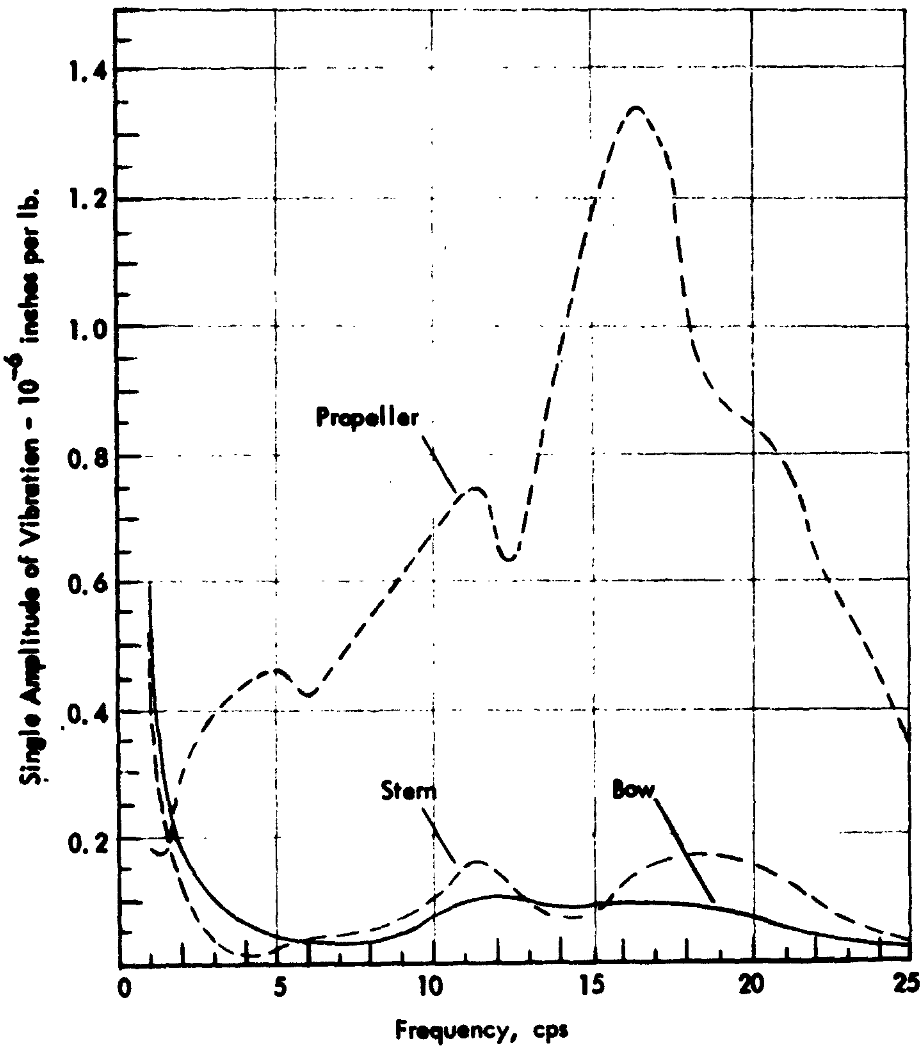


Figure 17 - Response Curves in Longitudinal Vibration

Case 3. Excitation at Propeller. 10% Hysteresis Damping
24 Uniformly Spaced Masses, Sprung Masses, Propulsion System

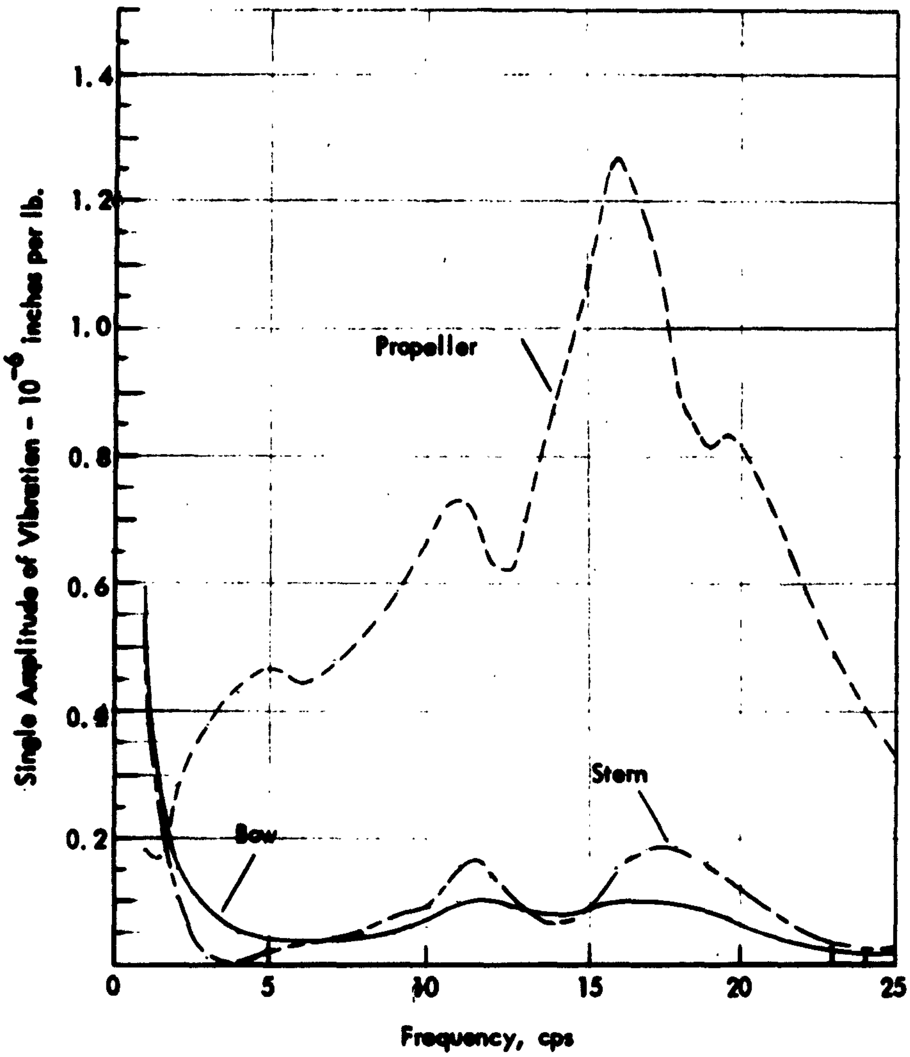


Figure 18 - Response Curves in Longitudinal Vibration

Case 4. Excitation at Propeller. 10% Hysteresis Damping
16 Concentrated Masses, Sprung Masses, Propulsion System

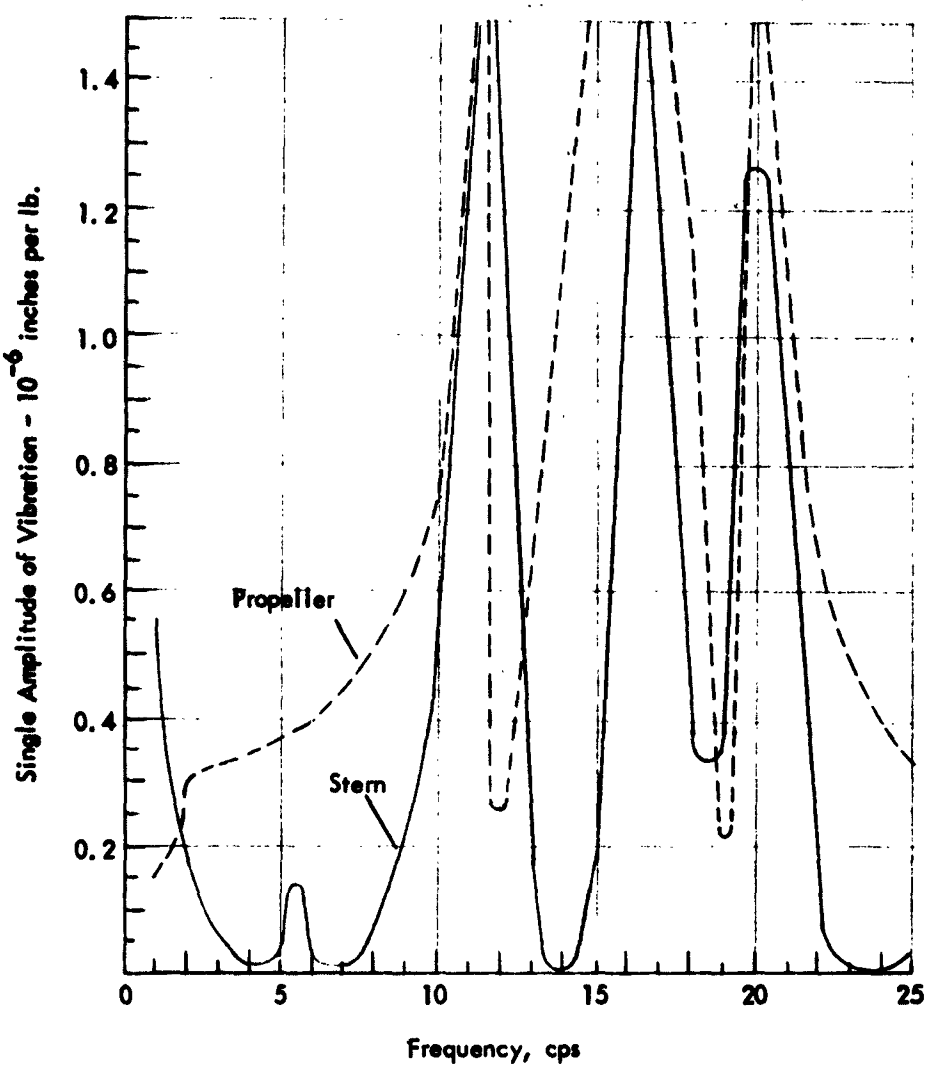


Figure 19 - Response Curves in Longitudinal Vibration

Case 6. Excitation at Propeller. No Hysteresis Damping
35 Concentrated Masses, Sprung Masses, Propulsion System

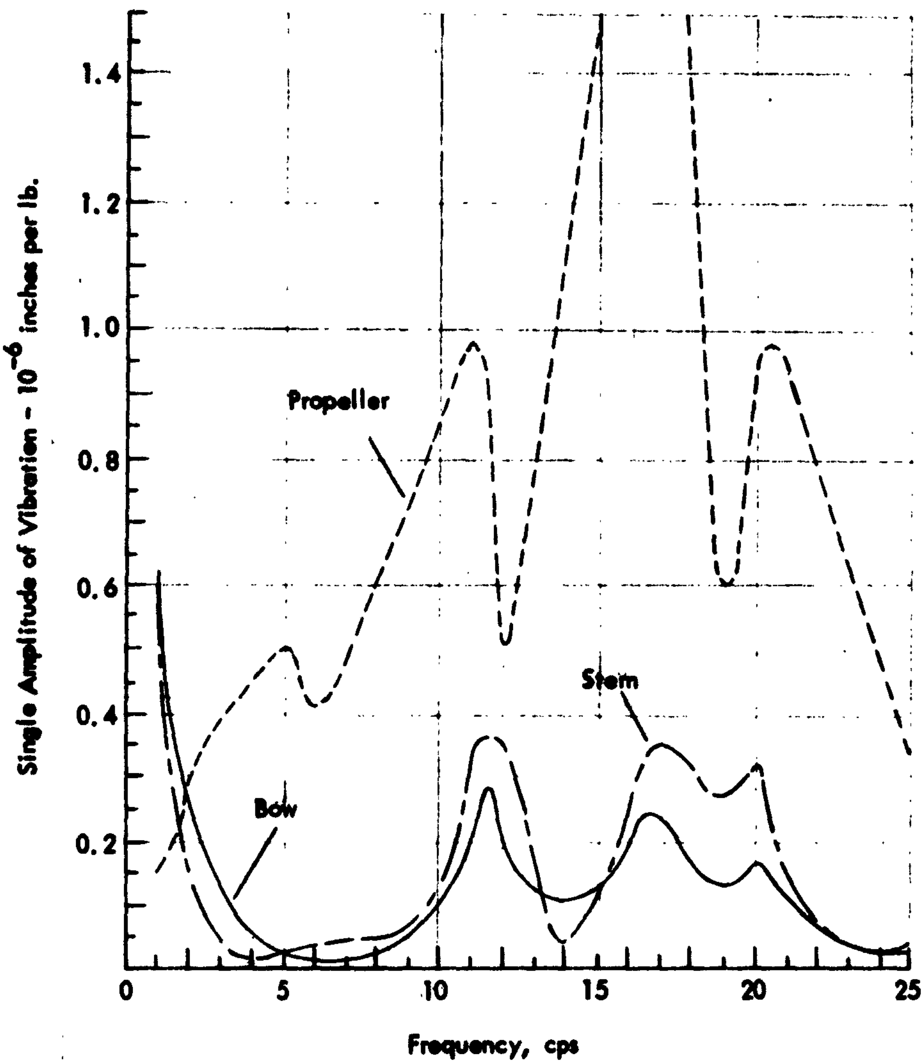


Figure 20 – Response Curves in Longitudinal Vibration

Case 6. Excitation at Propeller. 4% Hysteresis Damping
35 Concentrated Masses, Sprung Masses, Propulsion System

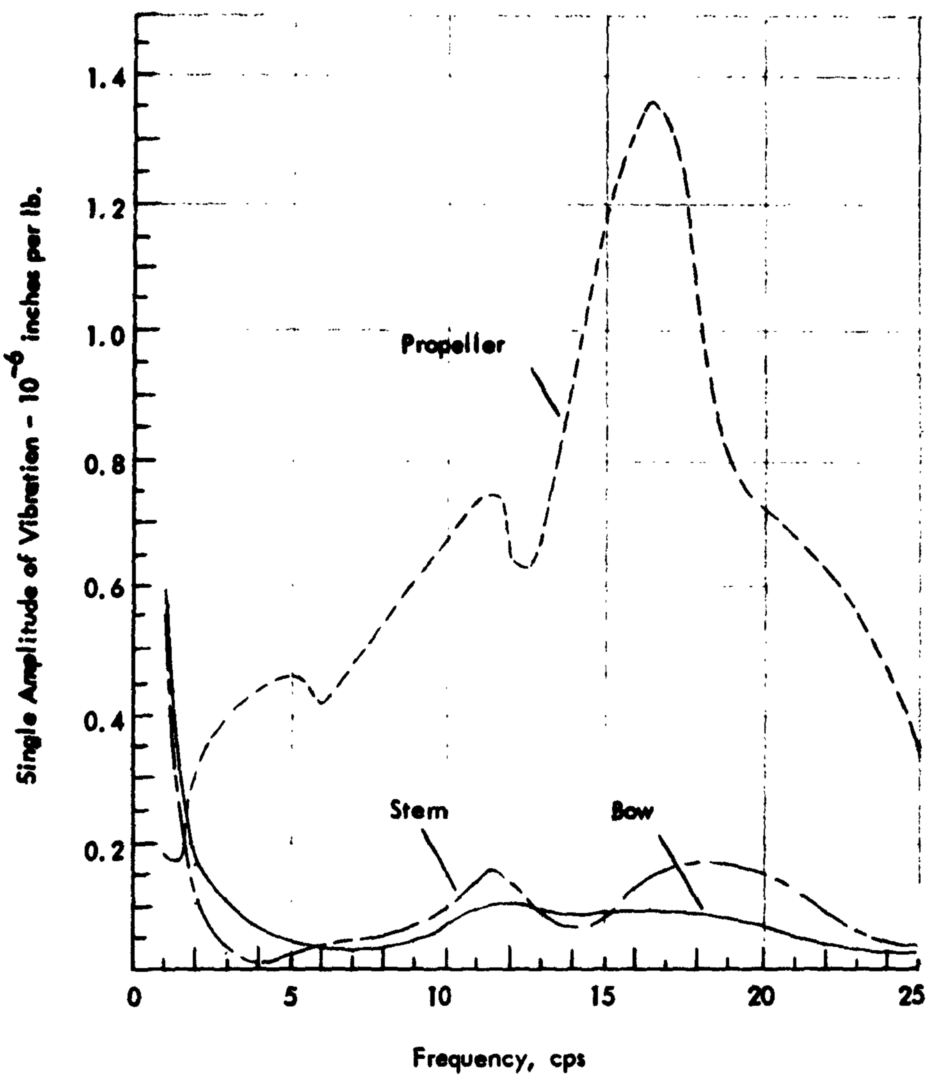


Figure 21 - Response Curves in Longitudinal Vibration

Case 7. Excitation at Propeller. 10% Hysteresis Damping
35 Concentrated Masses, Sprung Masses, Propulsion System

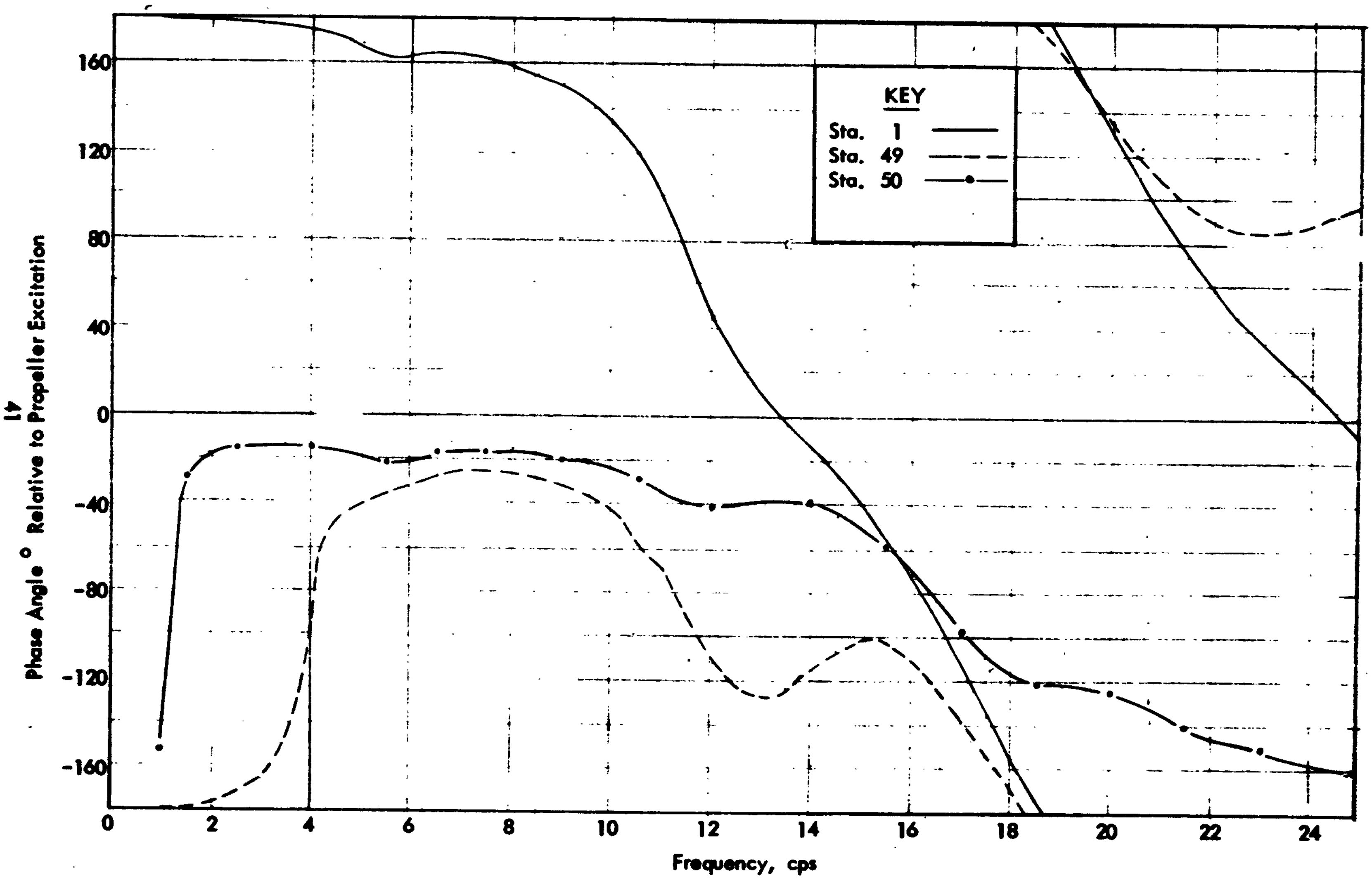


Figure 22 - Longitudinal Vibration - Phase Angle Relative to Propeller Excitation, Case 7

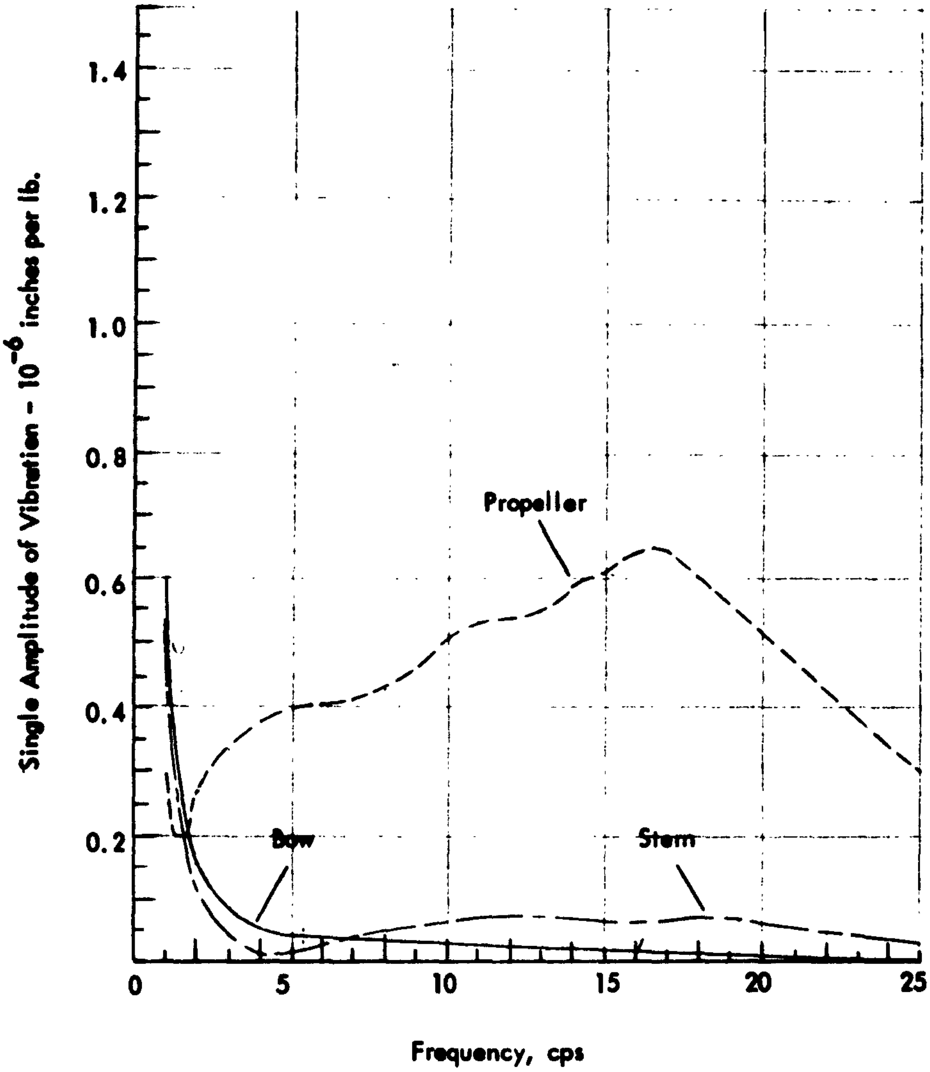


Figure 23 - Response Curves in Longitudinal Vibration

Case 8. Excitation at Propeller. 25% Hyesteresis Damping
35 Concentrated Masses, Sprung Masses, Propulsion System

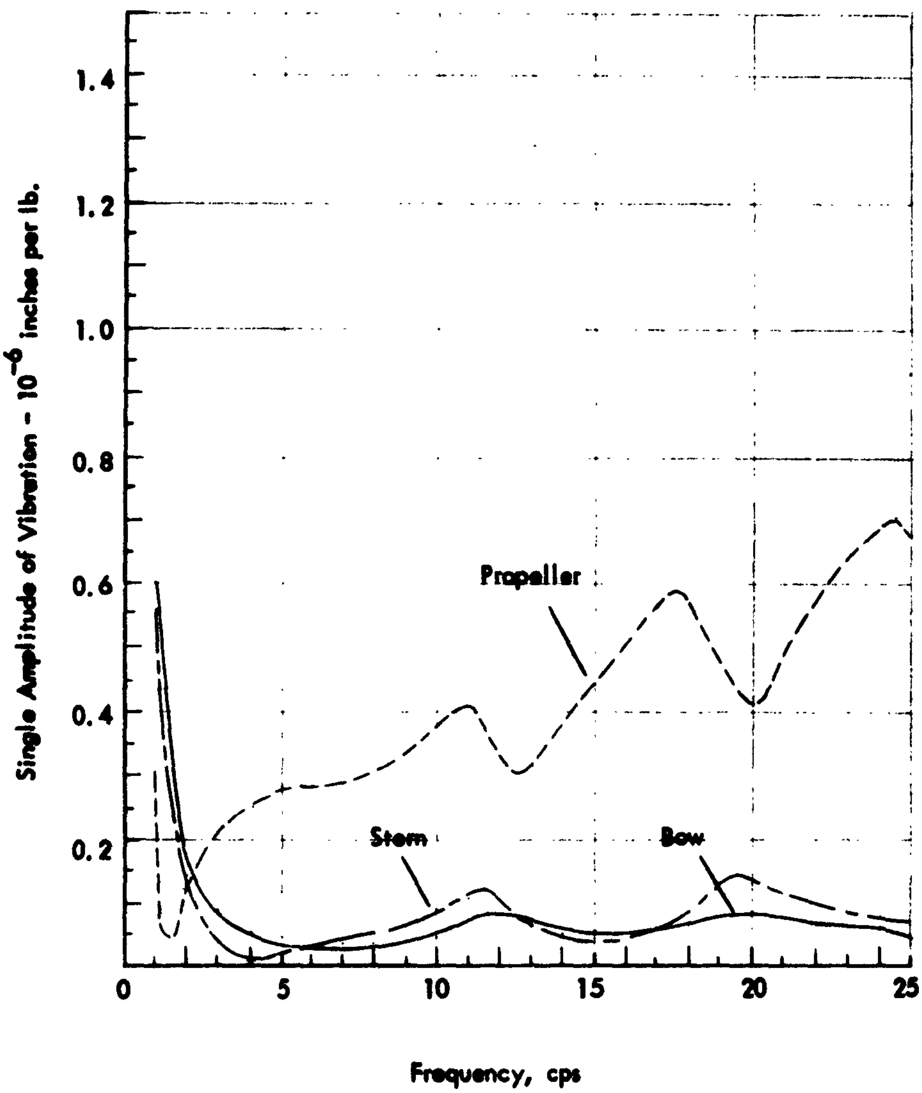


Figure 24 - Response Curves in Longitudinal Vibration

Case 9. Excitation at Propeller. 10% Hysteresis Damping
35 Concentrated Masses, Sprung Masses, Propulsion System, Stiffness of Connection between Hull and Thrust Bearing twice that of Case 7.

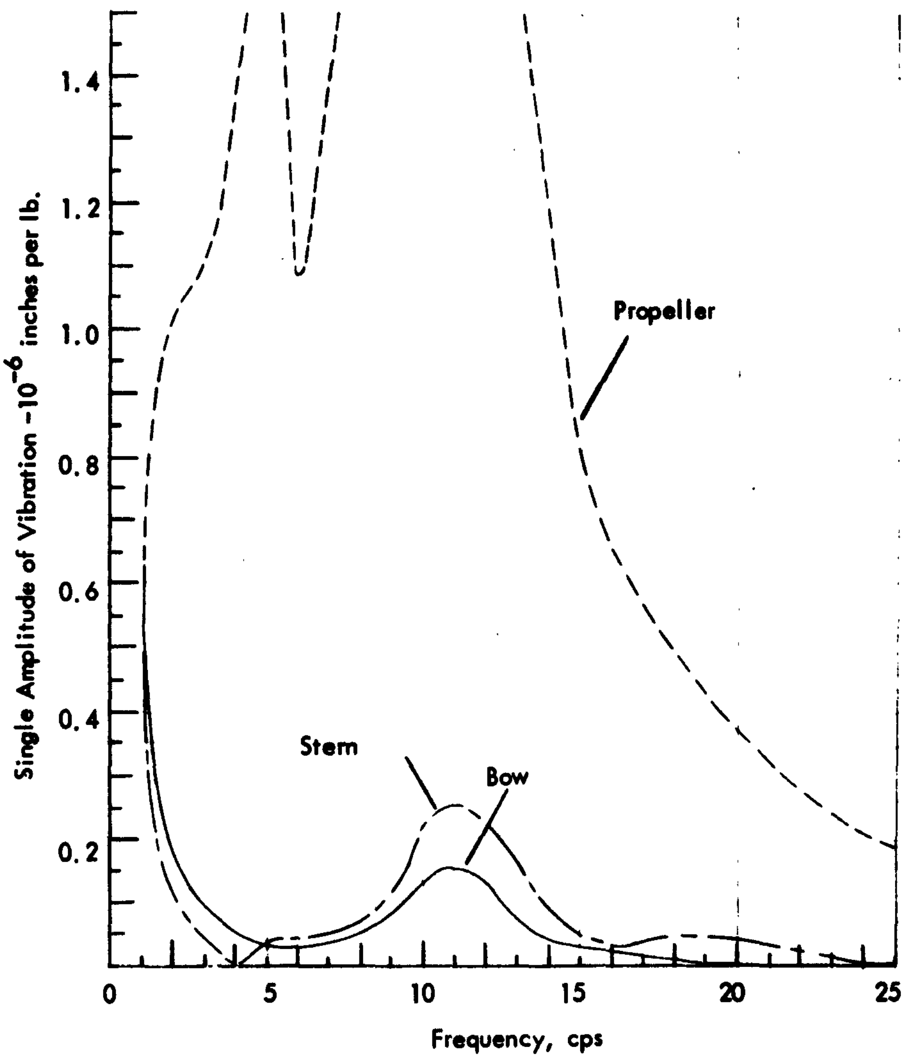


Figure 25 - Response Curves in Longitudinal Vibration

Case 10. Excitation at Propeller. 10% Hysteresis Damping

35 Concentrated Masses, Sprung Masses, Propulsion System, Stiffness of Connection between Hull and Thrust Bearing, 30% of that in Case 7.

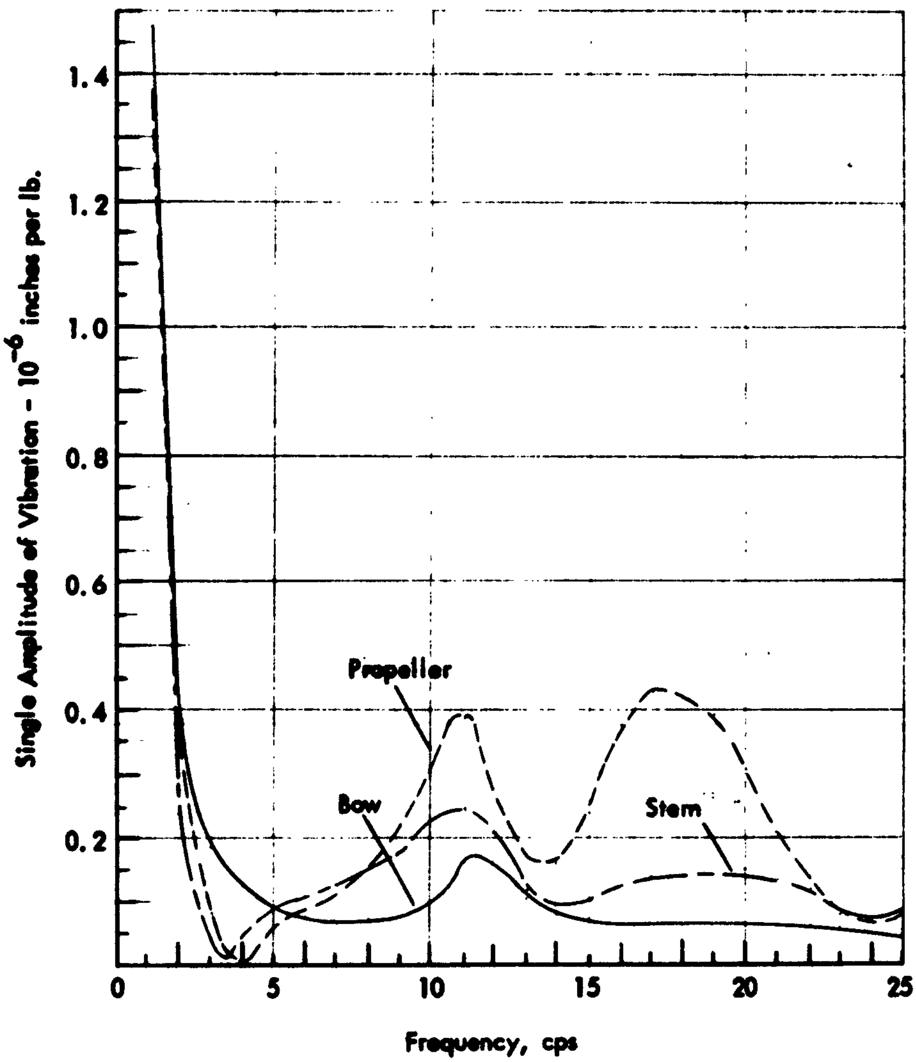


Figure 26 - Response Curves in Longitudinal Vibration

Case 11. Excitation at Stern. 10% Hysteresis Damping
35 Concentrated Masses, Sprung Masses, Propulsion System

amplitude of hull motion in Case 1. The resonant frequencies occur at about the same frequencies (except that the propeller in Case 7 shows a weak resonance at about 5 cycles that is not present in Case 1.) This corresponds to a mode in which the high speed parts of the system (turbines and high speed gears) vibrate against the propeller and the hull through the soft longitudinal stiffness of the bull gear web. The differences in hull amplitude can be explained by the addition of propeller damping and the damping of the sprung masses to the general hysteresis damping of the hull in Case 7.

A comparison of the effect of the number of stations used in defining the hull can be obtained by considering Cases 3 (24 masses), 4 (16 masses), and 7 (35 masses). The hull motion is the same in Cases 3 and 7 but differs some in Case 4. The propeller motion (which is really of secondary interest) is identical in Cases 3 and 7 except for a slight hump at about 20.5 cps in Case 3. There are slightly more differences between Case 4 and Cases 3 and 7 in the form of slightly sharpened resonances, but these are not large. It is probably better to use more than 16 stations in defining the hull.

When Cases 5, 6, 7, and 8 are compared, it becomes clear that the damping of the propeller and the sprung masses is not sufficient to keep the amplitude of hull vibration to reasonable limits. However, the addition of 4% of hysteresis damping is very effective. 10% and 25% damping make the submarine dead.

Cases 10, 7 and 9 illustrate the effects of the thrust bearing stiffness upon the hull response. In Cases 7 and 9 the effect of the thrust bearing stiffness in the amplitude of motion of the propeller is large but its influence on hull motion is small. However, Case 10 shows that when resonant frequency of the propeller on its thrust bearing matches a natural frequency of the hull, the amplitude of the hull motions can be increased. When this is recognized, it becomes clear that the amplitude of the hull motion caused by relatively high frequency (>15 cps) propeller excitation is strengthened by propulsion systems that have high axial resonant frequencies (i.e., stiff thrust blocks). It is probable that the very low natural frequency that can be obtained by supporting the thrust shoes on an air spring, could be very effective in attenuating the amplitude of hull vibration in the

axial modes.

Case 11 was run primarily for estimating the amplitude of hull motion under service conditions. However, when compared with Case 7, it illustrates that a harmonic force generated at the propeller will generally but not always give smaller ship motions than one generated on the surface of the hull.

2. Vertical Bending Vibration

The most conspicuous influence upon the bending vibration is the inclusion of the propulsion system as a sub-system in the calculations. This influence is first strongly shown even when the excitation is on the ship hull by comparing Cases 13 and 14 and when the excitation is transferred to the propeller, Case 15, results in a hull response that is entirely different from that obtained when the propulsion sub-system is not considered.

A comparison among Cases 15, 16 and 17 in which the hull is represented by 25, 16 and 35 concentrated masses shows that the predominant propeller frequency is much lower in Case 15 than in Case 16 or 17. This is because of the difficulty of tying the propulsion sub-system into the hull properly when required to work with regular station spacings.

In comparing Case 16 with Case 17, it will be noted that although the resonances occur at about the same frequencies, the response is different even at low frequencies. This result was not expected.

In comparing Cases 17 and 18 in which the treatment of the water inertia effects is changed (See Appendix C for an explanation of this) it is clear that no great change occurs because of the reduced inertia associated with the sinusoidal approximation but that the vibration does carry farther into the hull at the higher frequencies. Thus the amplitude of vibration at the bow at 16 cps is greater for Case 18 than for Case 17.

The effects of hull damping are shown in Cases 19 (no damping) 20 (4% damping) 18 (10% damping) and 21 (25% damping). Referring to Case 19 it is clear that the hull resonances are quite sharp and that the sprung masses and the propeller damping are

Text continues on Page 82

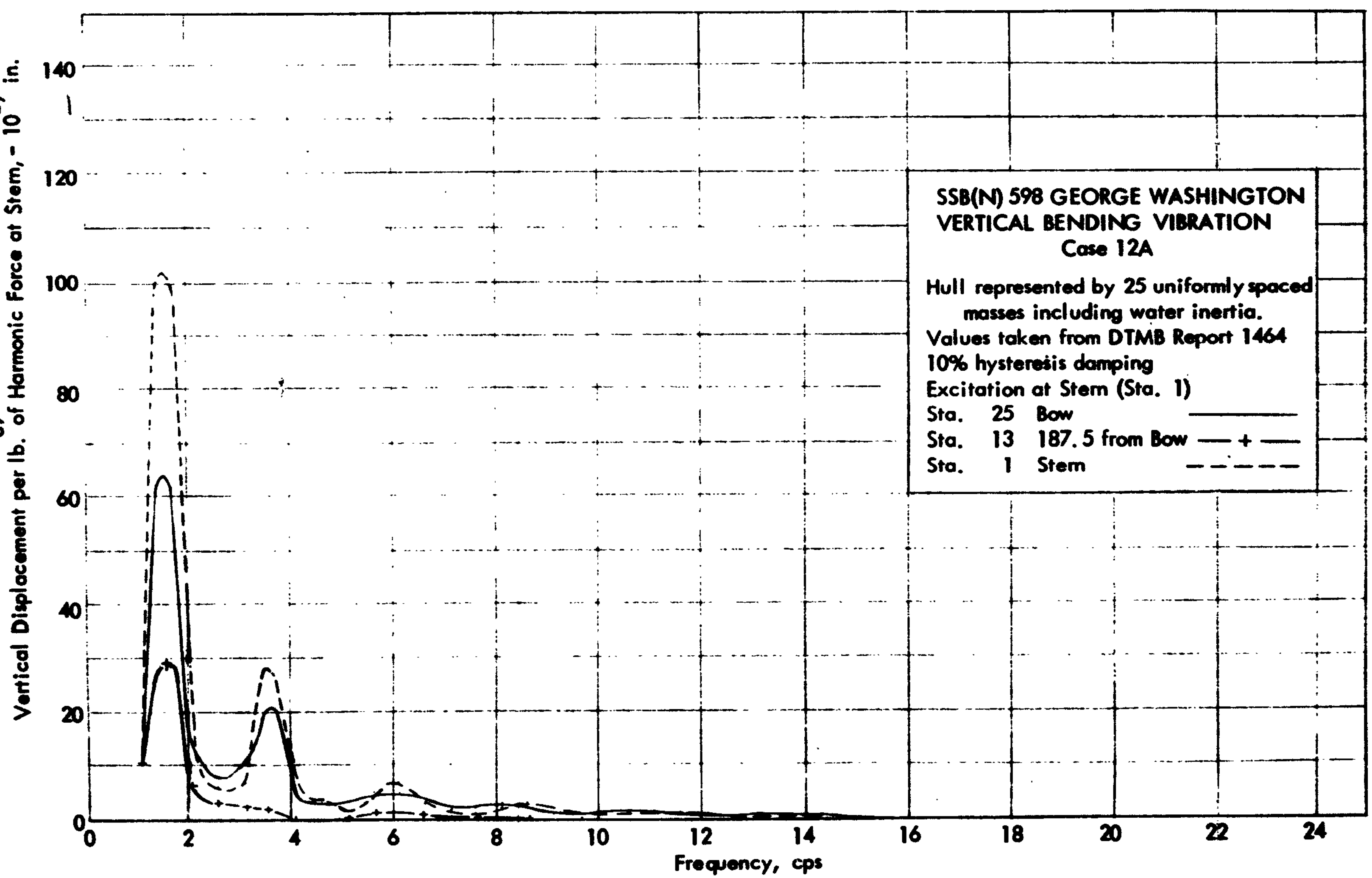


Figure 27 - Displacement in Vertical Bending for 1 lb. Excitation at Stem, Case 12A

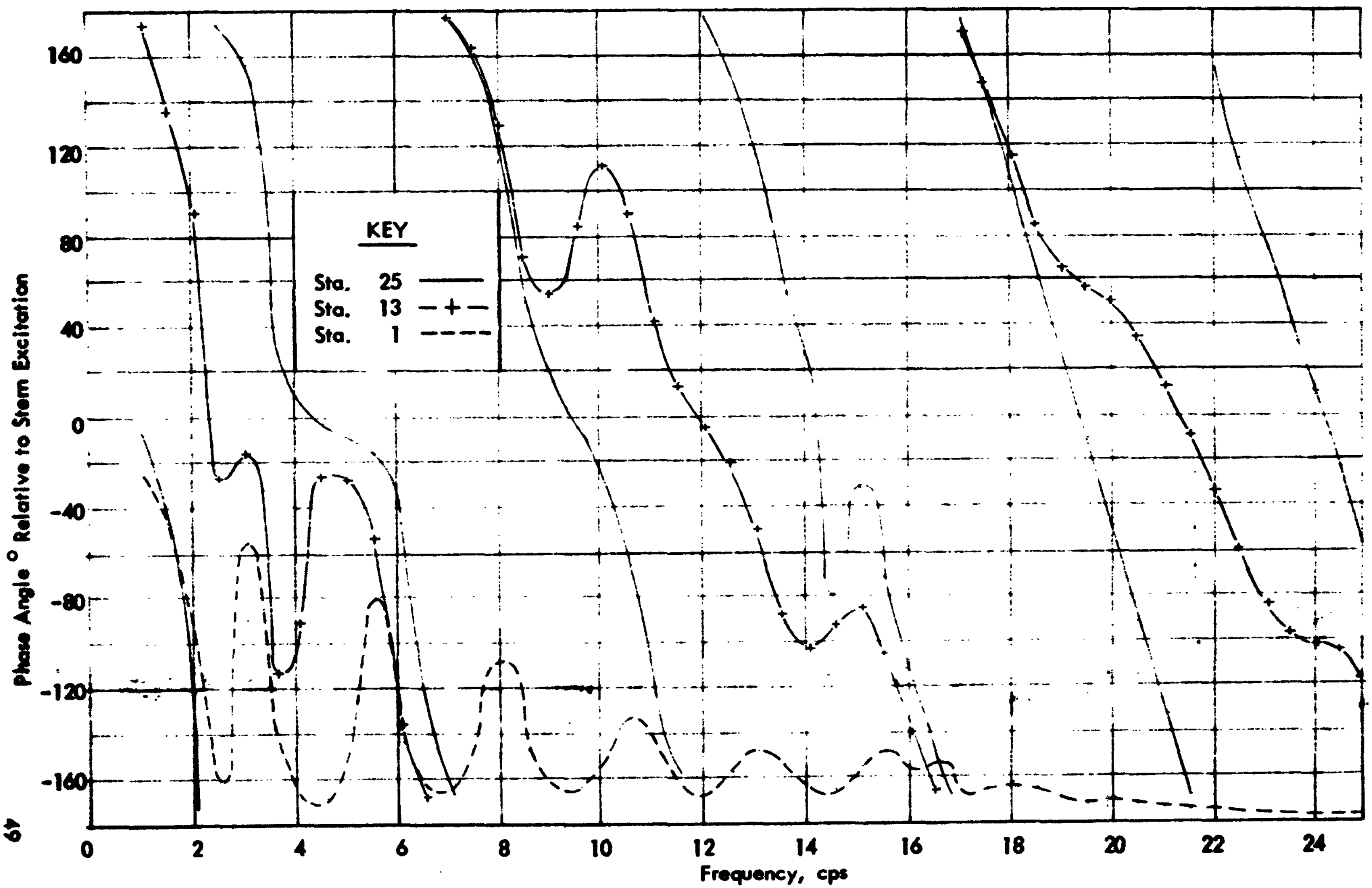


Figure 28-Vertical Bending Phase Angle Relative to Stem Excitation, Case 12A

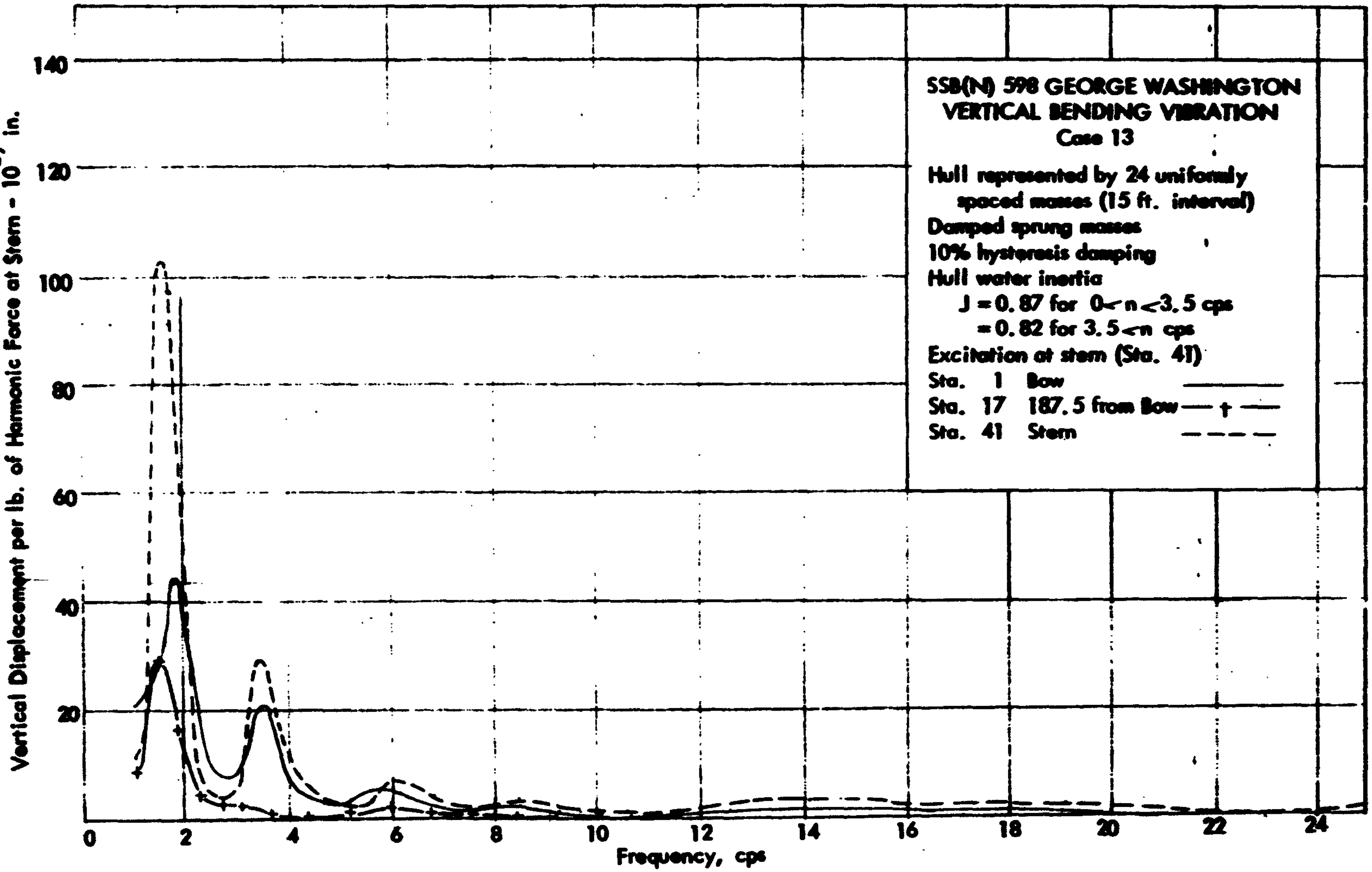


Figure 29-Displacement in Vertical Bending for 1 lb. Excitation at Stern, Case 13

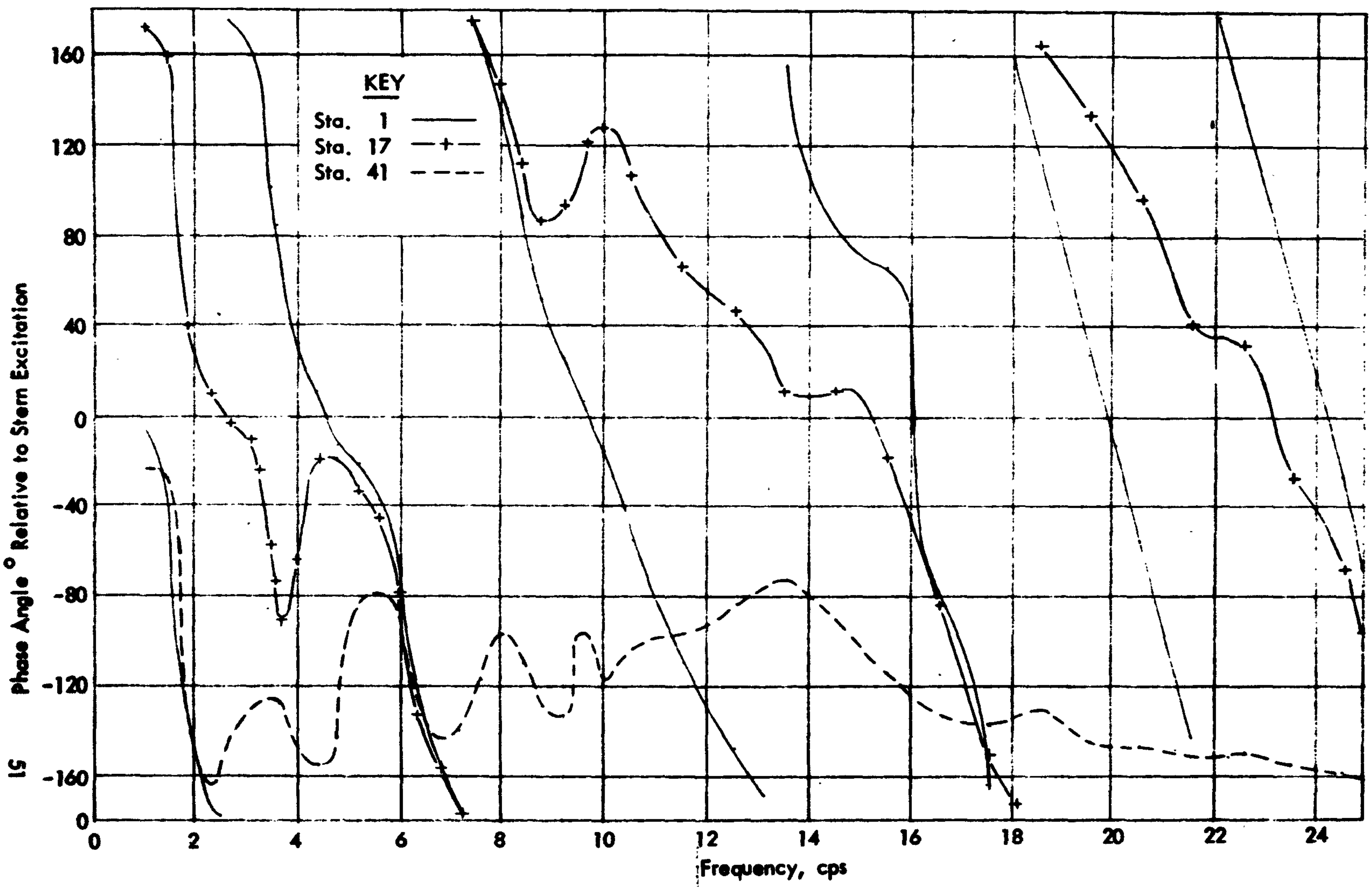


Figure 30-Vertical Bending Phase Angle Relative to Stem Excitation, Case 13

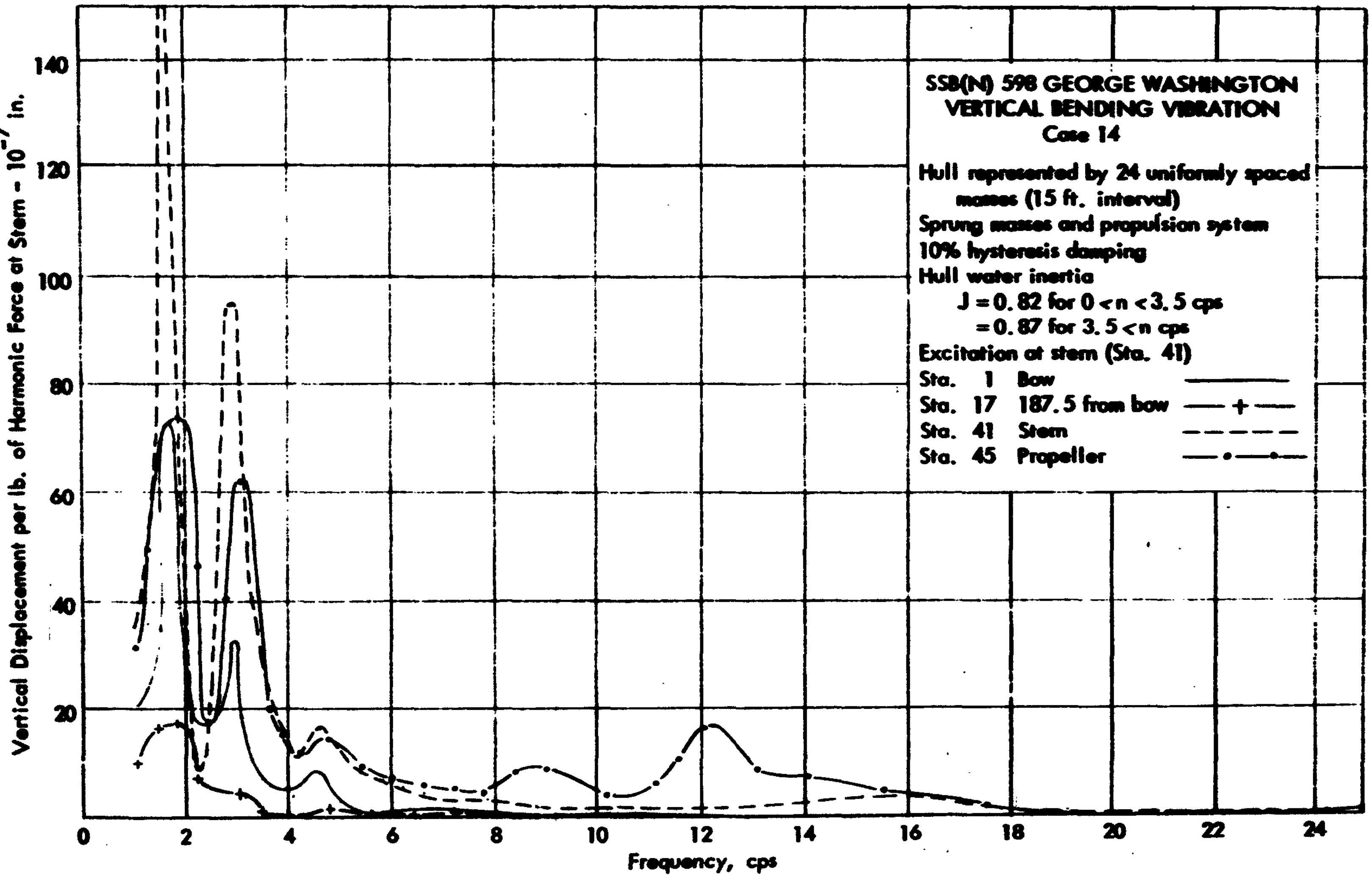


Figure 31 Displacement in Vertical Bending for 1 lb. Excitation at Stem, Case 14

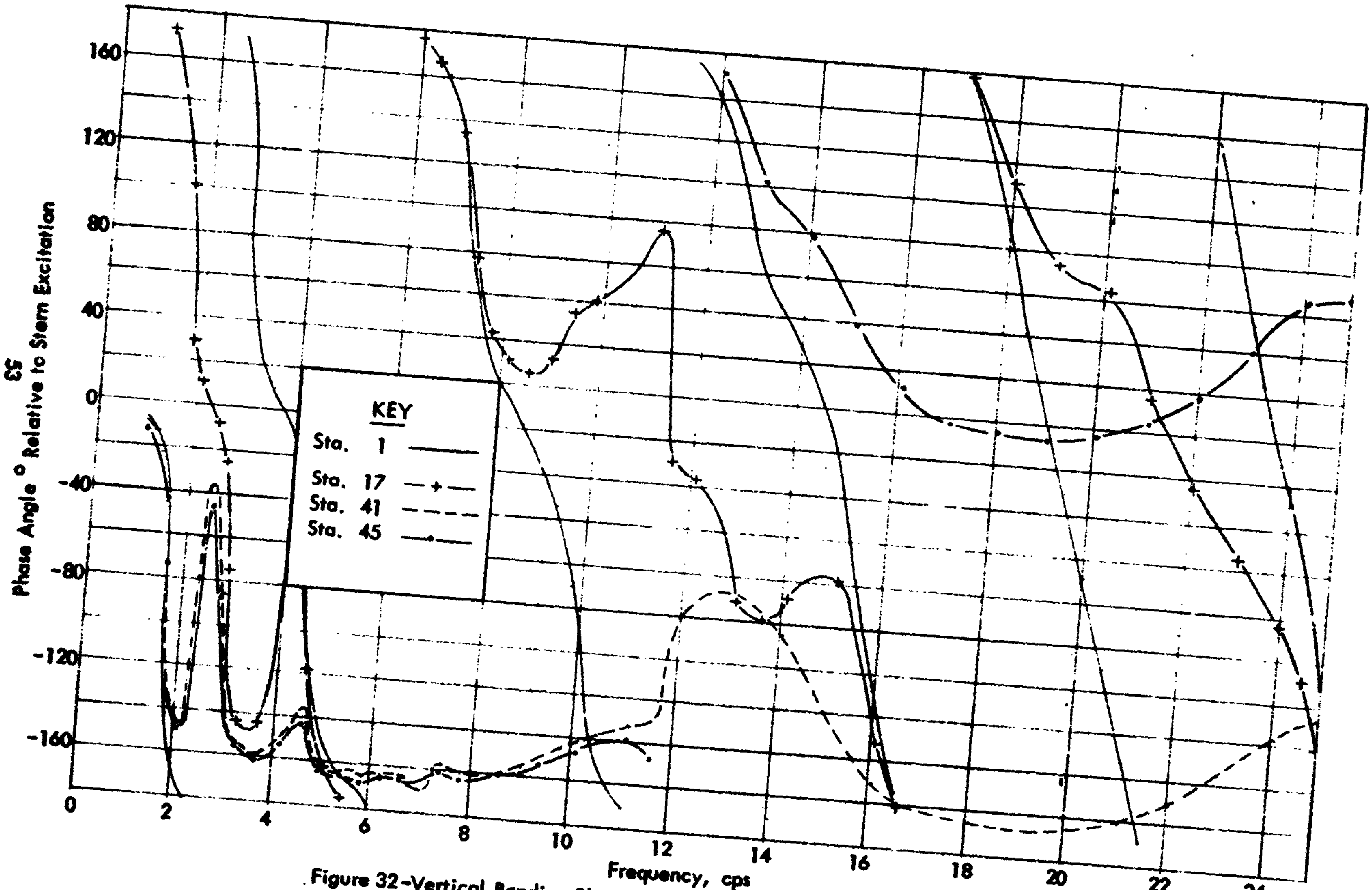


Figure 32-Vertical Bending Phase Angle Relative to Stem Excitation, Case 14

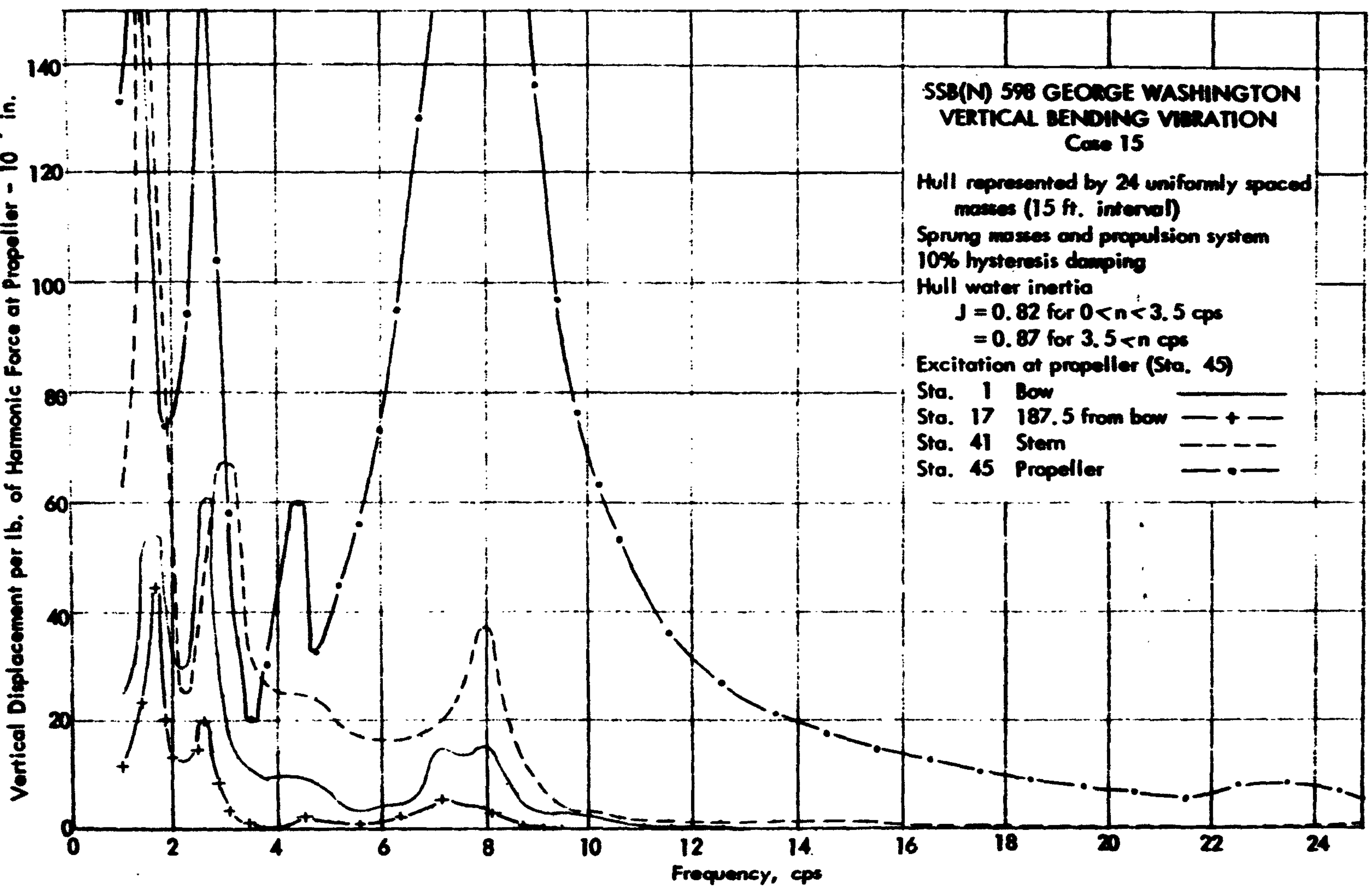


Figure 33 - Displacement in Vertical Bending for 1 lb. Excitation at Propeller, Case 15

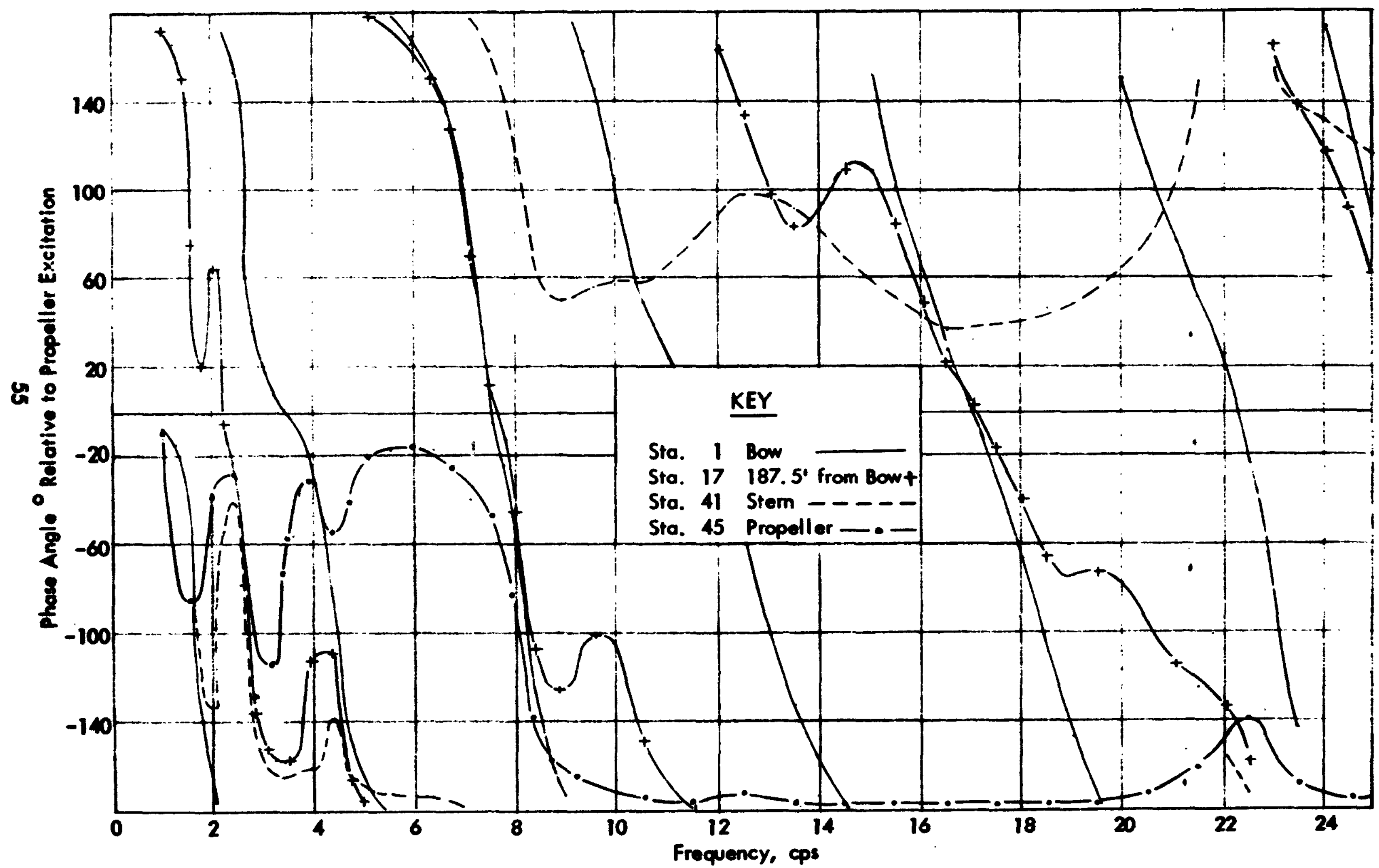


Figure 34 Vertical Bending Phase Angle Relative to Propeller Excitation, Case 15

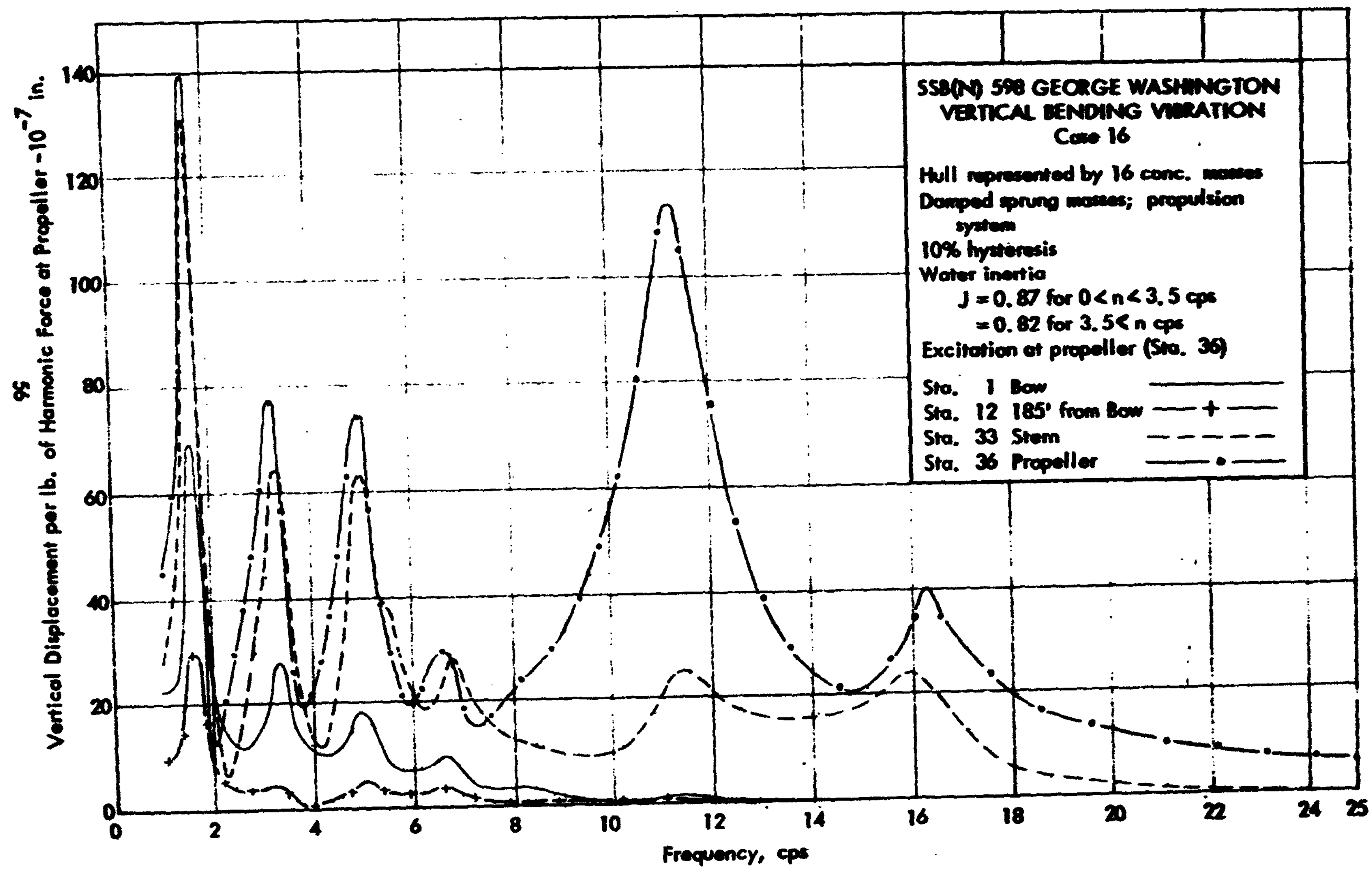


Figure 35–Displacement in Vertical Bending for 1 lb. Excitation at Propeller, Case 16

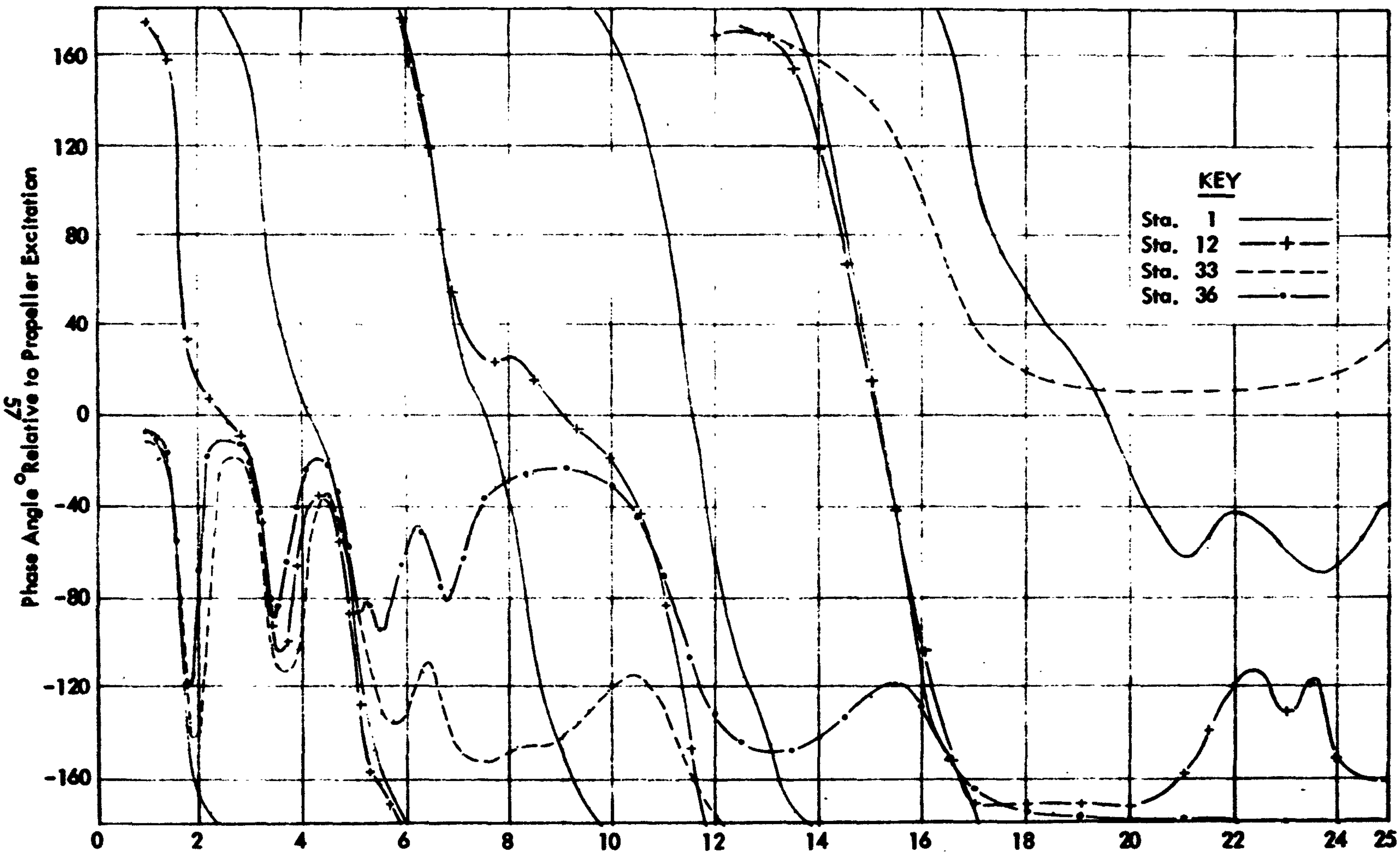


Figure 36–Vertical Bending Phase Angle Relative to Propeller Excitation, Case 16

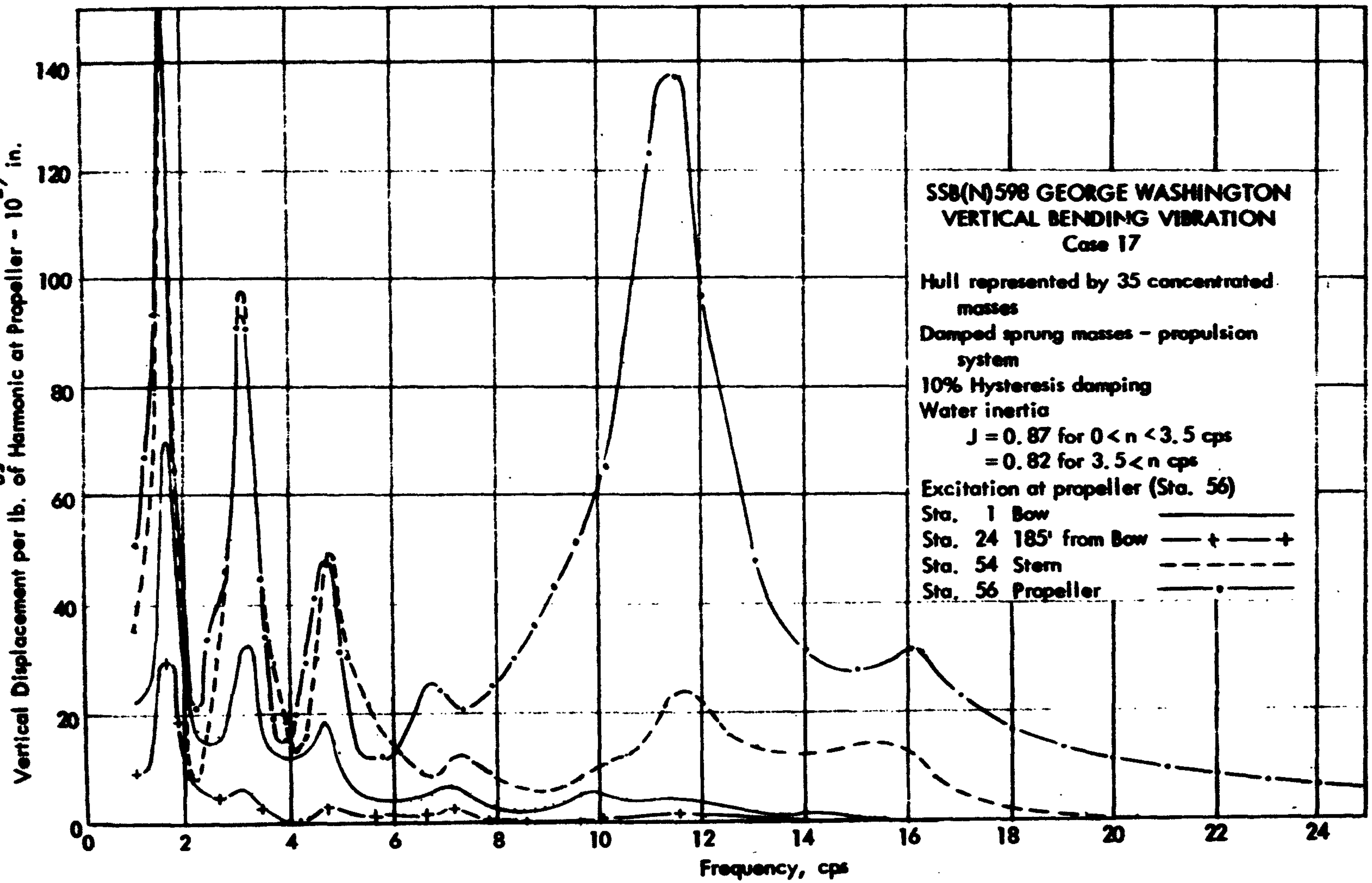


Figure 37–Displacement in Vertical Bending for 1 lb. Excitation at Propeller, Case 17

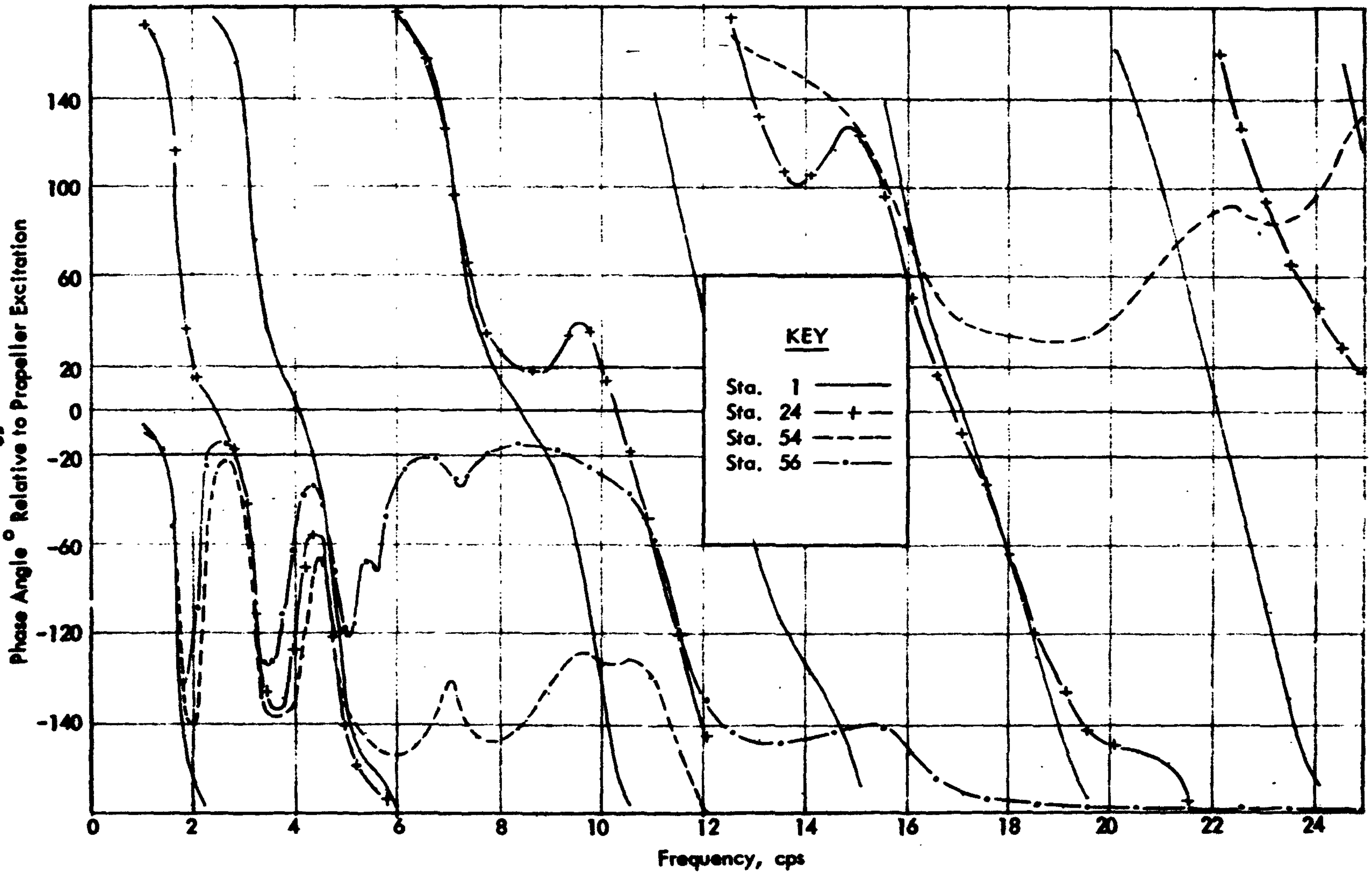


Figure 38 - Vertical Bending Phase Angle Relative to Propeller Excitation, Case 17

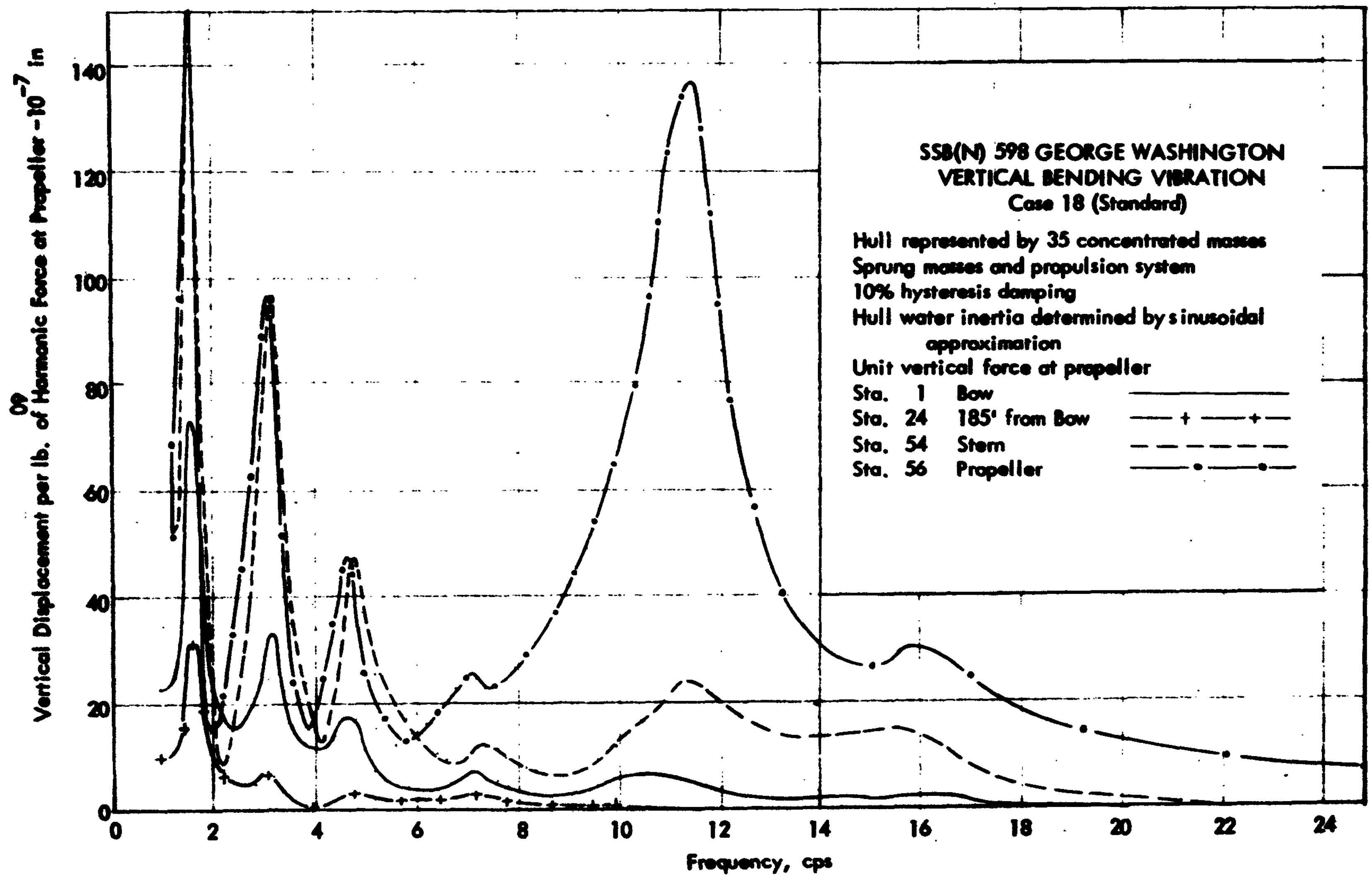


Figure 39 - Displacement in Vertical Bending for 1 lb. Excitation at Propeller, Case 18 (Standard)

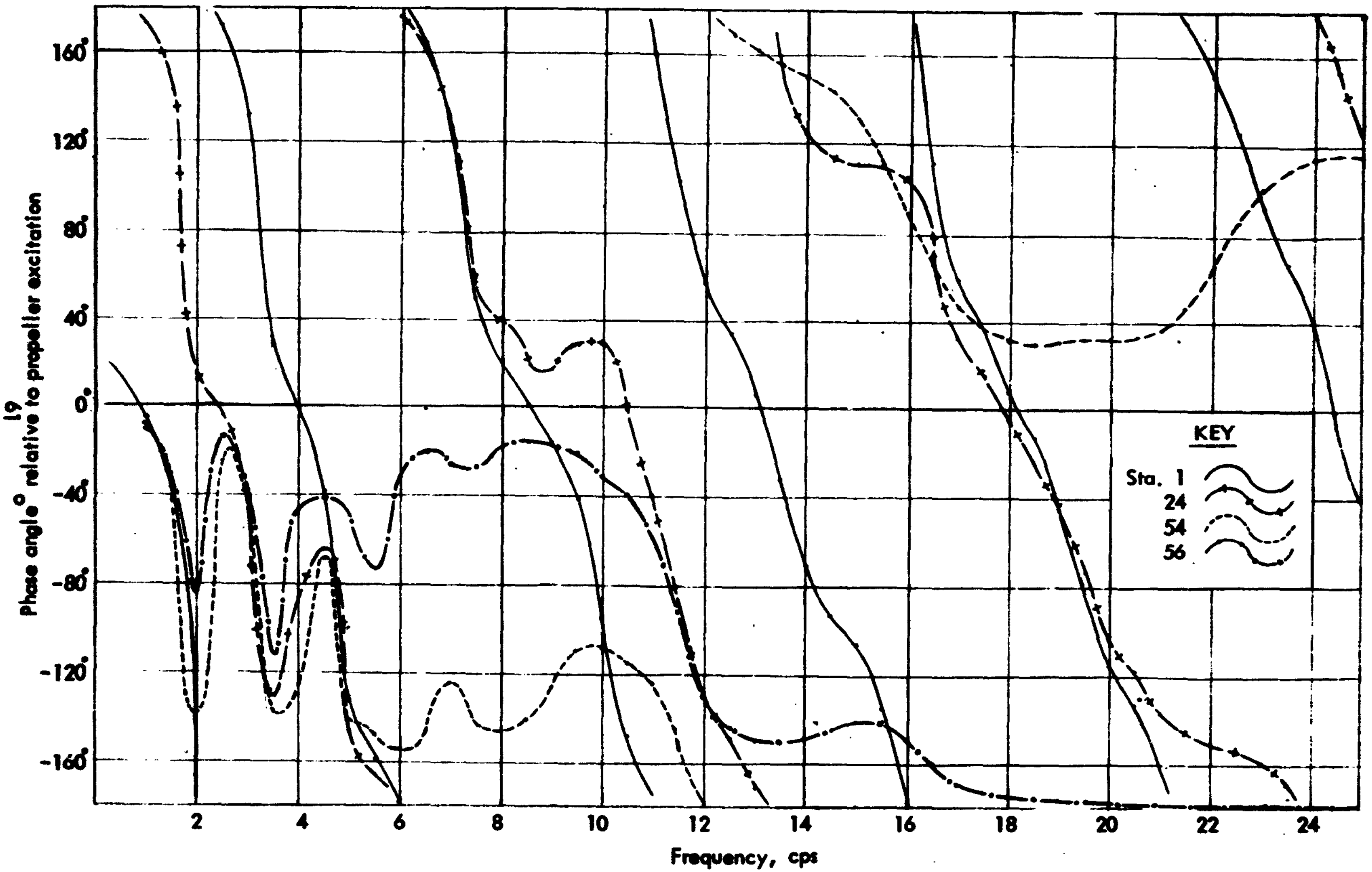


Figure 40 - Vertical Bending Phase Angle Relative to Propeller Excitation, Case 18 (Standard)

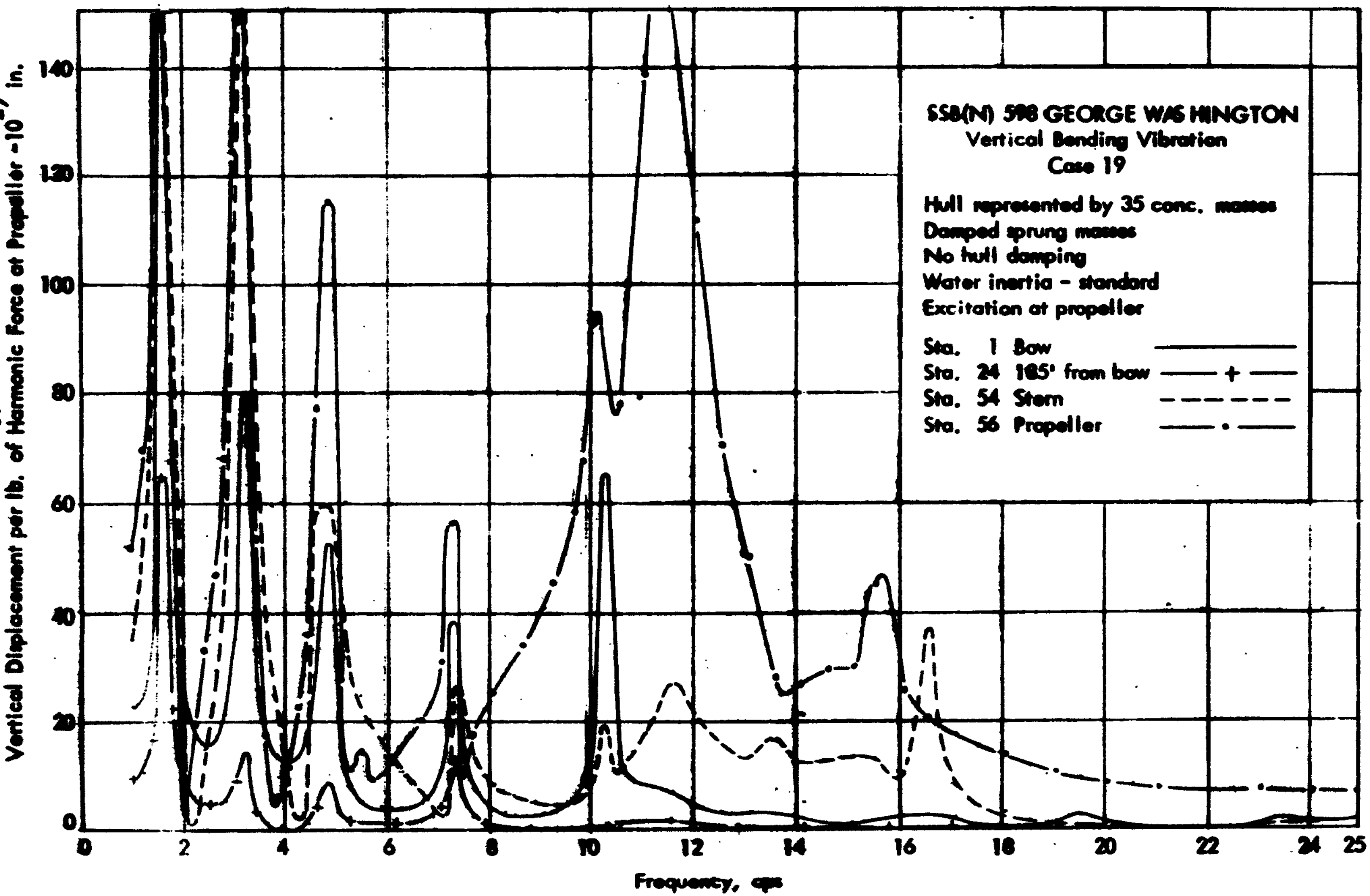


Figure 41 -Displacement in Vertical Bending for 1 lb. Force of Excitation at Propeller, Case 19

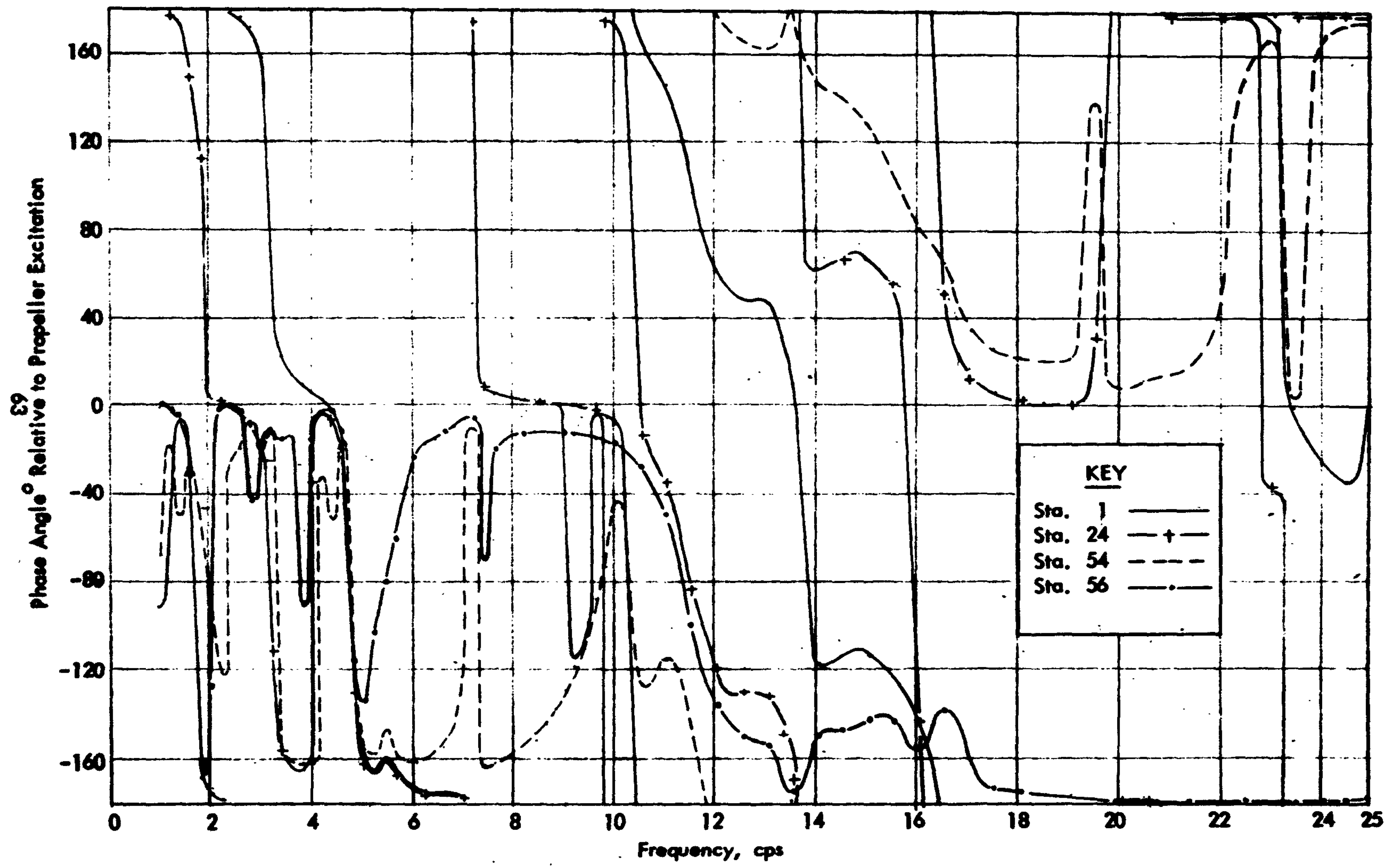


Figure 42 Vertical Bending Phase Angle Relative to Propeller Excitation, Case 19

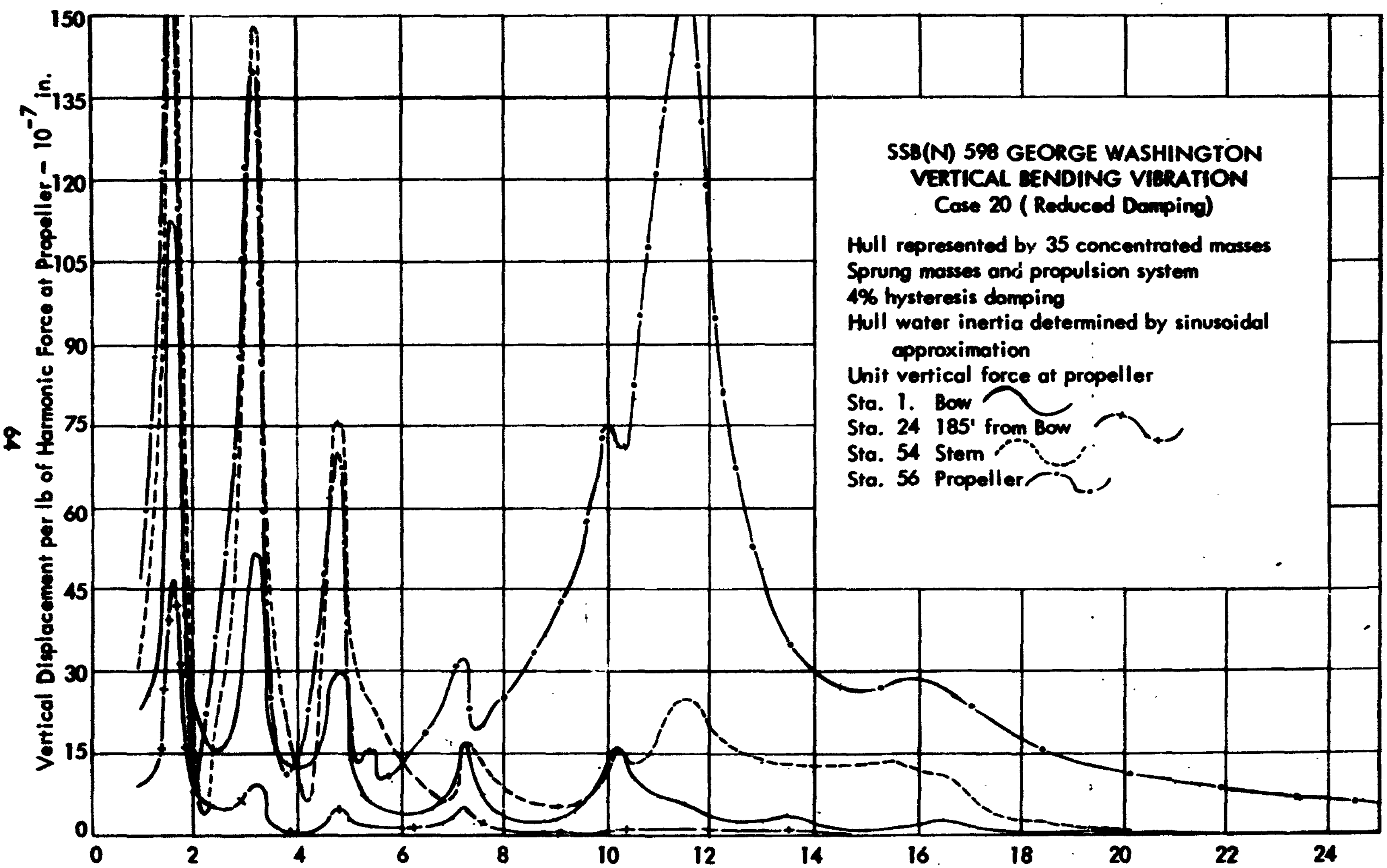


Figure 43- Displacement in Vertical Bending for 1 lb. Excitation at Propeller, Case 20 (Reduced Damping)

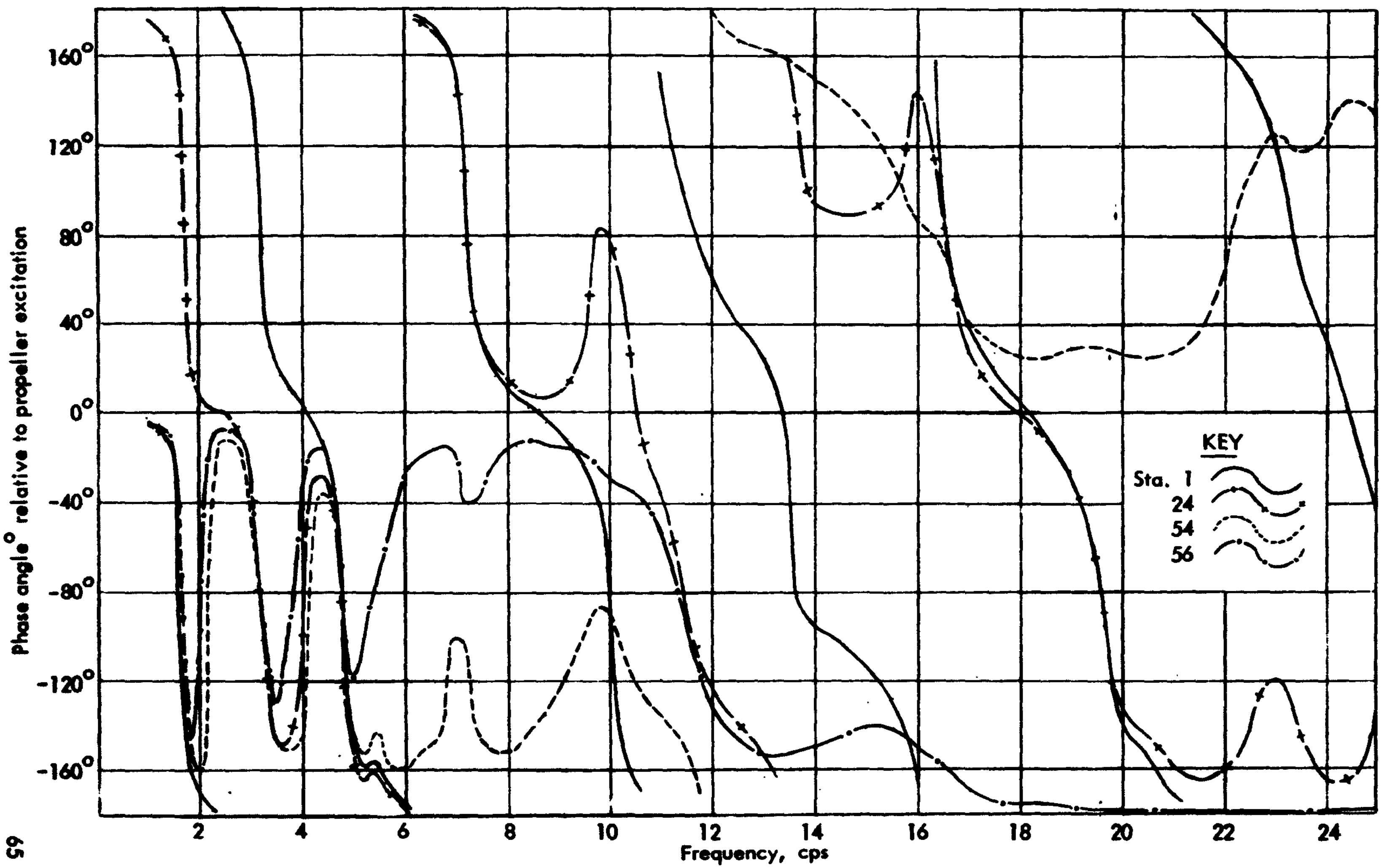


Figure 44–Vertical Bending Phase Angle Relative to Propeller Excitation, Case 20 (Reduced Damping)

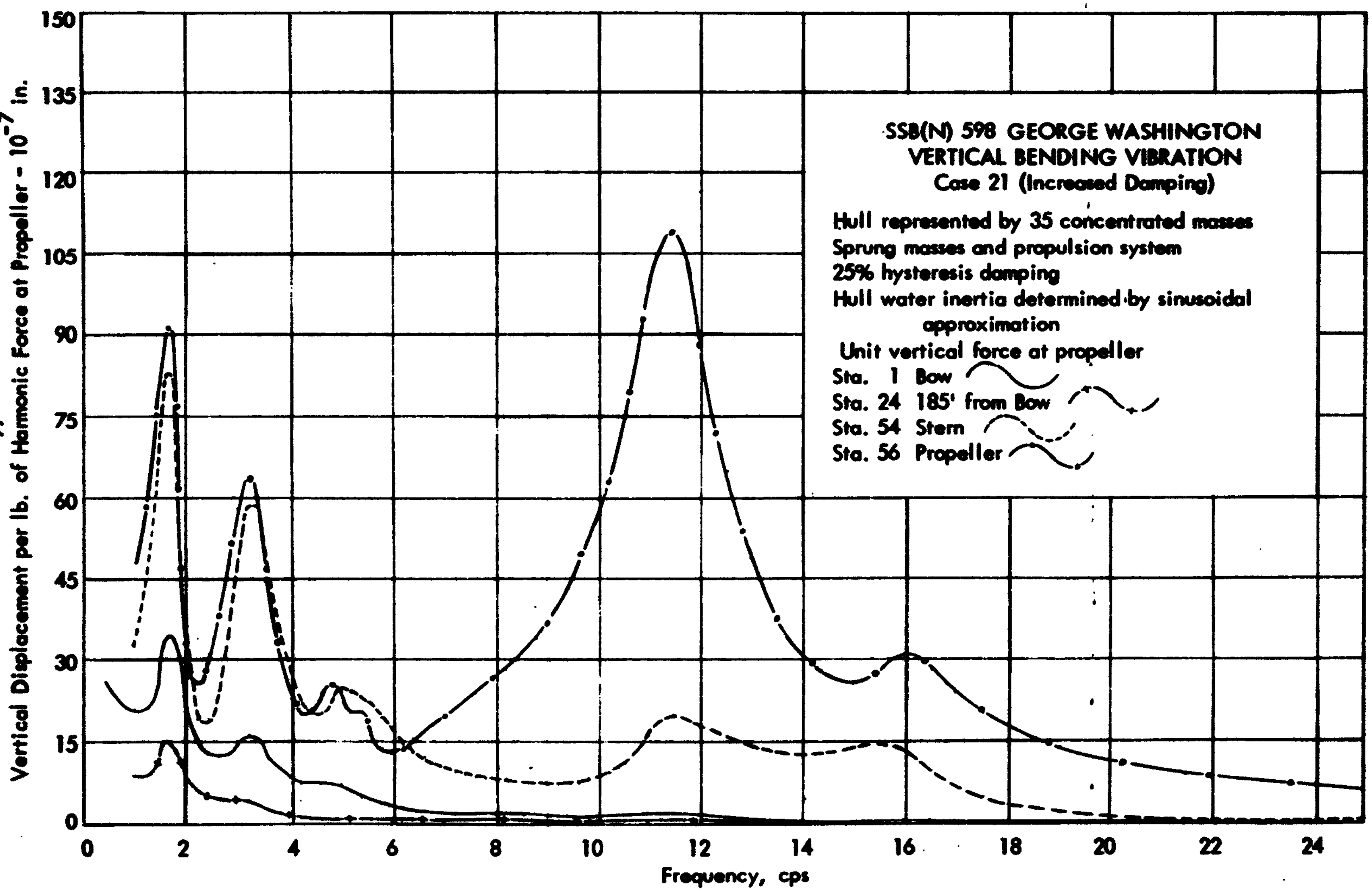


Figure 45 - Displacement in Vertical Bending for 1 lb. Excitation at Propeller, Case 21 (Increased Damping)

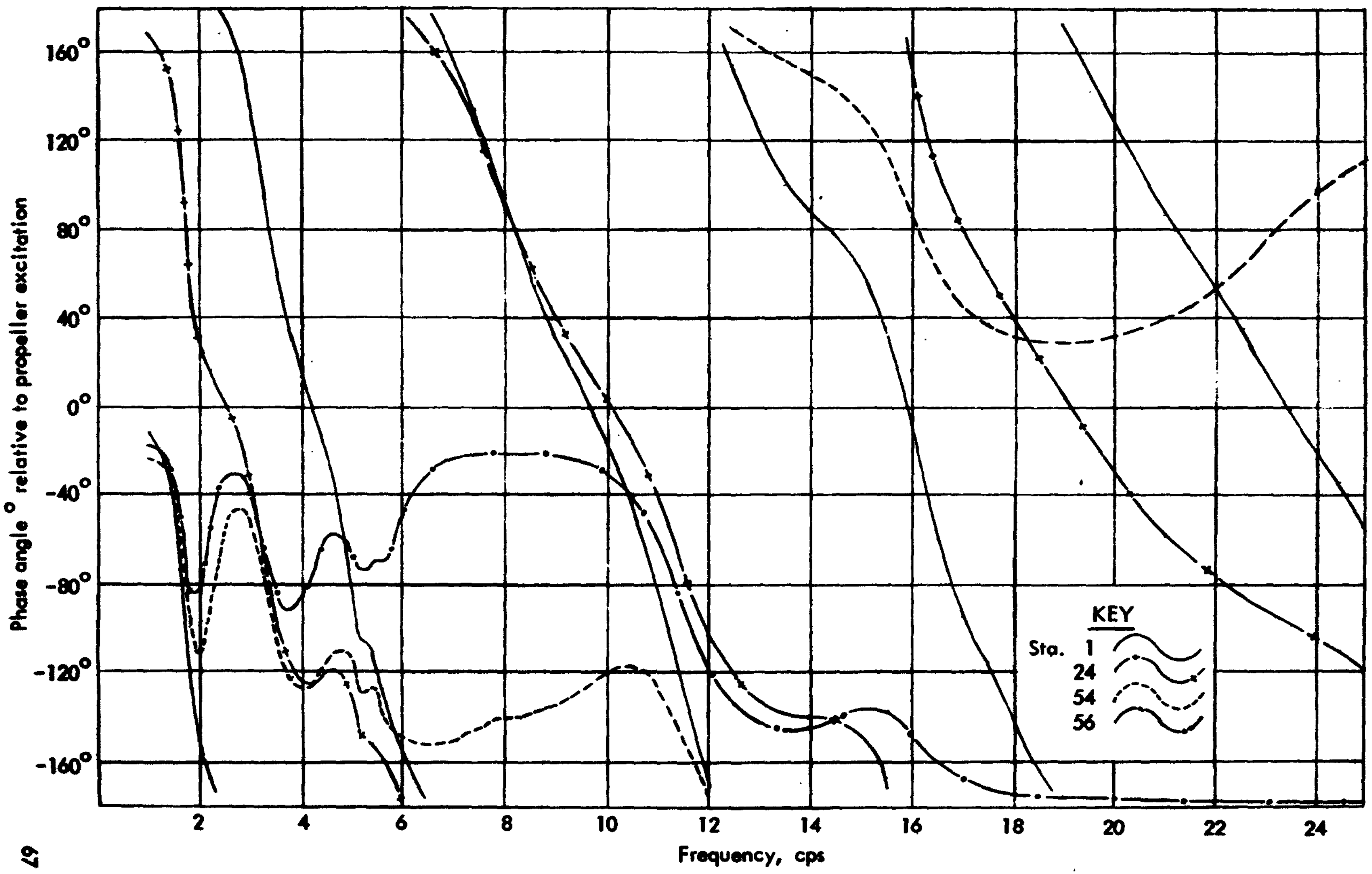


Figure 46 Vertical Bending Phase Angle Relative to Propeller Excitation, Case 21 (Increased Damping)

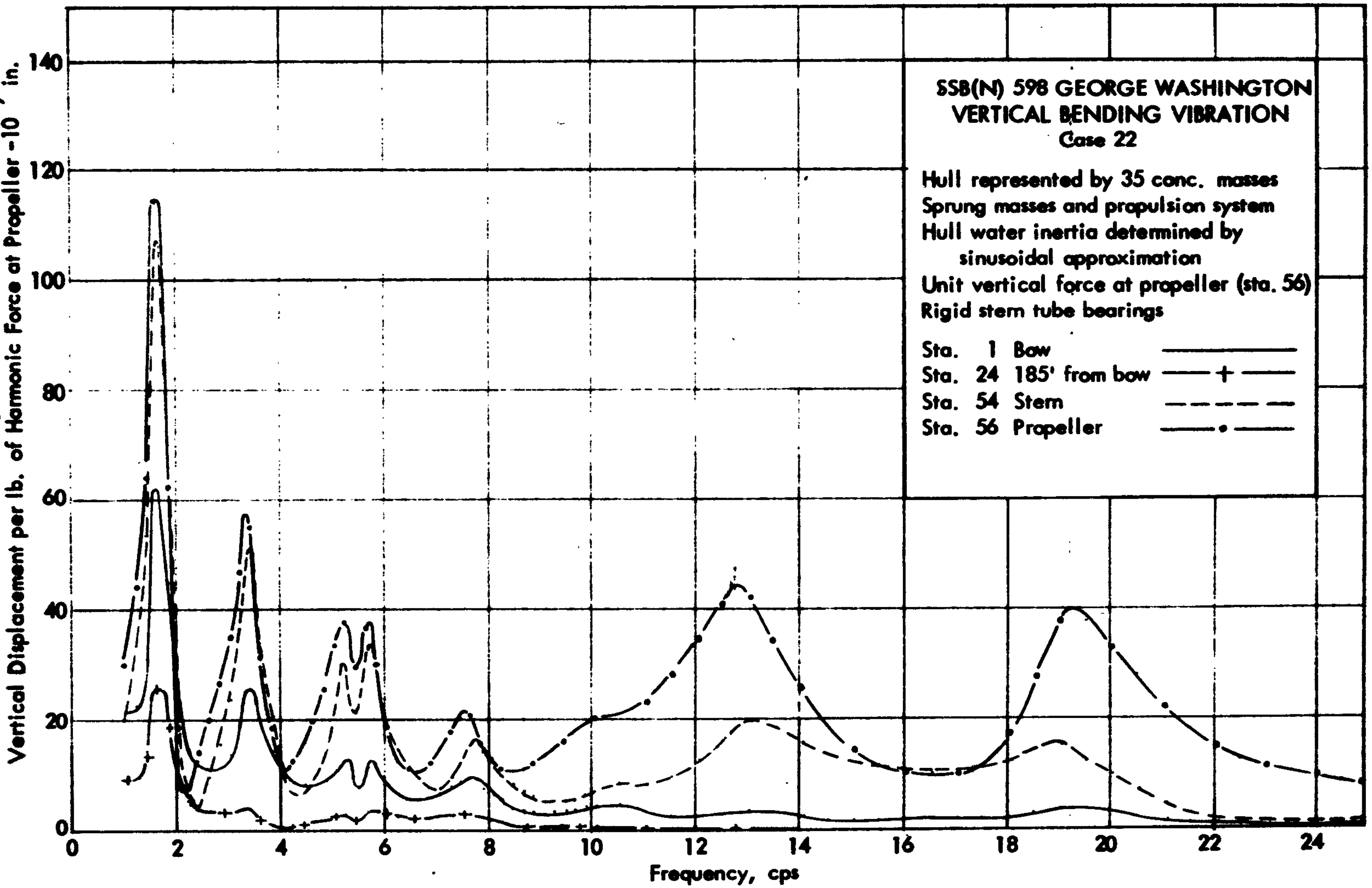


Figure 47-Displacement in Vertical Bending for 1 lb. excitation at Propeller, Case 22

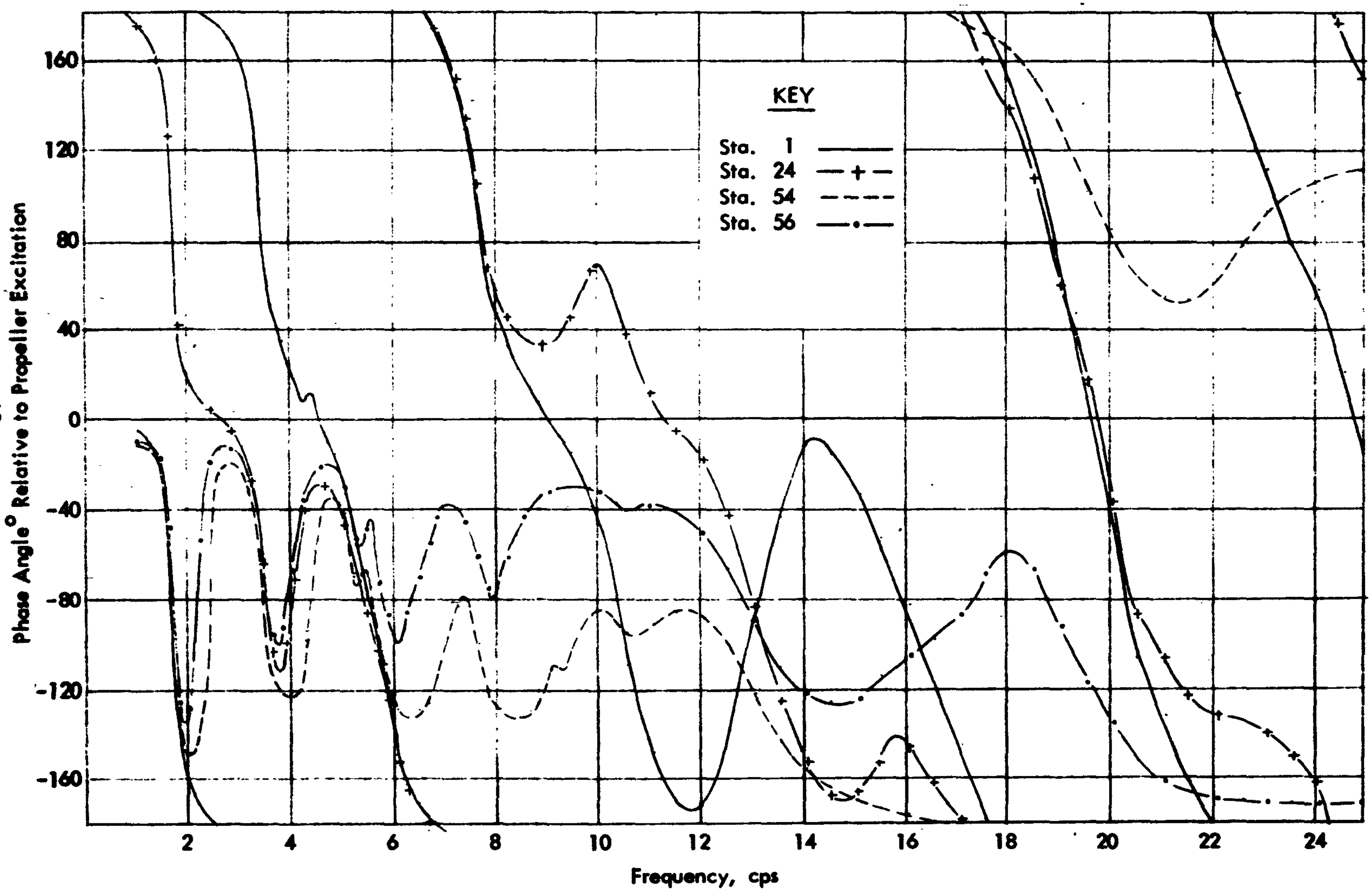


Figure 48 Vertical Bending Phase Angle Relative to Propeller Excitation, Case 22

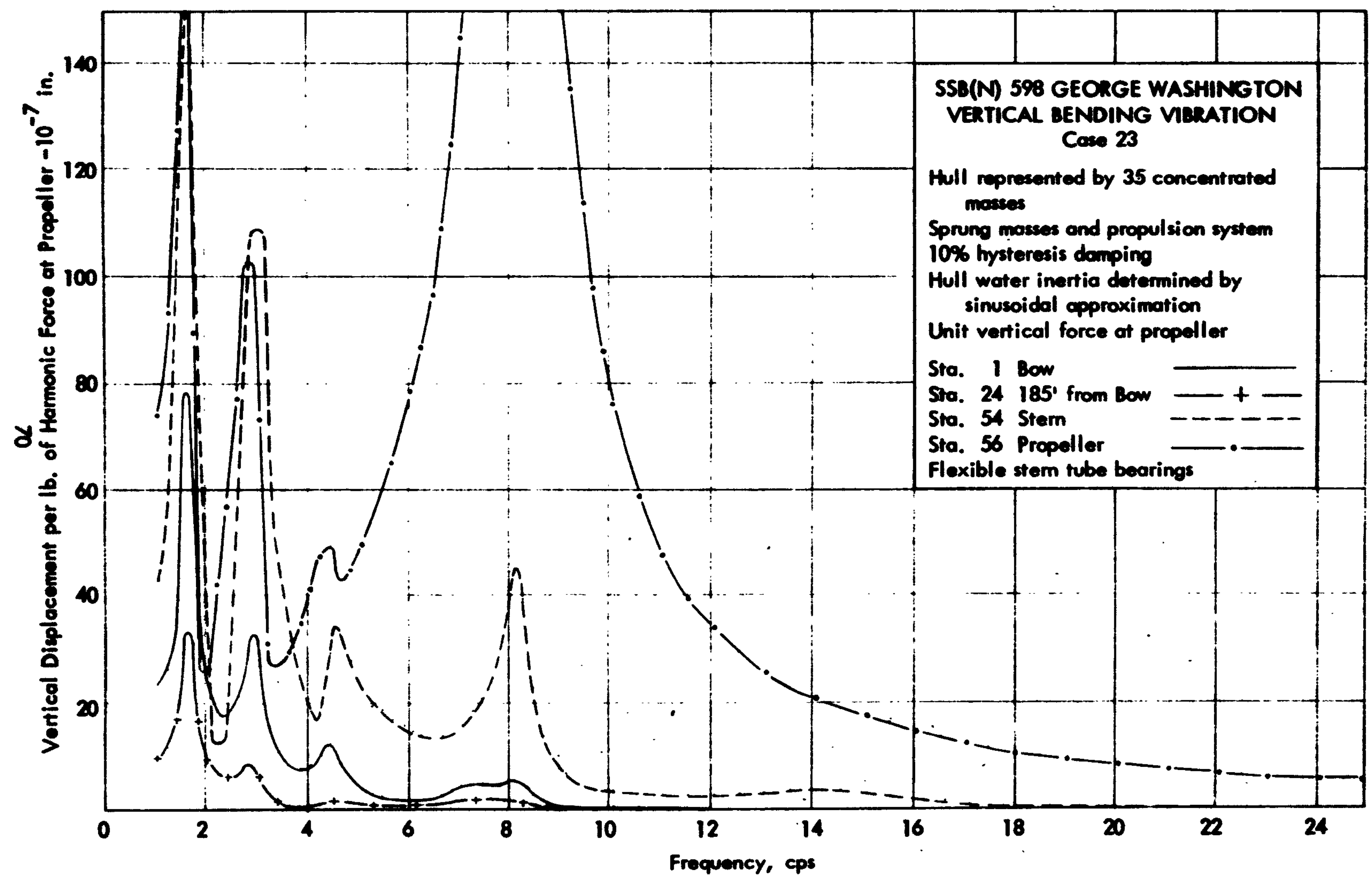


Figure 49–Displacement in Vertical Bending for 1 lb. Excitation at Propeller, Case 23

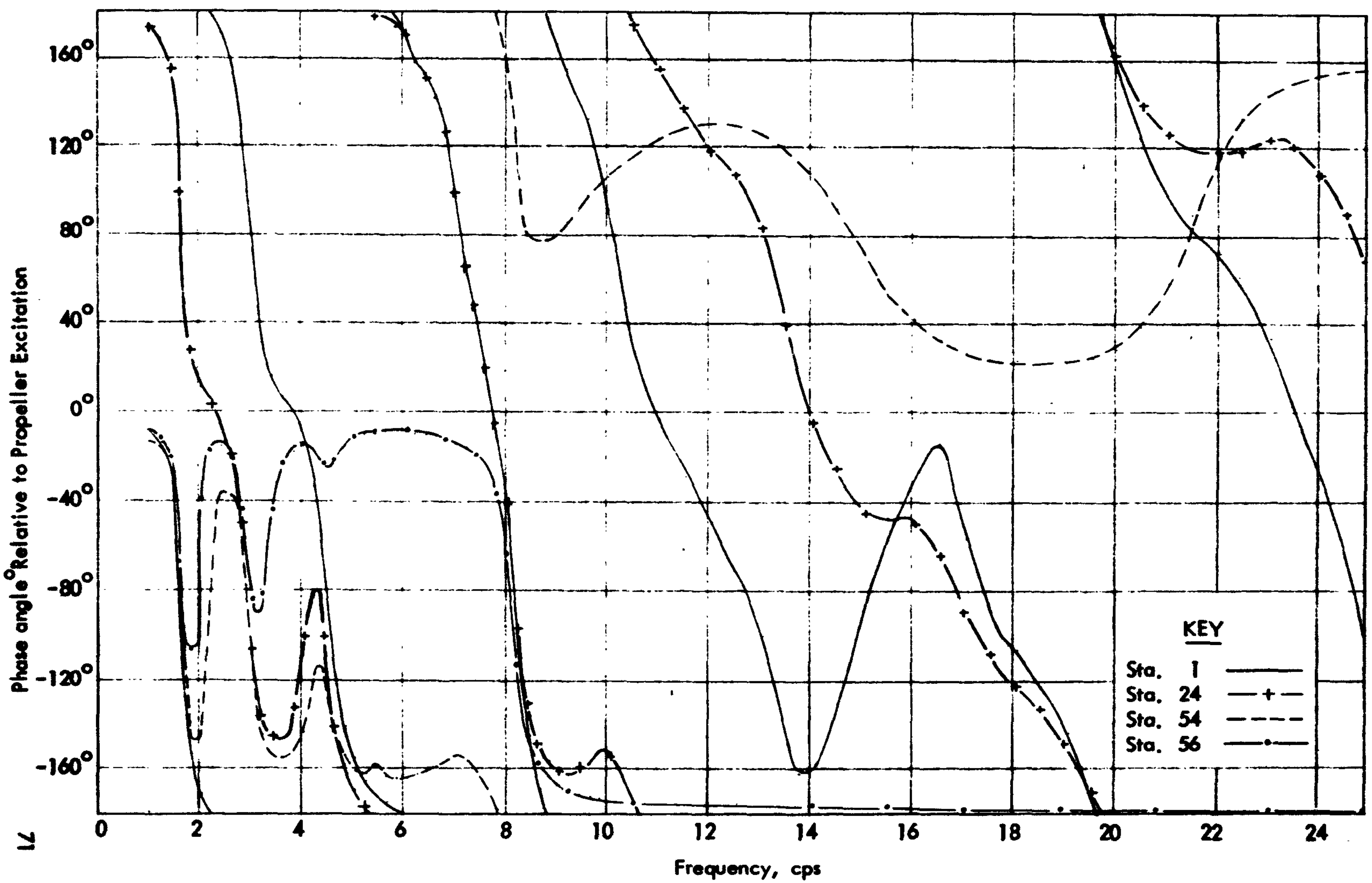


Figure 50–Vertical Bending Phase Angle Relative to Propeller Excitation, Case 23

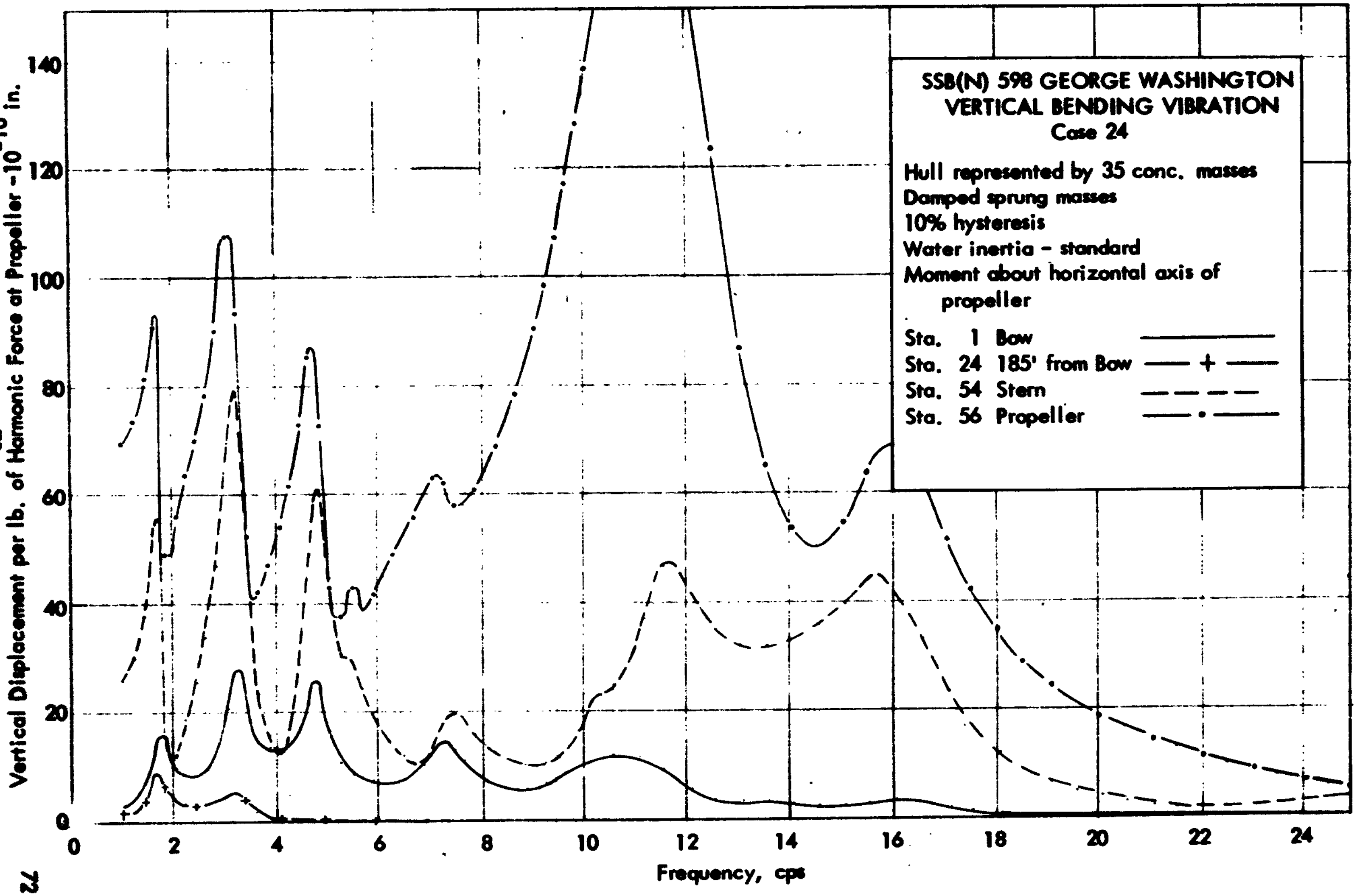


Figure 51 - Displacement in Inertial Bending for 1 lb. Excitation at Propeller, Case 24

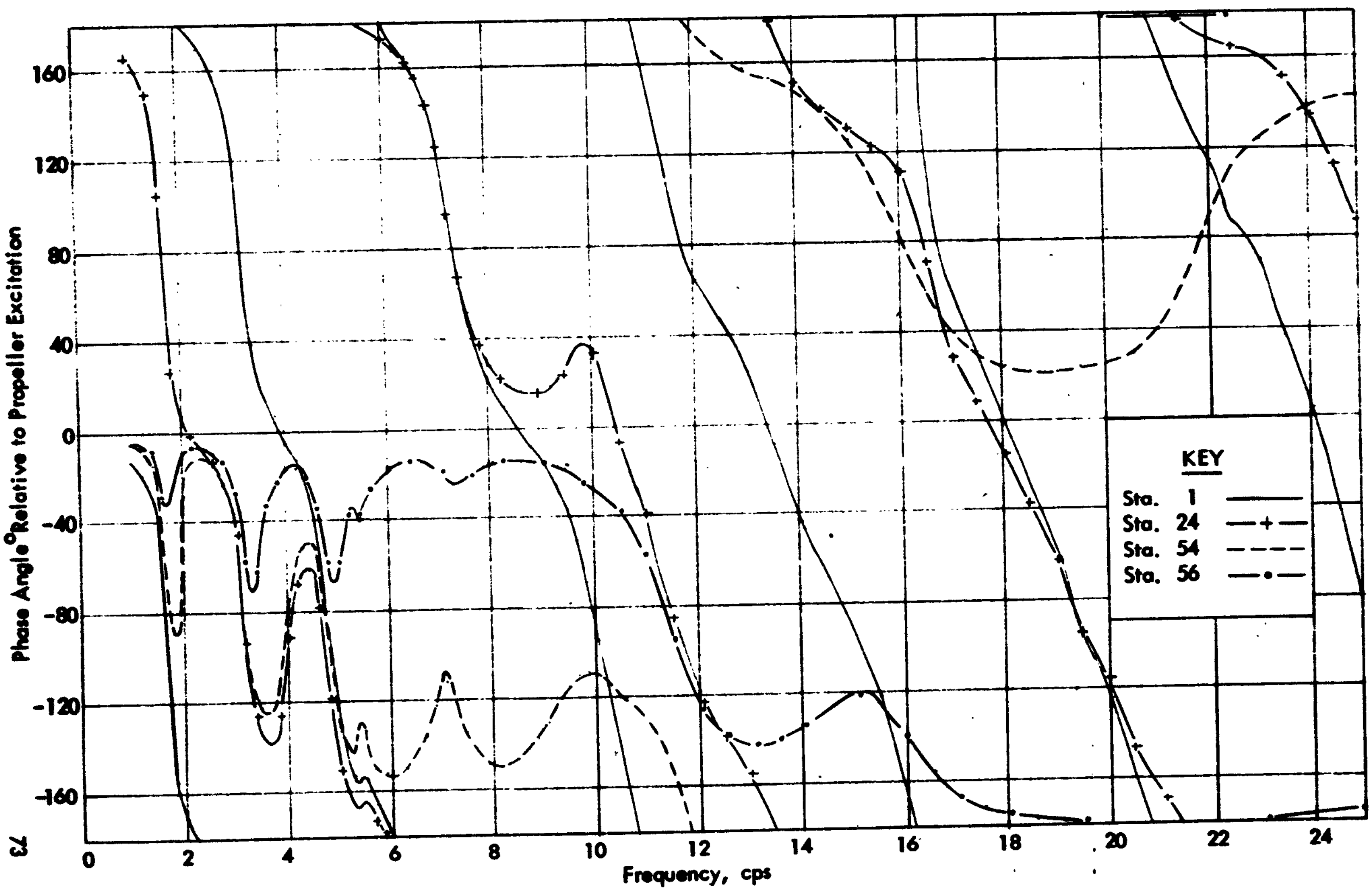


Figure 52 Vertical Bending Phase Angle Relative to Propeller Excitation, Case 24

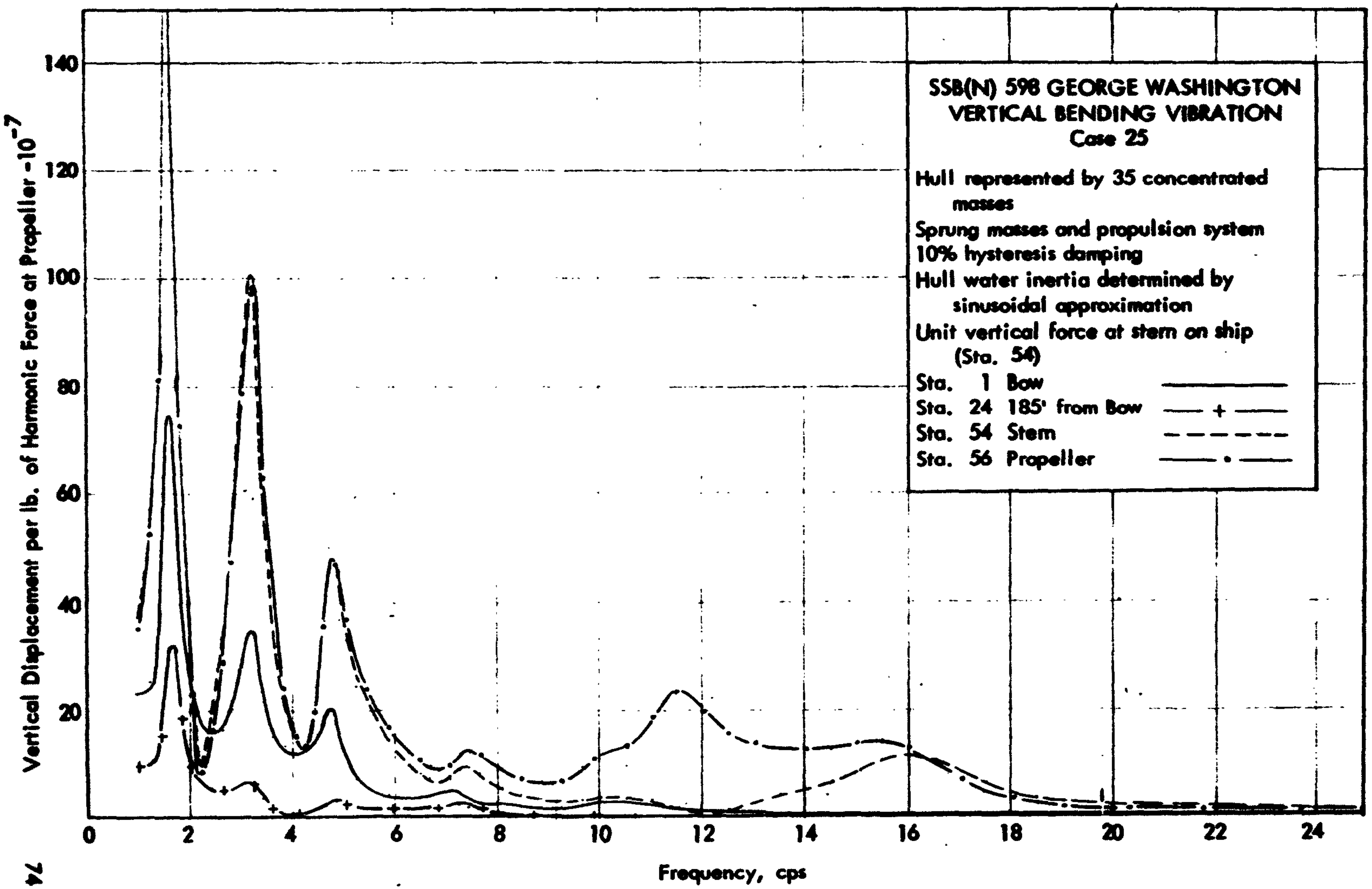


Figure 53-Displacement in Vertical Bending for 1 lb. excitation on hull, Case 25

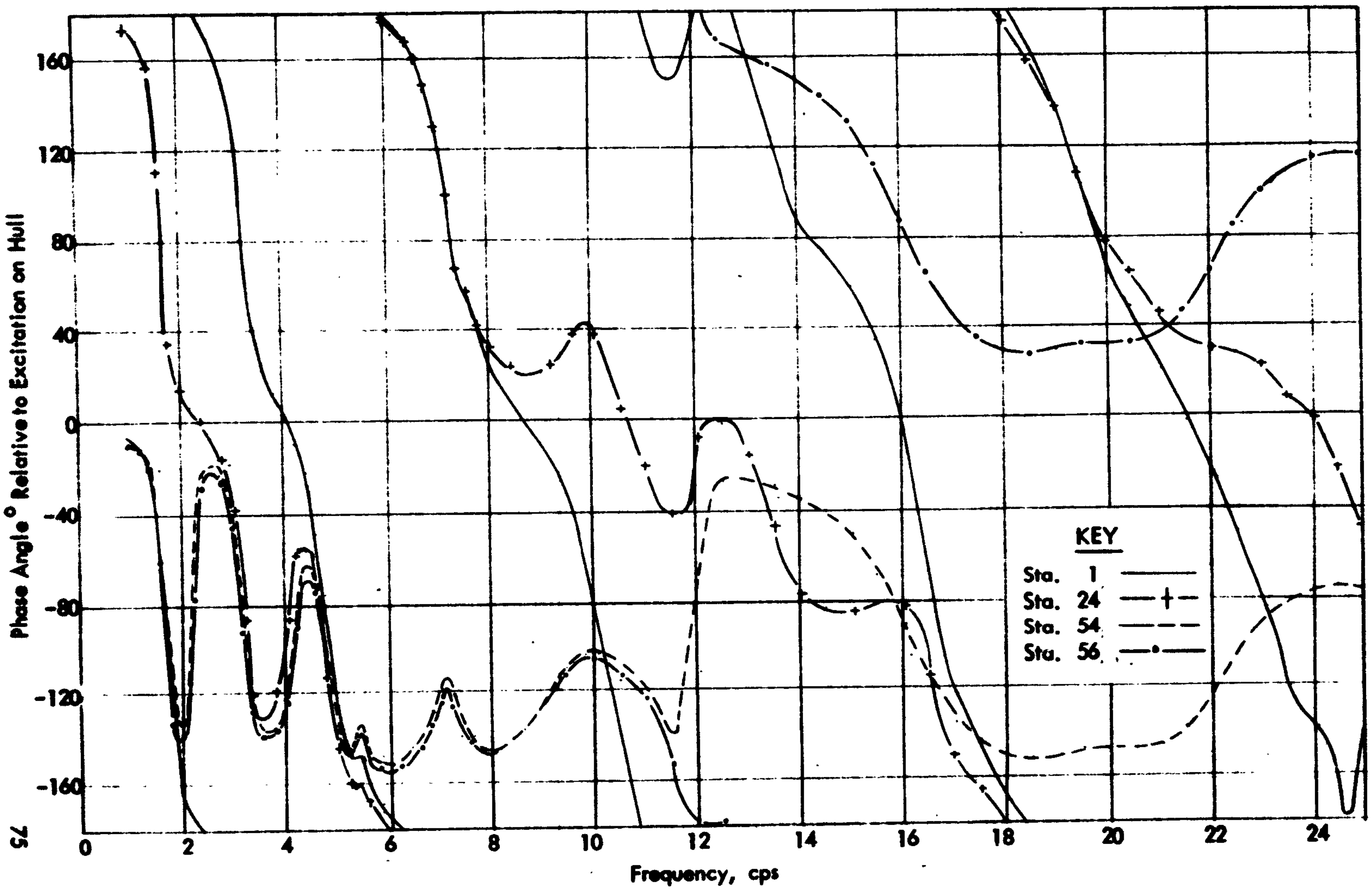


Figure 54 - Vertical Bending Phase Angle Relative to Excitation on Hull, Case 25

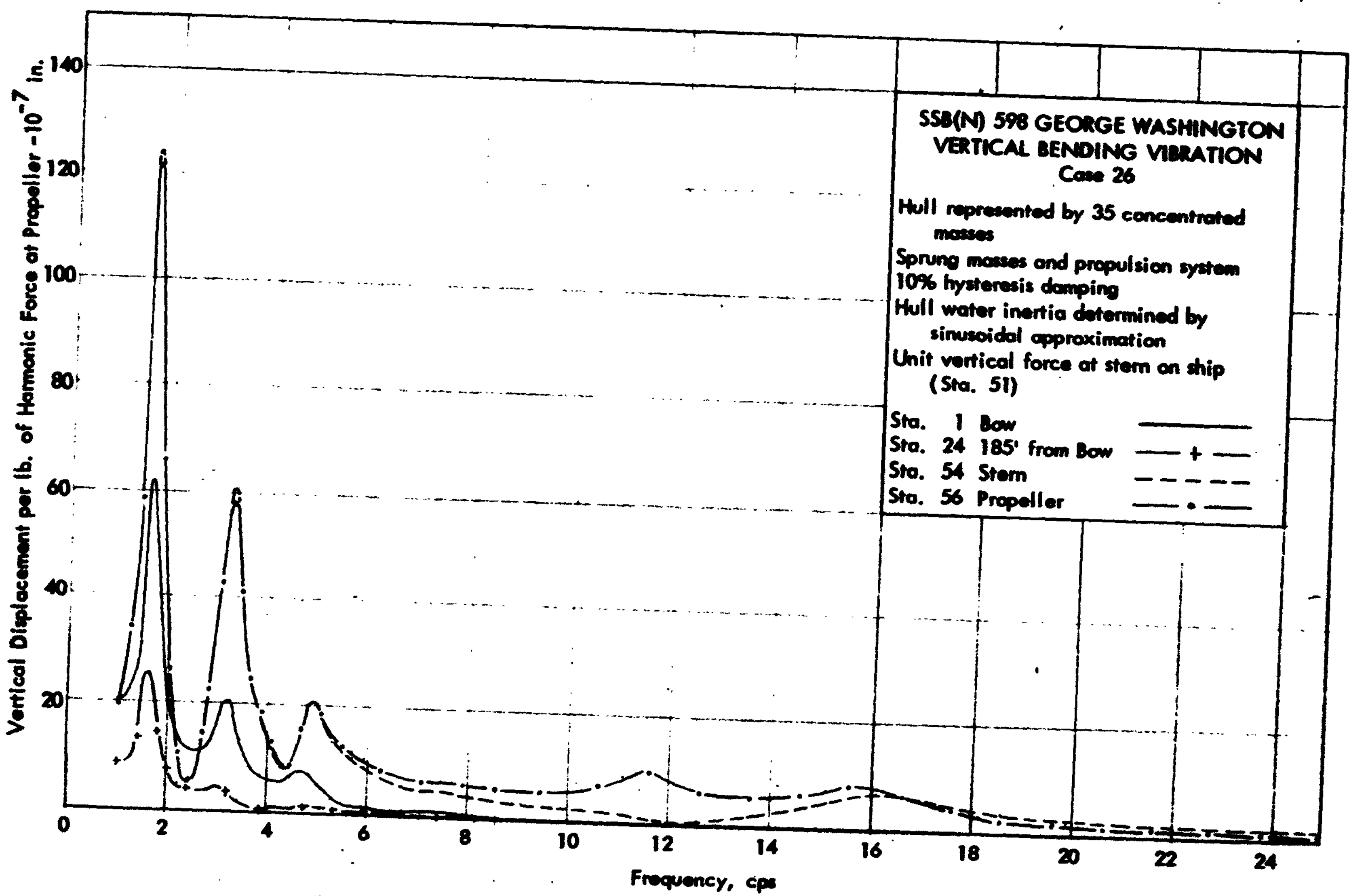


Figure 55 - Displacement in Vertical Bending for 1 lb. excitation on hull, Case 26

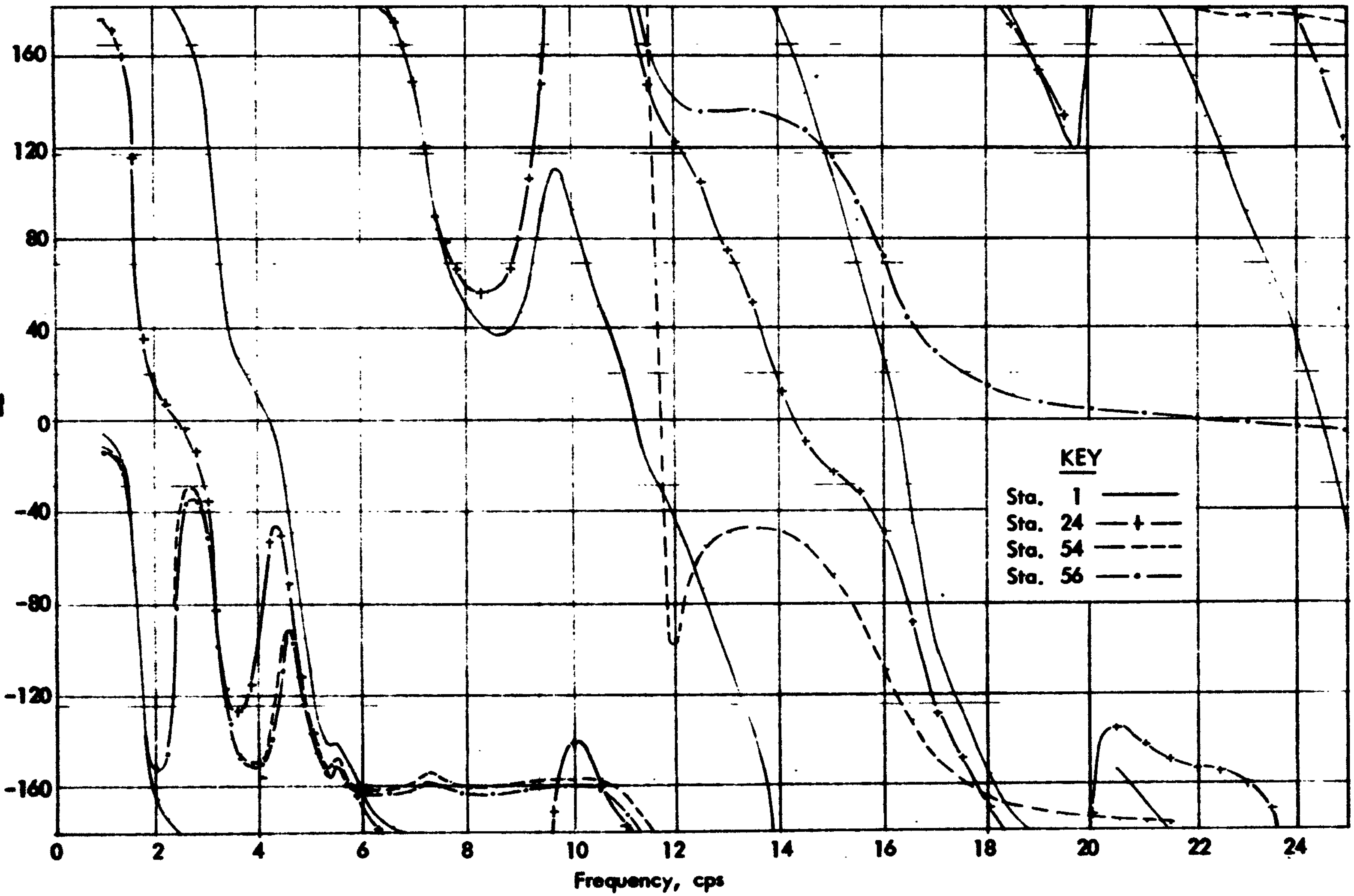


Figure 56 - Vertical Bending Phase Angle Relative to Excitation on Hull, Case 26

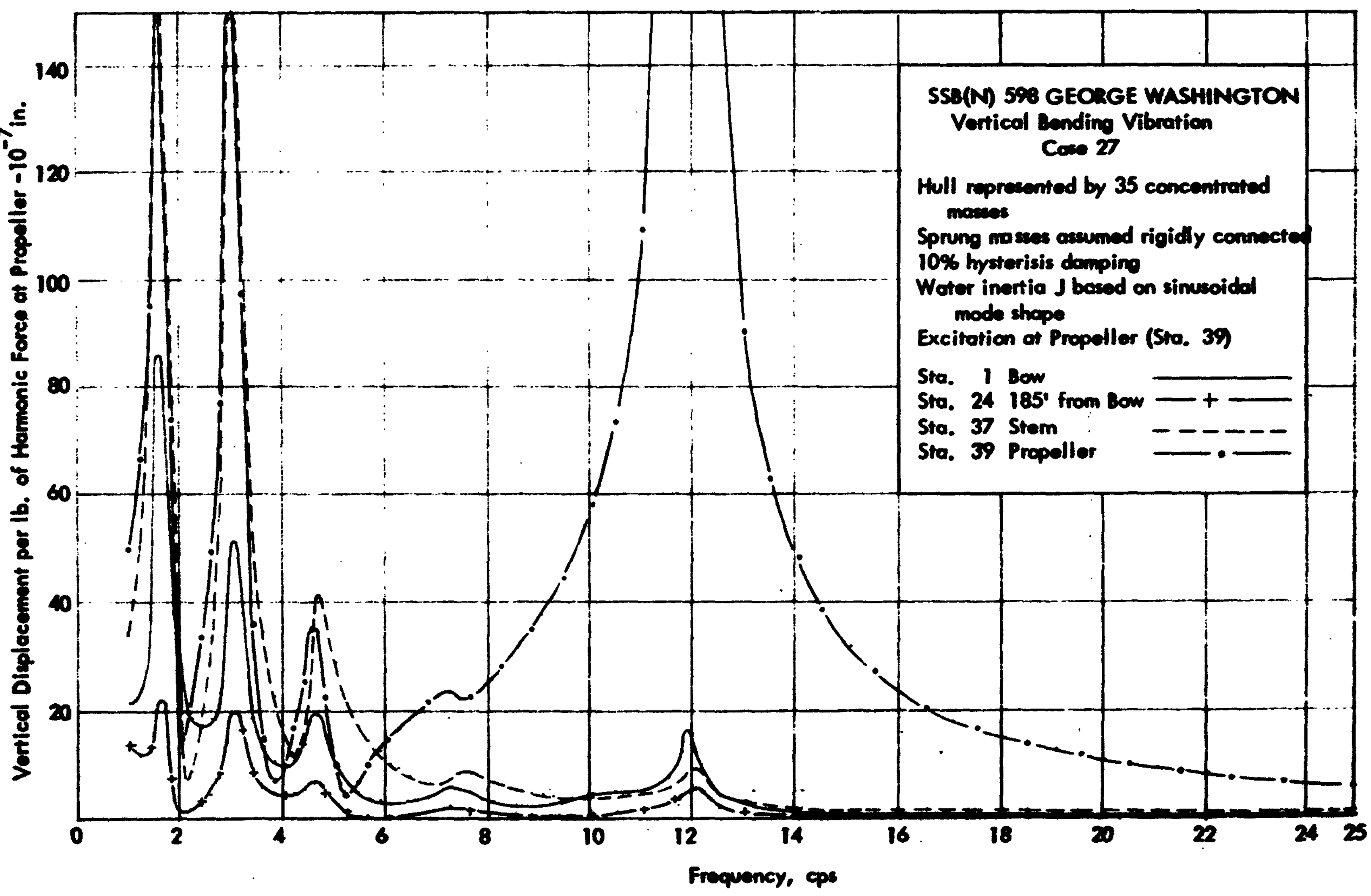


Figure 57–Displacement in Vertical Bending for 1 lb. Excitation at Propeller, Case 27

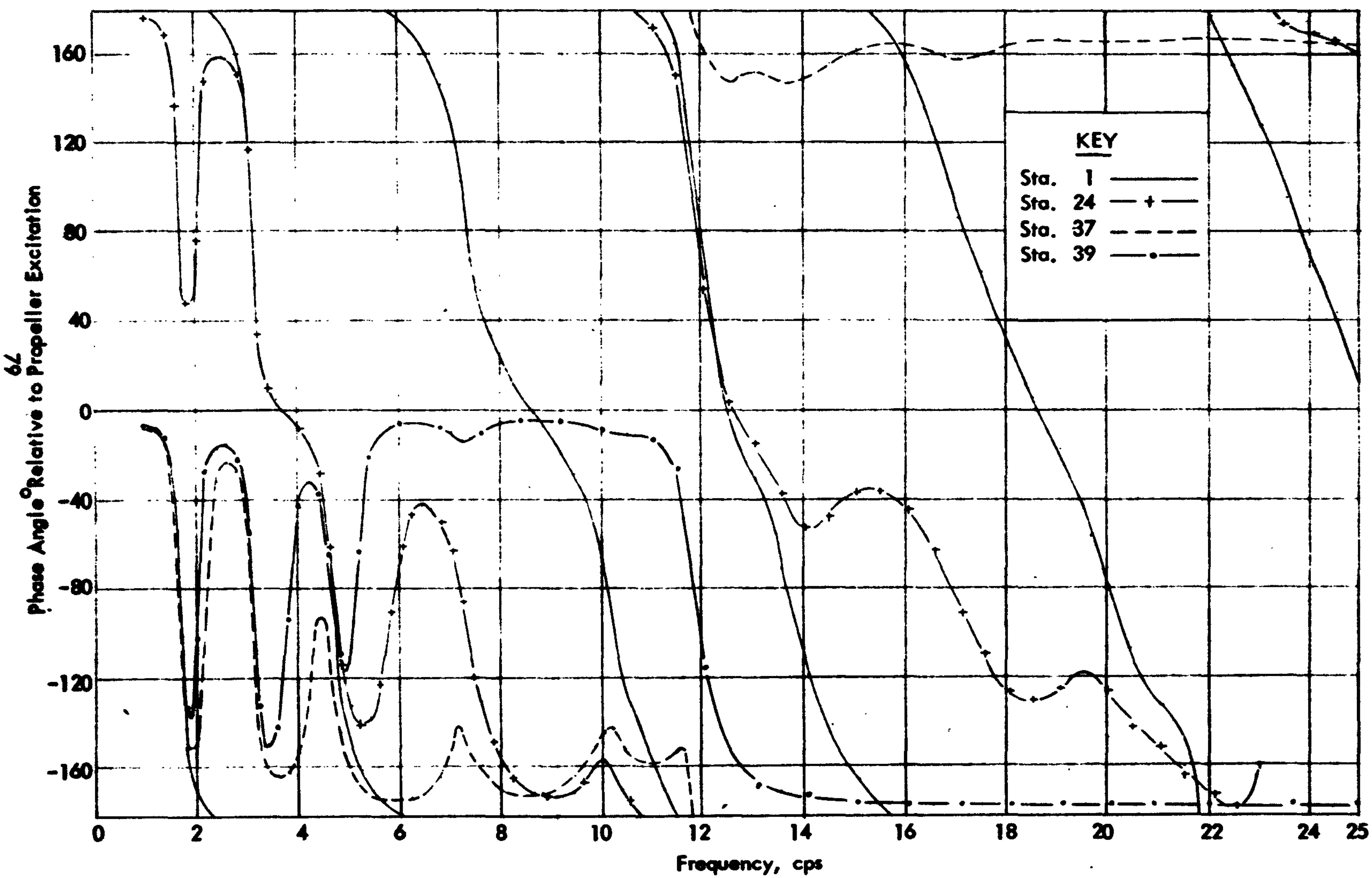


Figure 58-Vertical Bending Phase Angle Relative to Propeller Excitation, Case 27

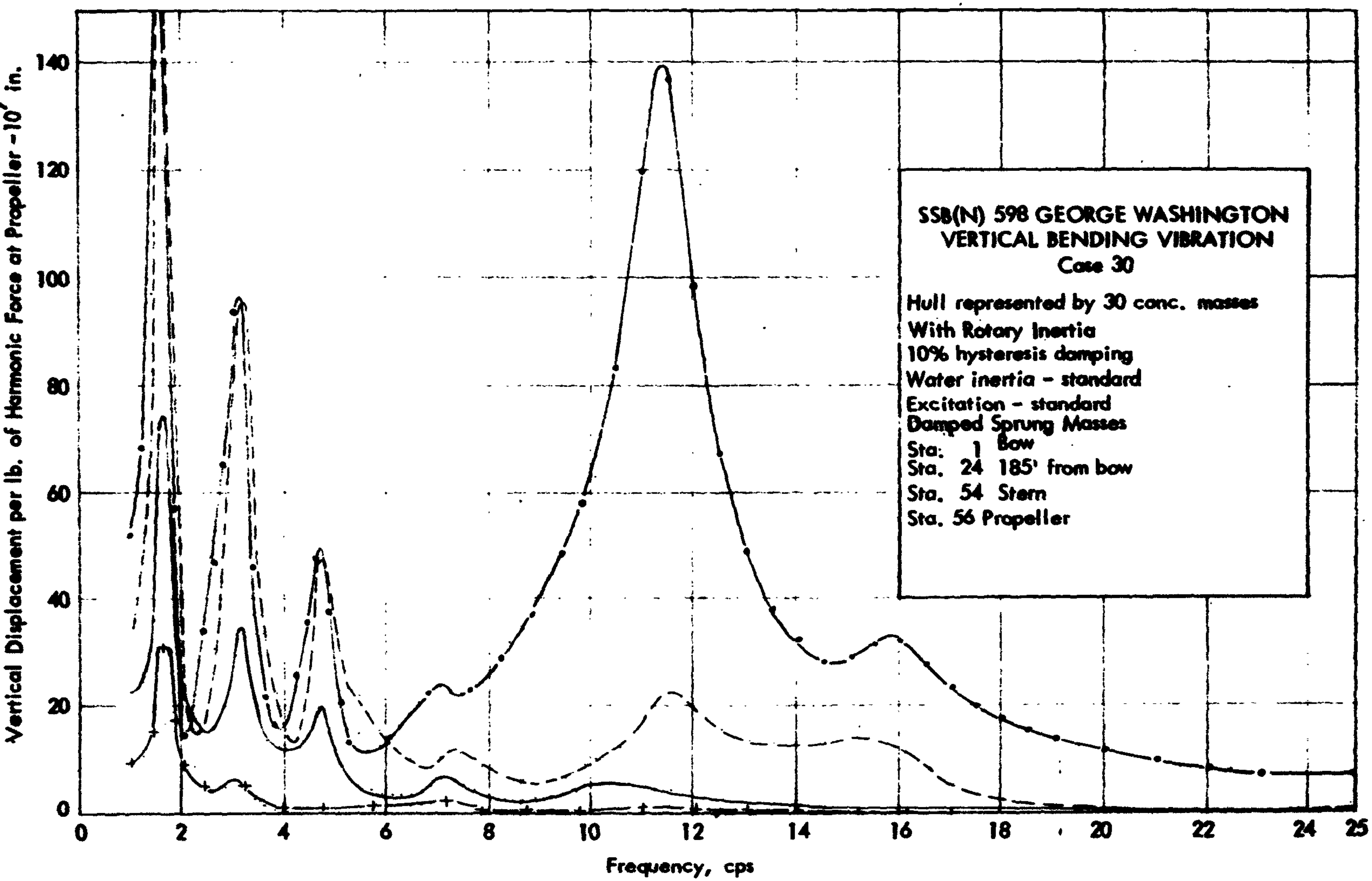


Figure 59 - Displacement in Vertical Bending for 1 lb. Excitation at Propeller, Case 30

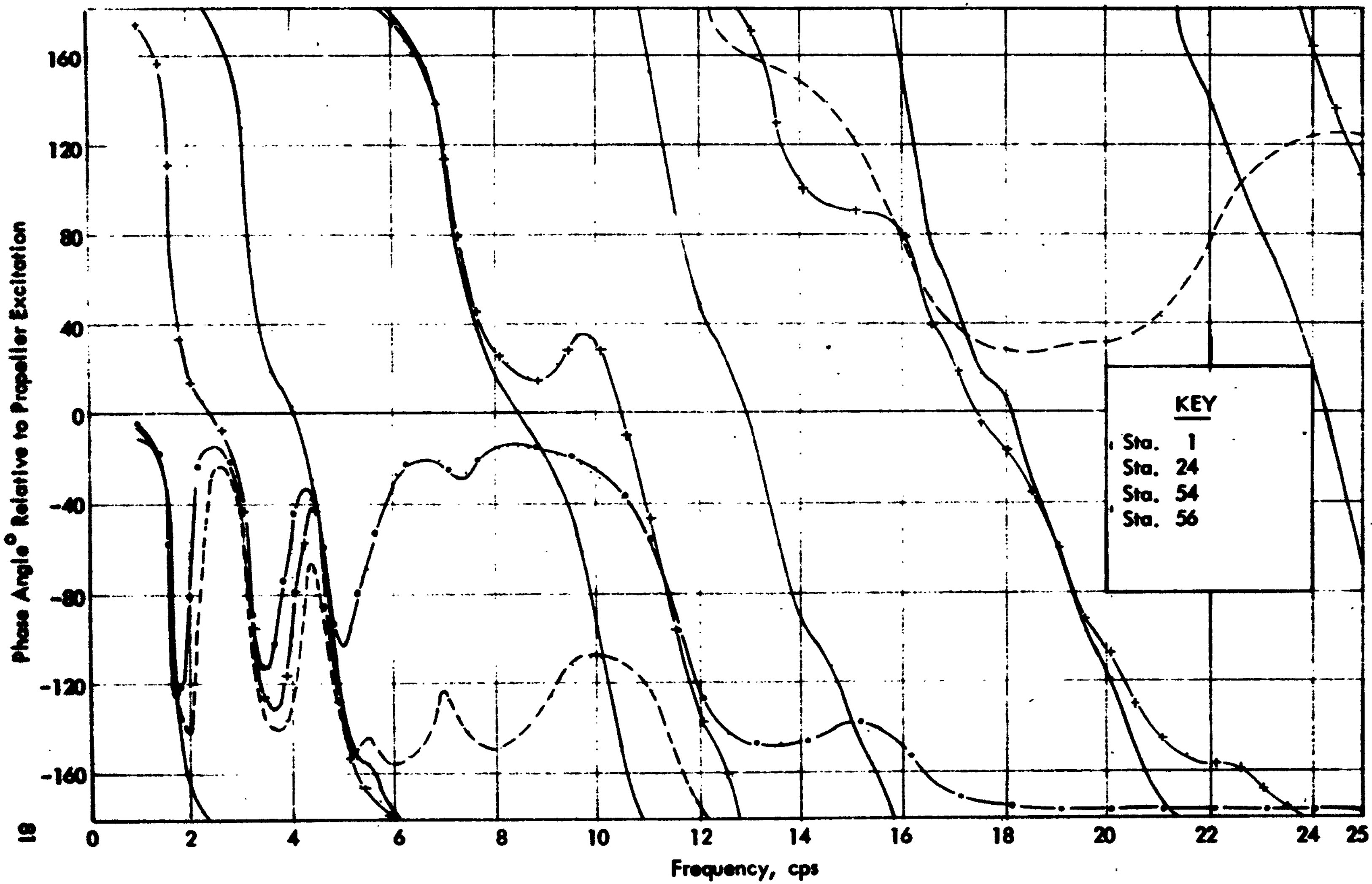


Figure 60–Vertical Bending Phase Angle Relative to Propeller Excitation, Case 30

sufficient to hold them within reasonable bounds even without any hull damping. Surely the effect of hull damping is two fold. (1) It reduces the amplitude of motion at the point of excitation and (2) it attenuates the relative motion of locations remote from the source of excitation. The value of damping which most closely matches that of the actual submarine will have to be obtained by comparison with the trials on the submarine. These calculations suggest that it requires less hull damping to match trial experience than was previously thought. This is a desirable result because although experience in other structures supports an assumption of 4% hysteresis damping, it is very difficult to explain the source of 10% or more of hysteresis damping.

When comparing Cases 22, 18 and 23, representing a 10 to 1 change in the stiffness of the stern tube bearing the change in the hull response is striking. This increased stiffness bearing (3 times over the standard) moves the propeller resonance from about 11.3 cps to about 19.5 cps as might be expected. What might not be expected is the large effect of this shift upon the hull response. The amplitudes of motion at the lower resonances are very much reduced and those at the higher resonances increased by this shift. The effects of reducing the stiffness of the stern tube bearing are entirely consistent with the above results. Although the amplitude of the propeller is increased this occurs at lower speeds and there is a large decrease in the hull response in the normal operating range. These results surely emphasize the importance of treating the propeller and shafting sub-system as precisely as possible.

Case 24 indicates that the hull is less responsive to a harmonic moment (a moment arm of more than 1000 inches would be required to give equivalence) than to a harmonic force. Cases 25 and 26 as compared with Case 18 indicate that except at the propeller resonance a harmonic force at the stern will generate a slightly higher hull response than the same force at the propeller. As the location of the harmonic force is moved forward Case 26 as compared to Case 25) the response on the hull falls off noticeably.

Case 27 as compared to Case 18 shows that for the lower hull frequencies the damping associated with the sprung masses has a significant effect. It also appears that

the inclusion of the sprung masses with their springing can change the hull frequencies by measureable amounts. However, some of the differences between Cases 27 and 18 are difficult to understand.

Finally, Case 30 as compared to Case 18, indicates that the effects of including the rotary inertia in the hull response calculations of a submarine are very small.

APPENDIX A

PROCEDURE USED IN DEFINING THE SHIP WEIGHT DISTRIBUTION

The basic information used in determing the weight distribution of the ship is given in the "Detailed Weight Summary," SSB(N) 599 dated 7/1/60, prepared by the Electric Boat Division of General Dynamics Corporation.[7] This weight summary is in the form of an IBM print-out. The summary breaks the ship weight into about 250 classifications. Under each classification it lists the plans covering the items contributing to the weight. For each plan it gives the total weight represented by the plan and the vertical and longitudinal center of gravity.

For many of the weights, particularly heavy concentrated items, this information can be used directly for estimating the longitudinal distribution. However, for some important weight elements, such as the hull plating, the plans cover such a large portion of the ship's length that it is necessary to make independent calculations to define the weight distribution. On many weight items, such as electrical wiring, piping and joiner work, the distribution of the component weights had to be estimated from the compartments served and the longitudinal position of the center of gravity.

From the Detailed Weight Summary a chart was prepared in which the weight in each of the several classifications was distributed throughout the ship. A typical section of this chart covering the portion of the ship length between 30 feet and 80 feet from the bow is shown in Figure A-1. It consists of distributed weights and concentrated weights. The distributed weights are added up at the several sections and a curve of distributed weight prepared. The concentrated weights are listed by location and are then distributed over a two foot length. Table A-1 presents a listing of the concentrated weights over the interval covered in Figure A-1. Figure A-2 shows the distributed mass, the spread-out concentrated masses, the captured water and the water ballast for the typical section of the ship between 30 feet and 80 feet from the bow. Finally, all of these curves are added and smoothed

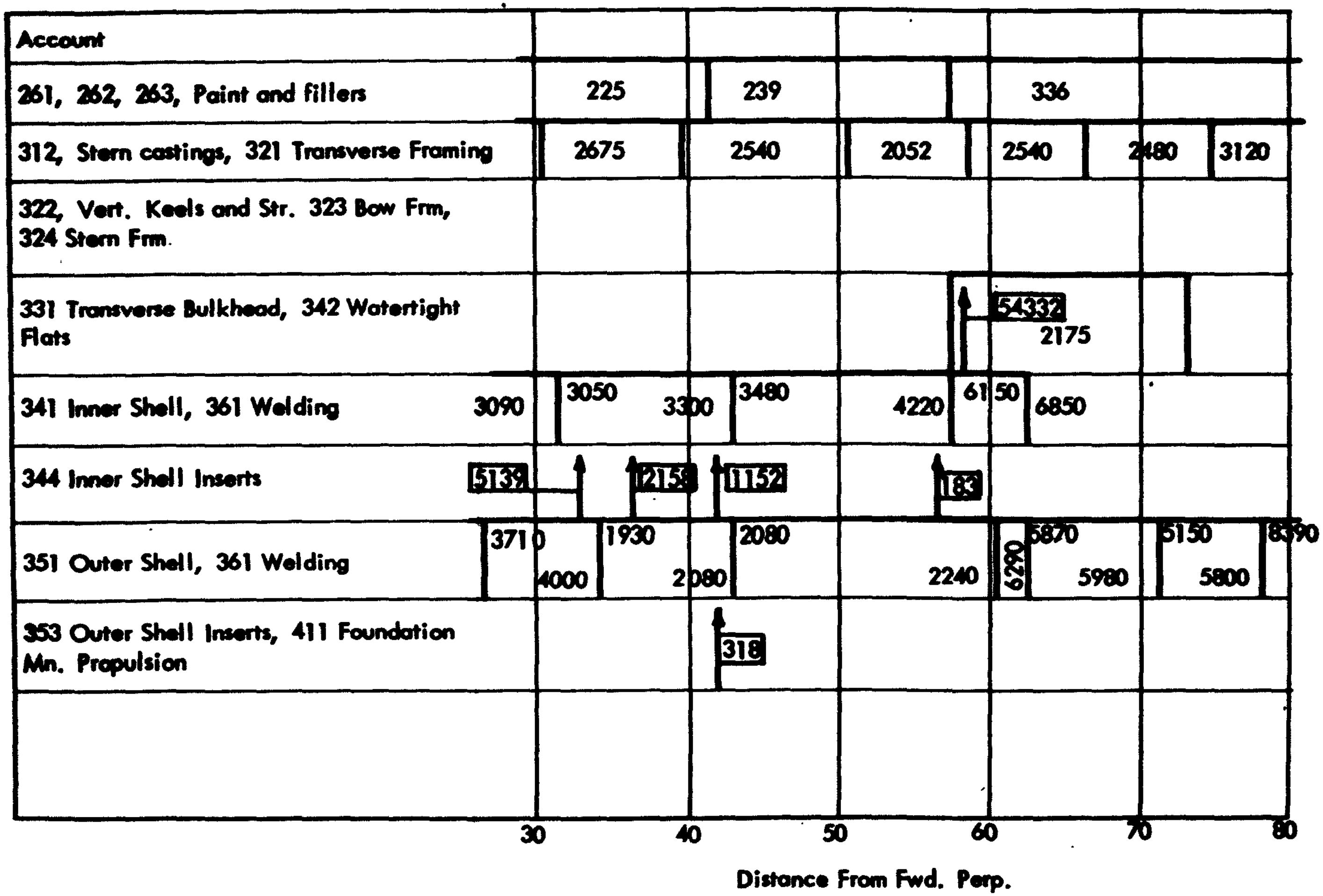


Figure A1 – Longitudinal Location of Hull Weights – Typical Section

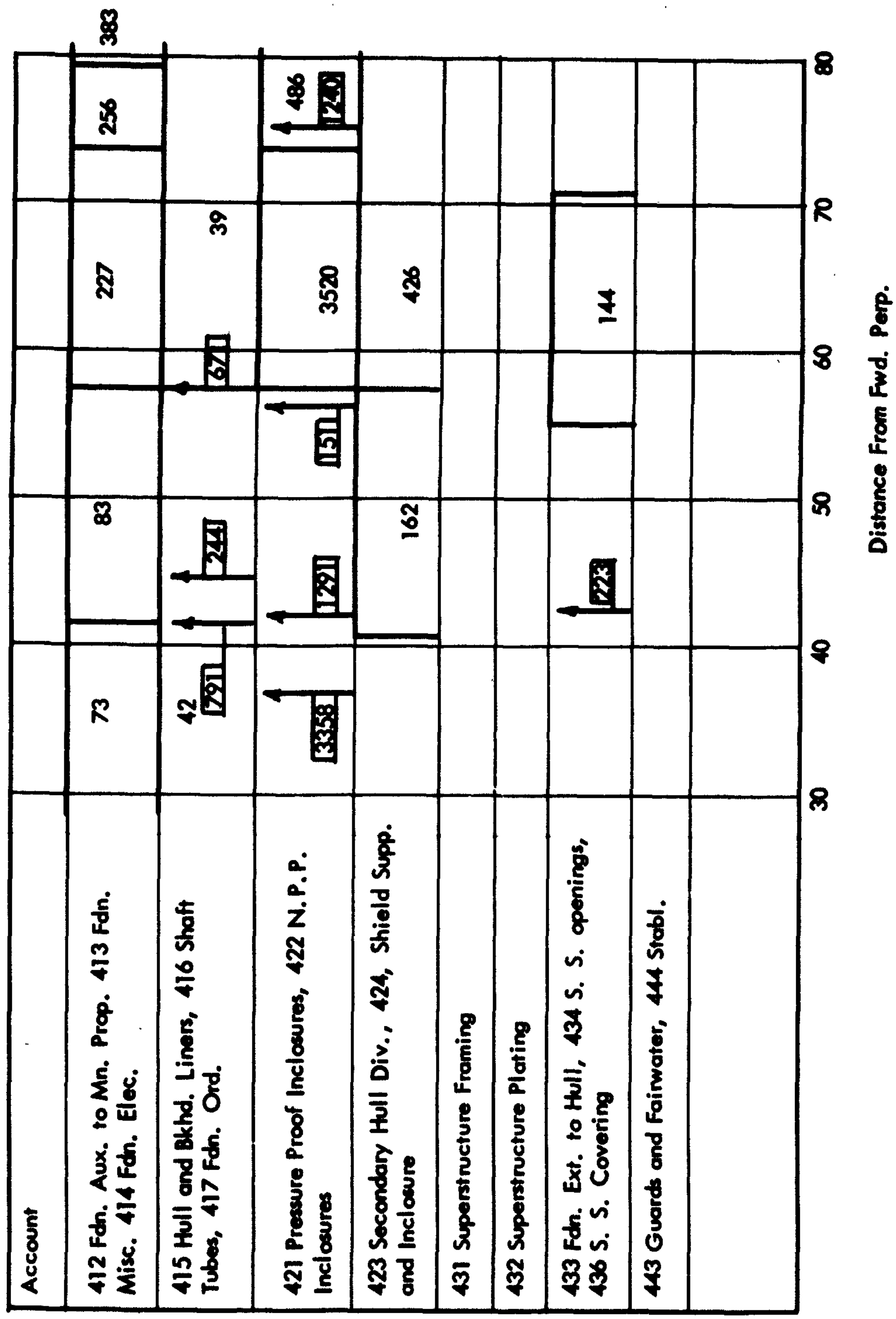


Figure A-1 (continued)

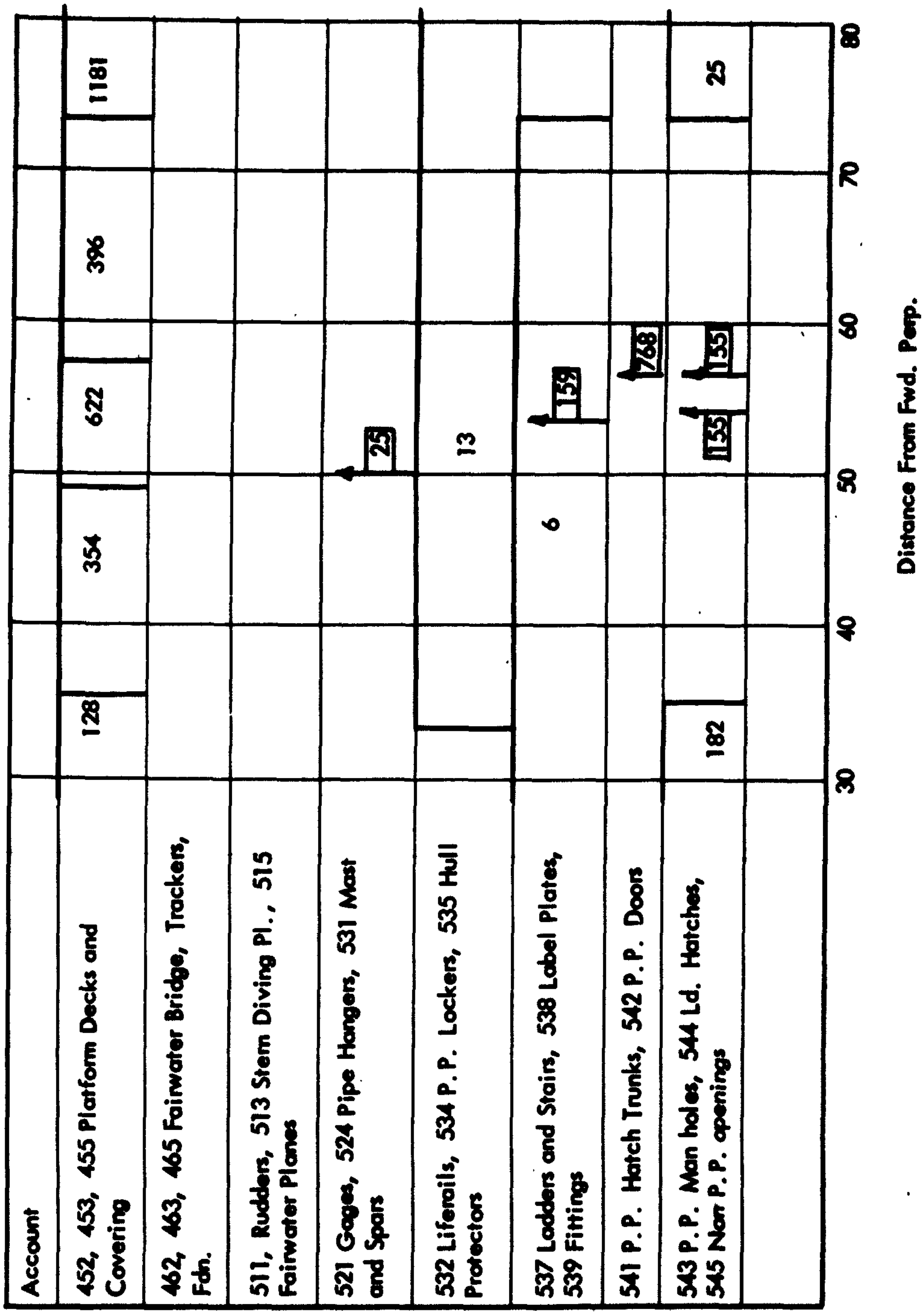


Figure A-1 (continued)

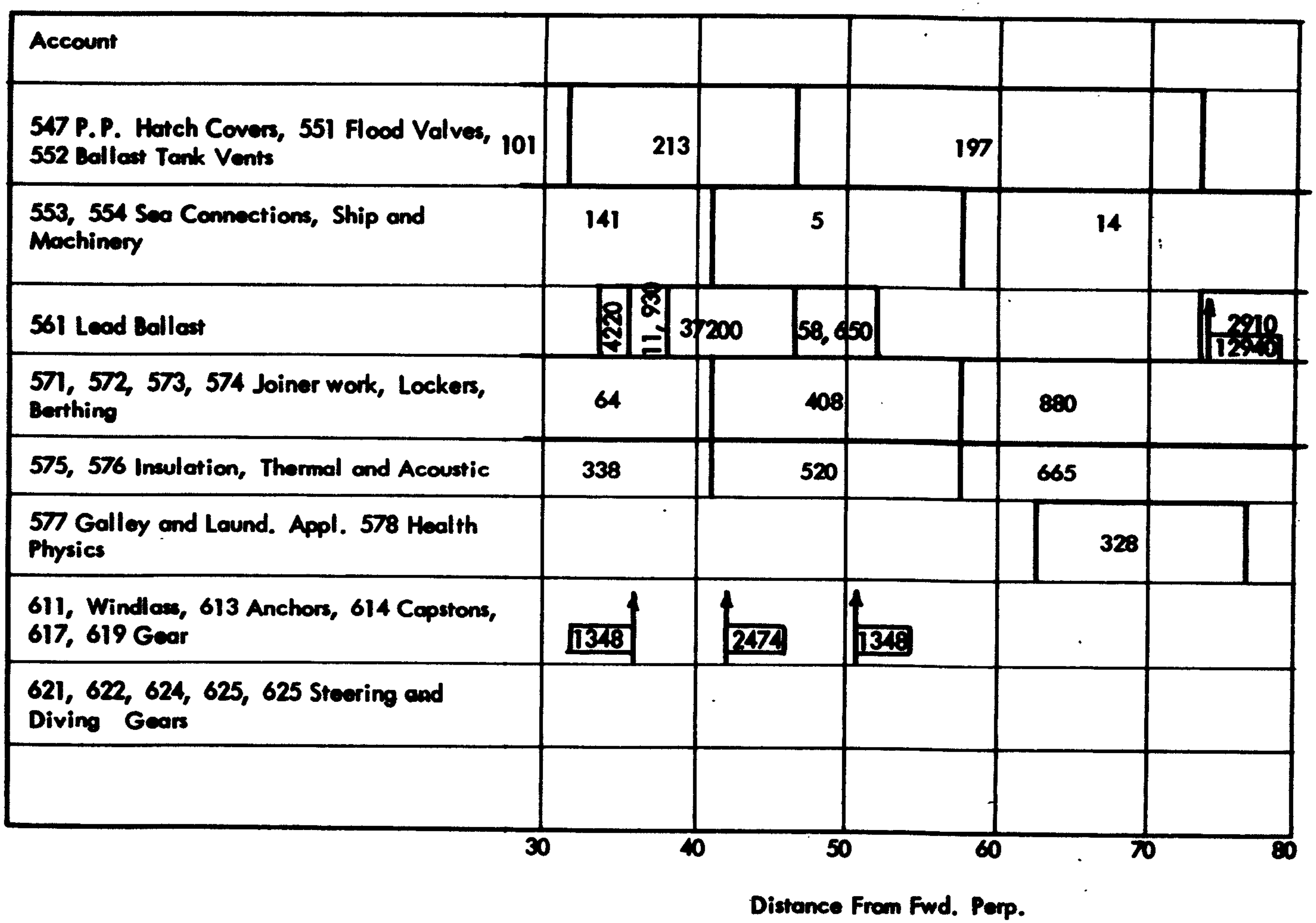


Figure A-1 (continued)

Account	30–40	40–50	50–60	60–70	70–80
627 Gyro. Stabil; 631, 632, 633 Periscopes; 634 Speed Ind.			$\boxed{522}$		2
636, 637 Mag. and Gyro Compass, 638 Nav. Masts, 639 Signalling					
641, 643 Battery and Ships Ventl. 644 Refrig.	19	24	$\boxed{520}$	$\boxed{1984}$	161
645 Air Cond. 646 Htg., 647 Gas Anal. and Burning		10		15	
648 Mn. Oxygen, 649 CO_2 Removal		19 $\boxed{11,500}$	5	1465	5
651→ 662 Trim and Dr., F. W., Distl., Plumbing, Cooling, F. O., L. O.	161	353	$\boxed{2937}$	211	
671→678 Comm. Announcing, Alarm, Elec. Ltg. control	67	124		242	
680→696 Radar, Sonar, etc.	72	134	$\boxed{205}$	63	
698 Mach. Shop, 711 Turbines, 712 Clutch, 713 Shafting, 714 Brgs., 715 Prop. 716 Grs.					

Distance From Fwd. Perp.

Figure A-1 (continued)

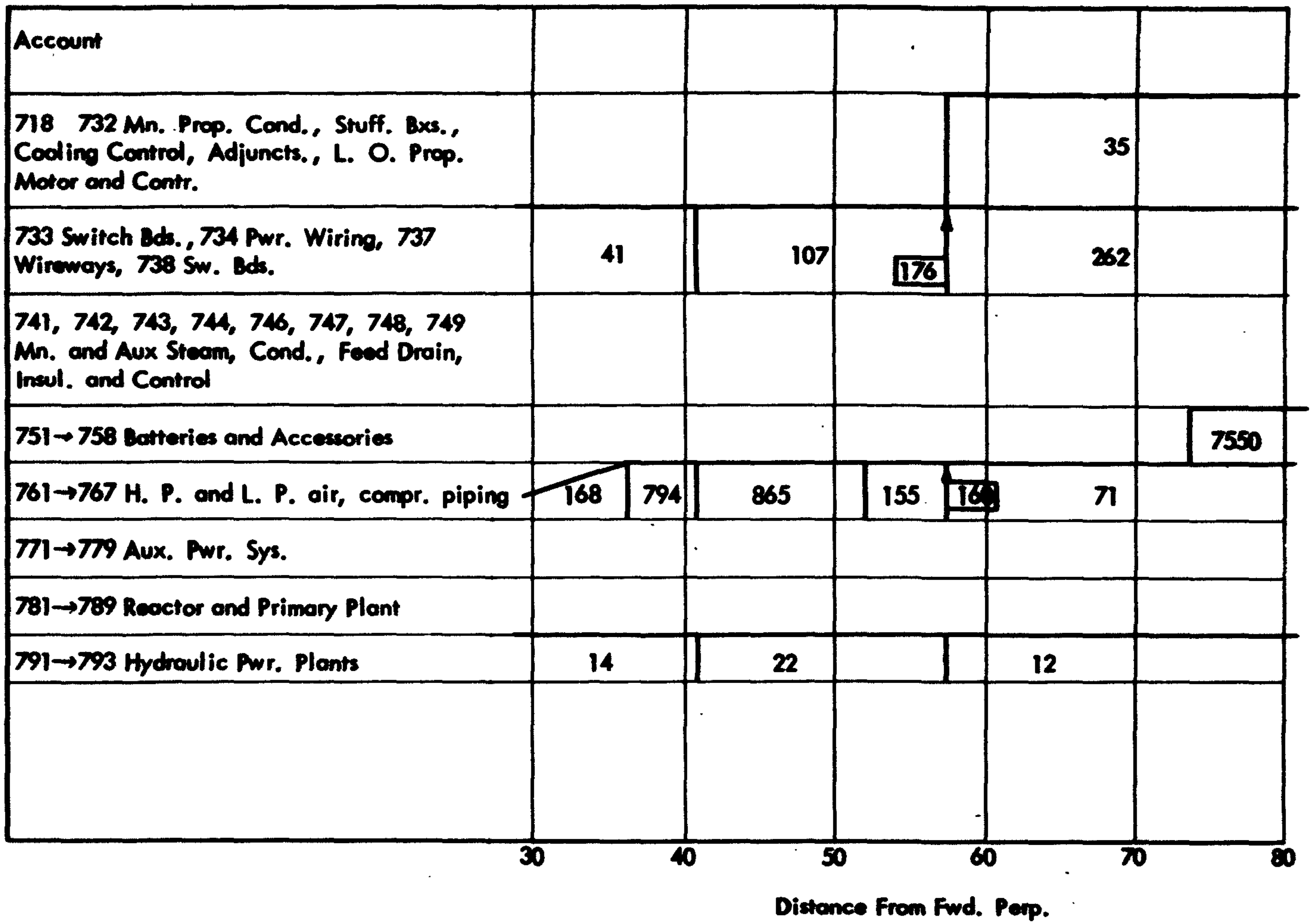


Figure A-1 (continued)

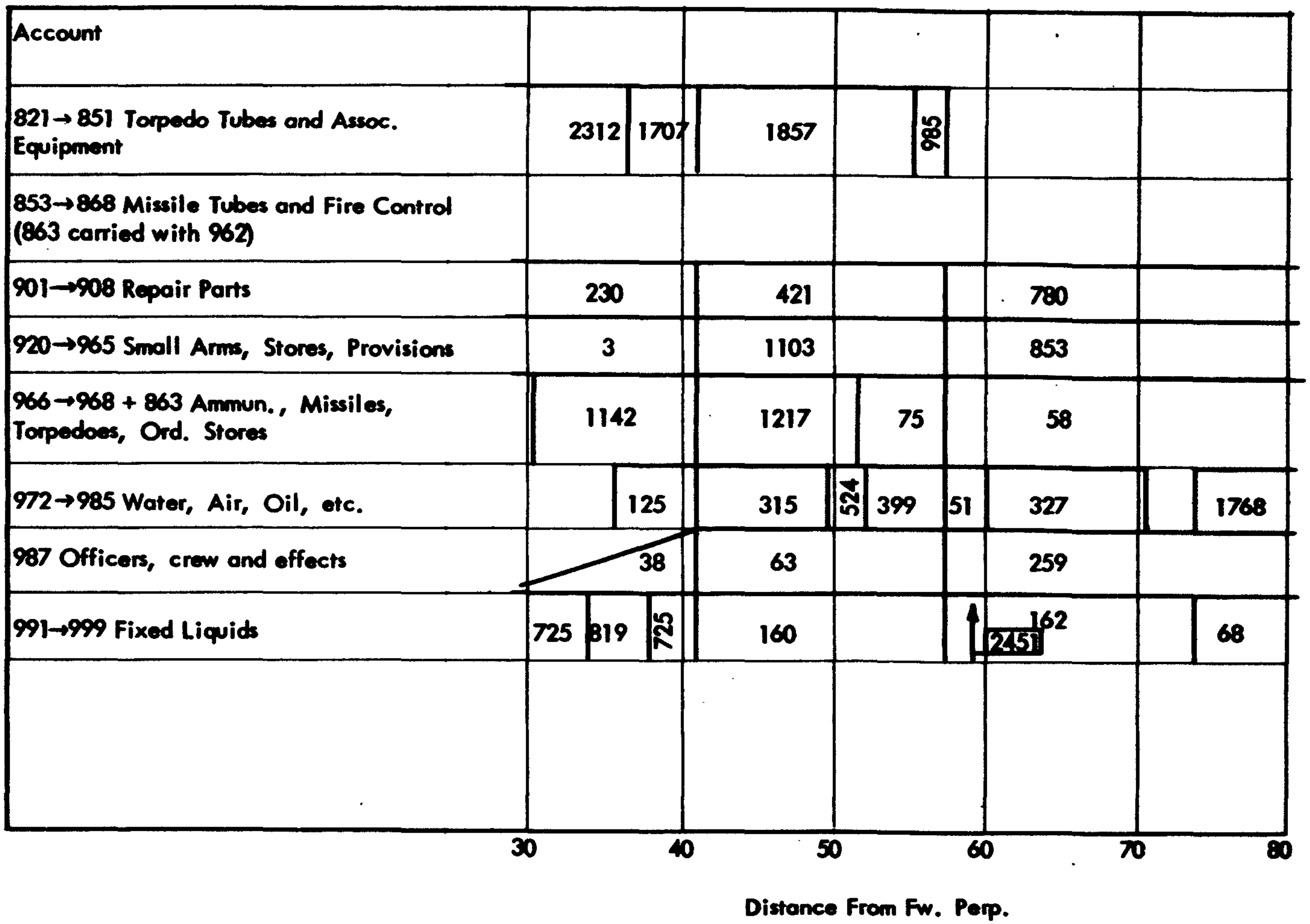


Figure A-1 (continued)

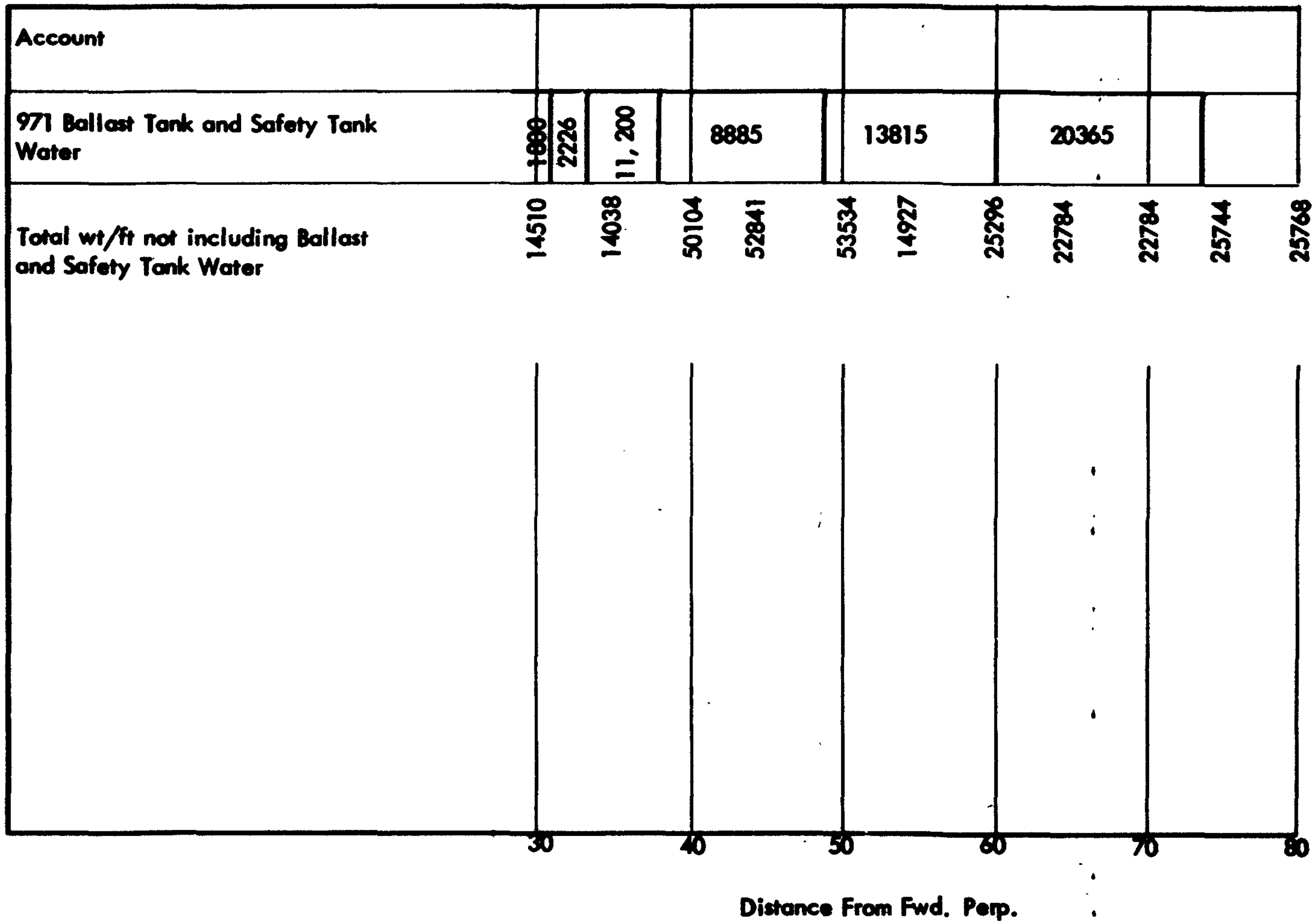


Figure A-1 (continued)

TABLE A-1 - SSB(N) 598 - WEIGHTS LOCATED BETWEEN 30' and 80' FROM THE BOW THAT ARE TREATED AS CONCENTRATED WEIGHTS

Distance from Bow ft.	Weight Account Number	Weight lbs		Remarks
30	682→696	1708		
30.5	541→542	9015		Fws. Escape Trunk
33	344	5139		
37	344	2158		
36	611→619	1348		
36.5	421	3358		
41.6	417	791		
42	344	1152		
42	353	318	5241	
42	421	1297		
42	611→619	2474		
42.5	433	223		
44.8	417	244		
50	421	25		
50.5	611→619	1348		
53.4	537	159	364	
53.4	682→696	205		
54	543→545	155		
54	641→644	520	12,175	
54	648→649	11,500		Oxy. Tanks and Piping
56	344	83		
56	422	151		
56	627→634	522	3693	
56	451→462	2937		
56.5	541→542	768	923	
56.5	543→544	155		

TABLE A-1 - (continued)

Distance from Bow ft.	Weight Account Number	Weight lbs		Remarks
57.5	415	671	1007	
57.5	733→738	176		
57.5	761→767	160		
58.6	331	54, 332		Trans. Bkhd.
59	991→999	2457		
61	641→644	1984		
74	561	12, 940		Lead Ballast
74.5	353	1383		
75	421	936		

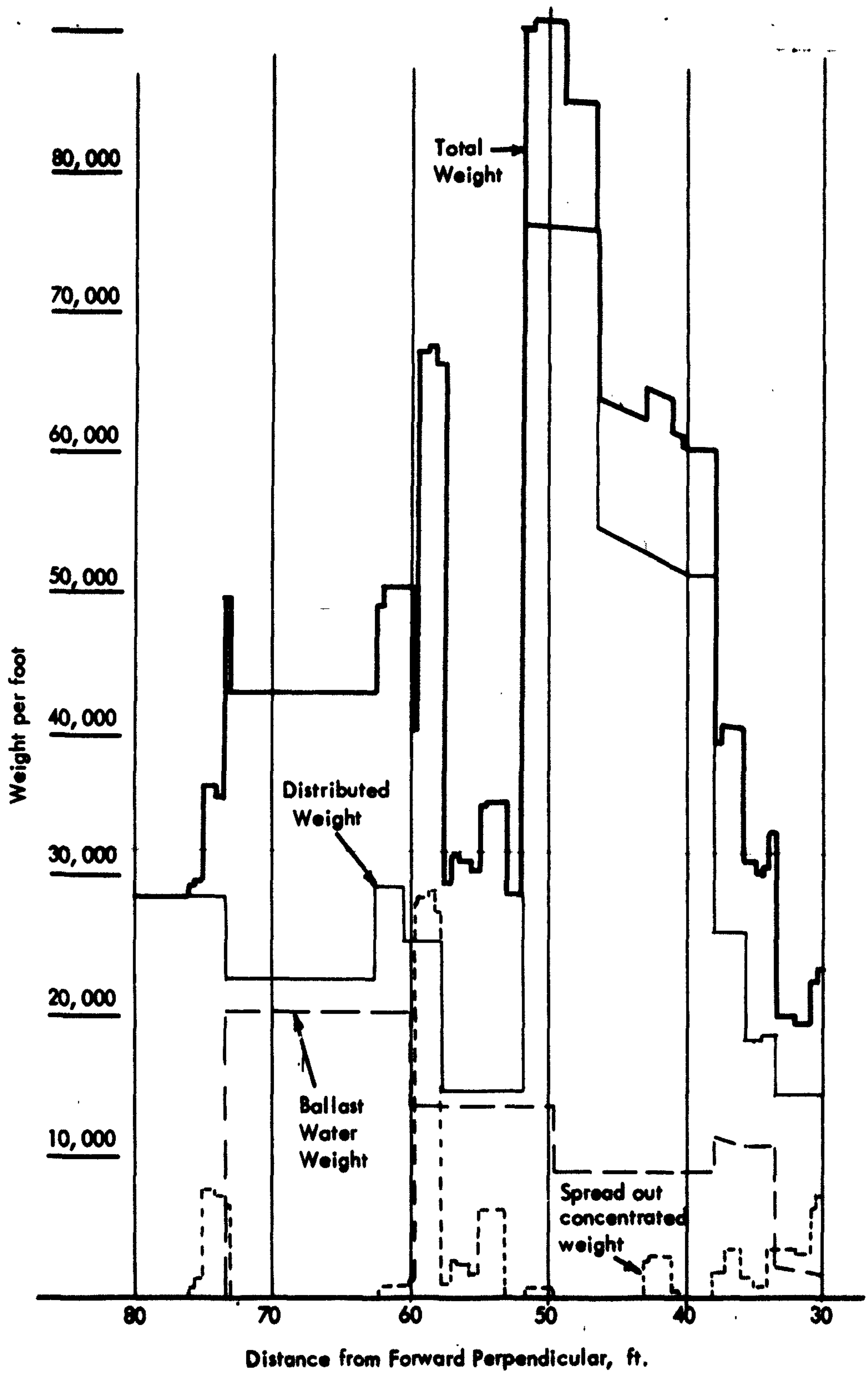


Figure A-2 - Section of Weight Distribution Curve

slightly and the resulting curve used to represent the weight distribution on the ship. This curve is given in Figure 3. For a check this curve is integrated and, with captive water deducted, compared with the submerged displacement of the ship.

With the general process explained, we will now discuss some of the details of the process of distributing the weights along the hull. For some of the main categories of hull weight it is necessary to compute the weight distribution along the hull. This is not particularly laborious in the case of the inner and outer hull shell plating, since the areas must be computed in the stiffness determination. Lead ballast is a very significant item of weight and can be concentrated within a small length. It is therefore well to define this weight closely from the plans. Also the water ballast is a very significant weight item. Normally, this is easily defined from the weight in a particular tank but in the case of the GEORGE WASHINGTON it was necessary to study the distribution of weight in the forward ballast tanks. Items like watertight flats are usually broken down into separate compartments and can be distributed quite easily by brief reference to the plans to ascertain the location, extent and shape. The framing is broken down in the Detailed Weight Summary into groups of four or five frames and the weights of these are quite easily distributed over their longitudinal span.

The distribution of the weights under some classifications such as furniture, wiring, and piping are more difficult to make. Where it is clear from the plan title that the weights represented by the plan fall within a certain span of length, they can be distributed either uniformly or by some simple distribution over the length of the span. For convenience in determining weight distribution for such weight categories as interior painting, electrical wiring, telephone wiring, etc., the interior of the hull was considered to be divided in ten sections as indicated in Table A-2. Using these sections it is possible to estimate a weight factor that will indicate what proportion of the weight will fall within each section. The application of this process to the interior paint, account 261, which weighs 28, 207 pounds is illustrated in Table A-3. The application of the process to the distribution of the weight of power wiring through the ship (weight about 45, 000 lbs) is illustrated in Table A-4. The

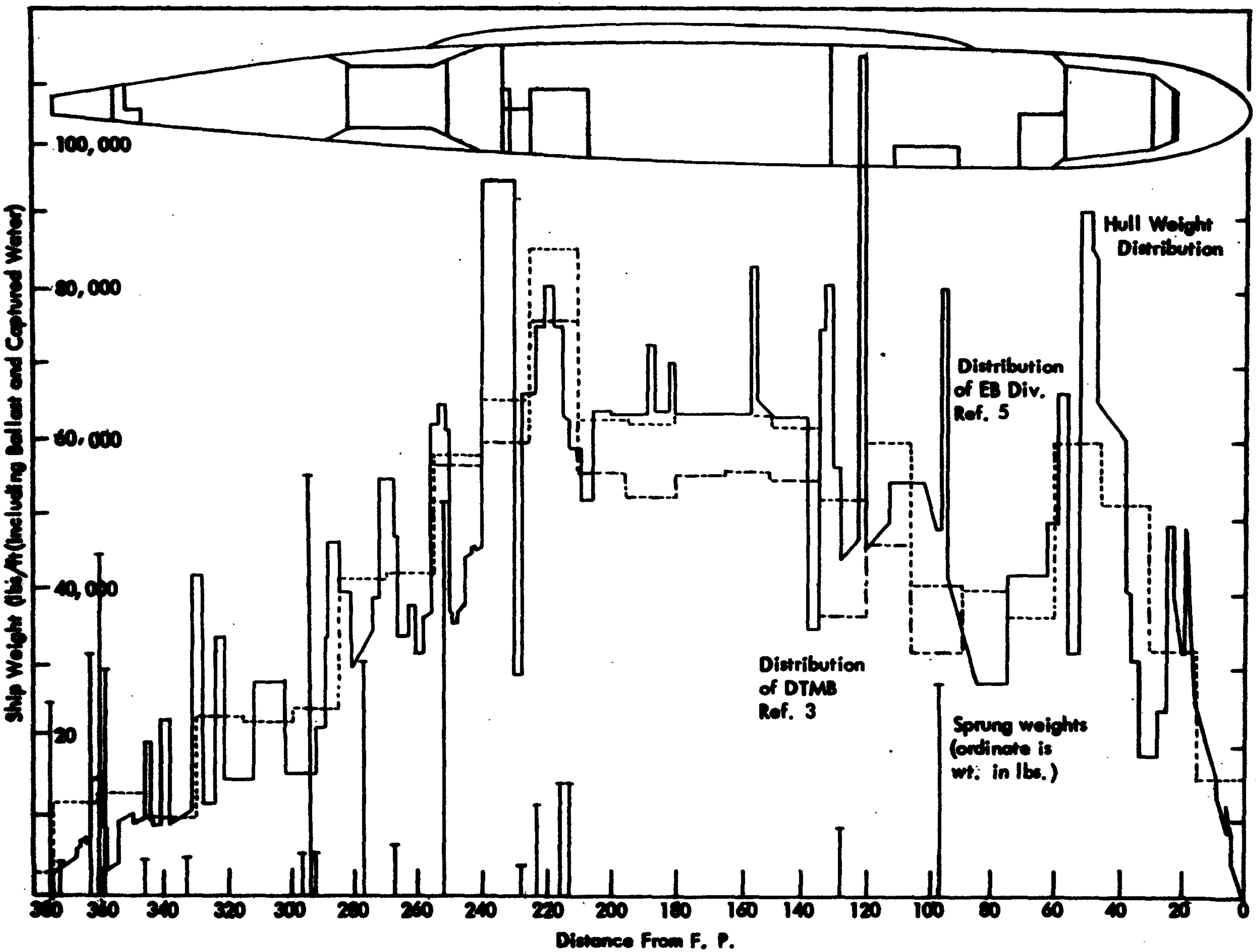


Figure 3 - Submerged Hull Weight SSB(N) 598 GEORGE WASHINGTON

TABLE A-2 - HULL DIVISIONS FOR ESTIMATING WEIGHT DISTRIBUTION

Section	Compartment	Fr. Nos.	Distances from F. P.	Length
0	Bow	0 → 13	0 → 25.25	25.25
1	Torpedo Rm. Fwd.	→ 20	→ 40.83	15.58
2	Torpedo Rm. Aft.	→ 26	→ 57.50	16.67
3	Living Qtrs.	→ M3	→ 92.75	35.25
4	C.O.C.	→ M17	→ 132.92	40.17
5	Missile Rm.	→ M44	→ 208.92	76.0
6	Missile Compt. Aft.	→ 44	→ 233.67	24.75
7	Reactor Compt.	→ 52	→ 253.67	20.00
8	Aux. Mach. Space	→ 64	→ 285.67	32.0
9	Engine Rm. Fwd.	→ 77	→ 320.17	34.50
10	Engine Rm. Aft.	→ 92	→ 353.67	33.50
11	Stern	→ 102	→ 373.50	19.83

TABLE A-3 - DISTRIBUTION OF WEIGHT OF INTERIOR POINT

Compartment	Length	Weighting Factor	Product	wt/ft
1	15.6	0.7	10.9	34
2	16.7	1	16.7	48
3	35	3	105	145
4	40	3	120	145
5	76	2	152	97
6	25	1	25	48
7	20	1	20	48
8	32	1.5	48	73
9	34	1.5	51	73
10	34	1	34	48
			583	

$$wt/ft = \frac{28,207}{583} \times \frac{product}{length} = 48.4 \quad \text{weighting factor}$$

TABLE A-4 - DISTRIBUTION OF WEIGHT OF POWER WIRING

Assume:

a	1 cable	from	compartment 2 to compartment 9
b	1 cable	from	compartment 3 to compartment 9
c	2 cables	from	compartment 4 to compartment 9
d	1 cable	from	compartment 5 to compartment 9
e	2 cables	from	compartment 6 to compartment 9
f	1 cable	from	compartment 7 to compartment 9
g	2 cables	from	compartment 8 to compartment 9
h	3 cables	in	compartment 9
i	3 cables	in	compartment 10 to compartment 9

wt/ft	compartment	a	b	c	d	e	f	g	h	i	Total
25	2	17									17
50	3	35	35								70
99	4	40	40	80							160
124	5	76	76	152	76						380
148	6	25	25	50	25	50					175
198	7	20	20	40	20	40	20				160
248	8	32	32	64	32	64	32	64			320
320	9	34	34	68	34	68	34	68	102	100	440
73	10									100	100
											1822

$$\text{wt/ft in compartment} = \frac{\text{Total}}{\text{Grand Total}} \times \frac{45,000}{\text{Length of compartment}}$$

application to other categories such as the announcing system, the telephone system, etc., can be understood.

Equipment weights which exceed 2000 pounds and are shock mounted are treated as sprung masses (See Appendix G). The fairwater planes, the stabilizers and the stern planes are especially studied as elastic structures in vertical vibration but are considered as rigid masses in the axial direction. The rudders are considered to be rigidly connected in both vertical and longitudinal vibration. Properly, the stabilizers and control surfaces should have been incorporated in the calculation program as elastic sub-systems but since the program was not developed when the hull was defined, this was not done.

The "captured" water, water in the free flooding sections of the ship, is treated as part of the mass of the ship. Consider, for example, the water between the superstructure and the hull. Beneath the superstructure the water has a relatively free flowing space. However, the air vent in the superstructure deck is only a very small percentage (less than 5%) of the superstructure deck area. Since it normally passes air this area is adequate for venting but water flowing through it would have to pass at more than 20 times the mean velocity in the tank. As a consequence for any vibration when the submarine is under water the water cannot flow through the superstructure vents in sufficient quantity to allow any significant free flow and the water is indeed "captured" and forced to move with the ship.

APPENDIX B

VIRTUAL INERTIA OF ENTRAINED WATER IN LONGITUDINAL MODES

1. Determination of an Ellipsoid to Represent the Hull

The distribution of the inertia of the entrained water on the SSB(N) 598 is assumed to be the same as that of the ellipsoid of revolution which most closely fits the hull. The first step in determining the inertia of the entrained water therefore is to find the dimensions of the ellipsoid which most closely matches the hull. The process that is used is the "Method of Least Squares" and is described in Reference 8.

Although the axes of the portions of the submarine that are surfaces of revolution are offset in the vertical direction and some of the cross-sections are not circular, it was assumed that the prolate ellipsoid which matches the breadth of the submarine most closely would adequately represent the submarine. The origin of the axes of coordinates was taken at the centerline at the bow and the x direction was taken along the centerline of the bow and stern sections. The equation of the ellipse that rotates into the ellipsoid is $\frac{(x-b)^2}{a^2} + \frac{y^2}{c^2} = 1$ where a, b and c are to be determined by the method of least squares to give the ellipsoid that most closely approximates the submarine.

Assume for a first approximation that a = b = 190' and c = 16.5'.

From the equation of the ellipse

$$y = c\left(1 - \frac{(x-b)^2}{a^2}\right)^{1/2}$$

Expand this value of y in a Taylor's series.

$$y = y_{a,b,c} + \frac{\partial y}{\partial a}_{a,b,c} \Delta a + \frac{\partial y}{\partial b}_{a,b,c} \Delta b + \frac{\partial y}{\partial c}_{a,b,c} \Delta c + \cdots$$

where:

$$\frac{\partial y}{\partial a} = c \left[1 - \frac{(x-b)^2}{a^2} \right]^{-1/2} \frac{(x-b)^2}{a^3}$$

$$\frac{\partial y}{ab} = c \left[1 - \frac{(x-b)^2}{a^2} \right]^{-1/2} \frac{(x-b)}{a^2}$$

$$\frac{\partial y}{\partial c} = \left[1 - \frac{(x-b)^2}{a^2} \right]^{1/2}$$

Letting s equal the sum of the squares of the differences, v_i, between the actual value of y, $\tilde{y}_i$, and the value computed by the equation for y.

$$s = \sum^{n} v_i^{\,2} = \sum^{n} (y - \tilde{y}_i)^2$$

$$= \sum^{n} \left\{ c\left[1 - \frac{(x-b)^2}{a^2} \right]^{1/2} + c \left[1 - \frac{(x-b)^2}{a^2} \right]^{-1/2} \frac{(x-b)^2}{a^3} \Delta a \right.$$

$$\left. + c \left[1 - \frac{(x-b)^2}{a^2} \right]^{-1/2} \frac{(x-b)}{a^2} \Delta b + \left[1 - \frac{(x-b)^2}{a^2} \right]^{1/2} \Delta c - \tilde{y}_i \right\}^2$$

The values of Δa, Δb and Δc which will minimize s are found by setting

$$\frac{\partial s}{\partial(\Delta a)} = \frac{\partial s}{\partial(\Delta b)} = \frac{\partial s}{\partial(\Delta c)} = 0 .$$

This yields the normal equations:

$$2\sum^{n} v_i\, c \left[1 - \frac{(x-b)^2}{a^2}\right]^{-1/2} \frac{(x-b)^2}{a^3} = 0 \qquad B(1)$$

$$2\sum^{n} v_i\, c \left[1 - \frac{(x-b)^2}{a^2}\right]^{-1/2} \frac{(x-b)}{a^2} = 0 \qquad B(2)$$

$$2\sum^{n} v_i \left[1 - \frac{(x-b)^2}{a^2}\right]^{1/2} = 0 \qquad B(3)$$

After substituting the value of v_i, these equations reduce to:

$$c\sum^{n} (x-b)^2 + \frac{c}{a^3}\,\Delta a \sum^{n} \left[1 - \frac{(x-b)^2}{a^2}\right]^{-1} (x-b)^4$$

$$+ \frac{c}{a^2}\,\Delta b \sum^{n} \left[1 - \frac{(x-b)^2}{a^2}\right]^{-1} (x-b)^3 + \Delta c \sum^{n} (x-b)^2$$

$$- \sum^{n} y_i \left[1 - \frac{(x-b)^2}{a^2}\right]^{-1/2} (x-b)^2 = 0 \qquad B(1a)$$

$$c\sum^{n} (x-b) + \frac{c}{a^3}\,\Delta a \sum^{n} \left[1 - \frac{(x-b)^2}{a^2}\right]^{-1} (x-b)^3$$

$$+ \frac{c}{a^2}\,\Delta b \sum^{n} \left[1 - \frac{(x-b)^2}{a^2}\right]^{-1} (x-b)^2 + \Delta c \sum^{n} (x-b)$$

$$- \sum^{n} y_i \left[1 - \frac{(x-b)^2}{a^2}\right]^{-1/2} (x-b) = 0 \qquad \text{B(2a)}$$

$$c \sum^{n} \left[1 - \frac{(x-b)^2}{a^2}\right] + \frac{c}{a^3} \; \Delta a \sum^{n} (x-b)^2$$

$$+ \frac{c}{a^2} \; \Delta b \sum^{n} (x-b) \; + \; \Delta c \sum^{n} \left[1 - \frac{(x-b)^2}{a^2}\right]$$

$$- \sum^{n} y_i \left[1 - \frac{(x-b)^2}{a^2}\right]^{1/2} = 0 \qquad \text{B(3a)}$$

To find the values of Δa, Δb and Δc, tables are formed for the quantities

$$\sum^{n} (x-b), \; \sum^{n} (x-b)^2$$

$$\sum^{n} \left[1 - \frac{(x-b)^2}{a^2}\right]$$

$$\sum^{n} \left[1 - \frac{(x-b)^2}{a^2}\right]^{-1} (x-b)^2, \; \sum^{n} \left[1 - \frac{(x-b)^2}{a^2}\right]^{-1} (x-b)^3, \; \sum^{n} \left[1 - \frac{(x-b)^2}{a^2}\right]^{-1} (x-b)^4$$

$$\sum^{n} y_i \left[1 - \frac{(x-b)^2}{a^2}\right]^{-1/2}, \; \sum^{n} y_i \left[1 - \frac{(x-b)^2}{a^2}\right]^{-1/2} (x-b)$$

$$\sum^{n} y_i \left[1 - \frac{(x-b)^2}{a^2}\right]^{-1/2} (x-b)^2$$

Values of y_i are picked from the graph of the half breadths at the 15' – 9-7/8" water line as plotted in Figure B-1 for six values of x.

The results are given in Table B-1.

Substituting the values from Table B-1 into the equations B(1a), B(2a) and B(3a) yields the following:

$$17,380\ \Delta a - 5690\ \Delta b + 88,700\ \Delta c = 146,000 \qquad B(1b)$$
$$0.2138\ \Delta a - 0.00456\ \Delta b + 3.543\ \Delta c = 2.0 \qquad B(2b)$$
$$29.95\ \Delta a - 132\ \Delta b + 10\ \Delta c = 2152 \qquad B(3b)$$

From a simultaneous solution of these:

$\Delta a = 0.517'$	a improved =	190'	+ 0.517"	=	190.52'
$\Delta b = -16.12'$	b improved =	190'	- 16.12'	=	173.88'
$\Delta c = 0.515'$	c improved =	16.50'	+ 0.515'	=	17.015'

Therefore the ellipsoid that most closely matches the submarine is:

$$\frac{(x-173.88)^2}{(190.52)^2} + \frac{(y^2+z^2)}{(17.015)^2} = 1$$

The curve of this quantity is plotted in Figures B-1 and B-2. It is obvious that although this ellipsoid matches the ships hull as closely as is possible by any single ellipsoid, there is still a considerable discrepancy at the ends.

Because of the complexity of dealing with a closer simulation of the ship, the simple ellipsoid has been considered to be an adequate representation at the present time. The methods of Reference 9 would have to be developed further before they would be applicable to the problem of a shape differing from an ellipsoid.

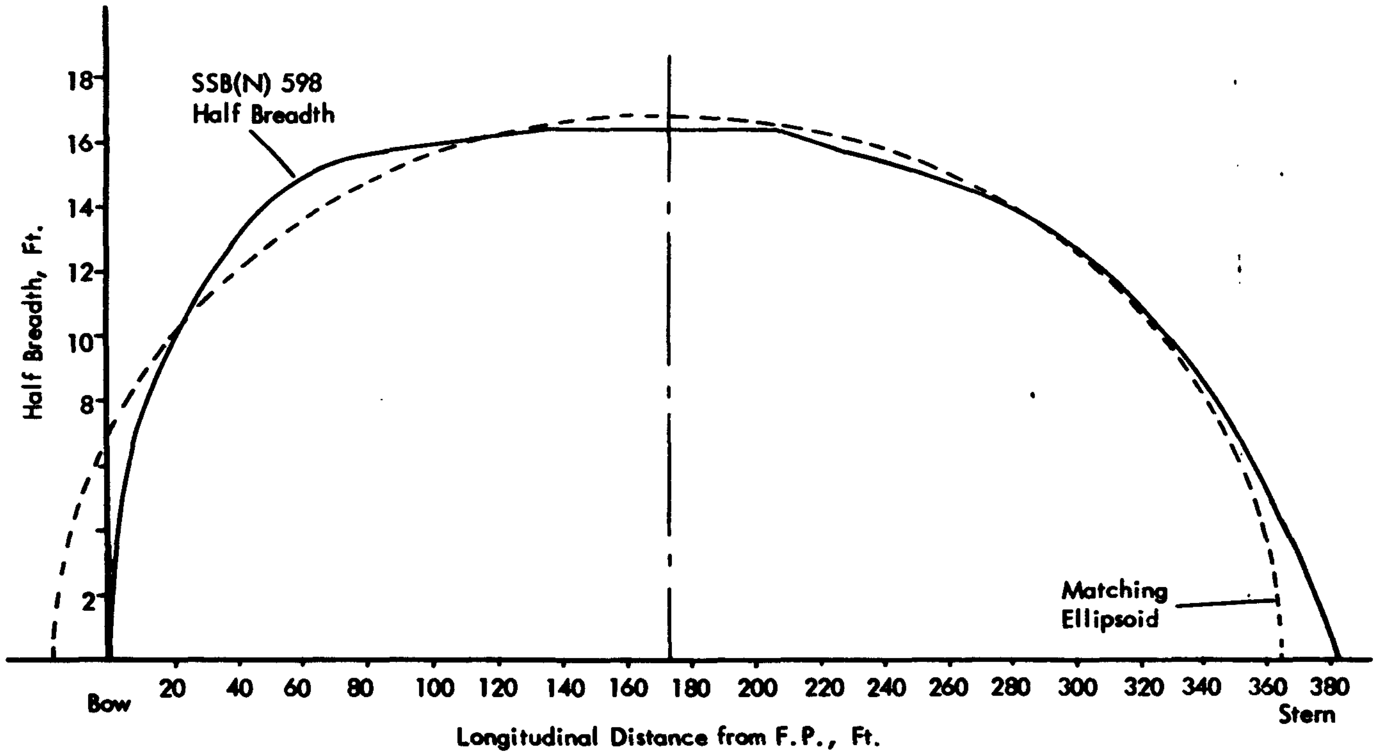


Figure B-1 - Half Breadth of SSB(N) 598 at 15' - 9-7/8" Water Line and Radius of Matching Ellipsoid

TABLE B-1 - MEAN SQUARE DETERMINATION OF ELLIPSOID

x	y_1	(x-b)	$(x-b)^2$	$\left[1-\frac{(x-b)^2}{a^2}\right]$	$\left[\;\right]^{-1}(x-b)^2$	$\left[\;\right]^{-1}(x-b)^3$	$\left[\;\right]^{-1}(x-b)^4$
20	10.0	-170	28,900	0.20	144,500	-24.6×10^6	$4,180\times10^6$
60	15.0	-130	16,900	0.531	31,780	-4.13×10^6	537×10^6
150	16.5	-40	1,600	0.956	1,670	-0.066×10^6	2.7×10^6
250	15.4	+60	3,600	0.902	3,990	$+.244\times10^6$	14.4×10^6
300	12.9	+110	12,100	0.664	18,200	2.00×10^6	220×10^6
350	7.1	+160	25,600	0.290	88,400	14.12×10^6	$2,260\times10^6$
		-10	88,700	3.543	288,540	-12.43×10^6	$7,214\times10^6$

x	y_1	$\left[1-\frac{(x-b)^2}{a^2}\right]^{1/2}$	$y_1\left[\;\right]^{1/2}$	$y_1\left[\;\right]^{-1/2}(x-b)$	$y_1\left[\;\right]^{-1/2}(x-b)^2$
20	10.0	.447	4.47	-3800	646,000
60	15.0	.730	10.93	-2670	347,000
150	16.5	.978	16.15	-675	27,000
250	15.4	.950	14.62	+973	58,300
300	12.9	.815	10.51	1,740	191,300
350	7.1	.538	3.82	2,115	338,000
			60.50	-2,317	1,607,600

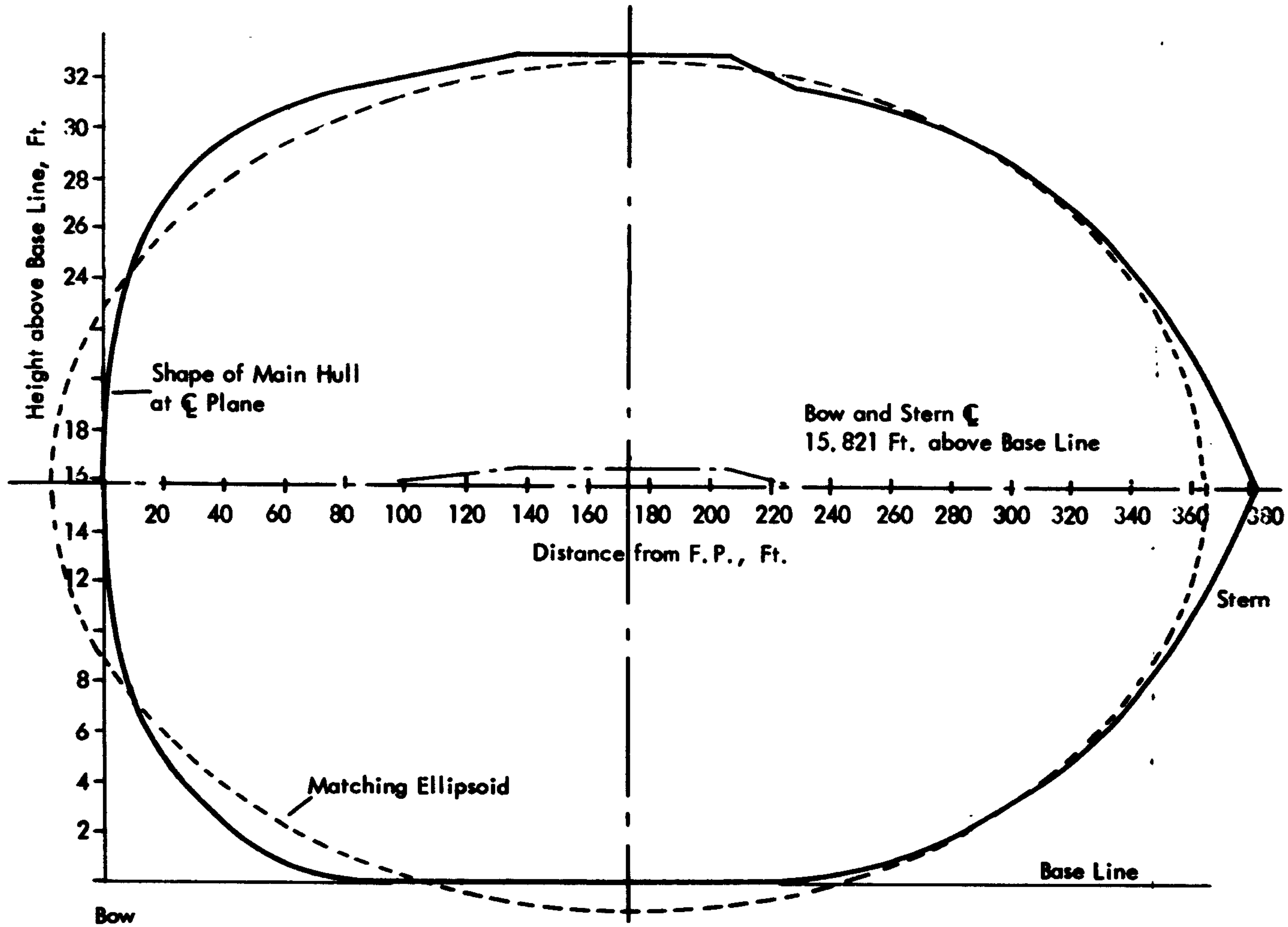


Figure B-2 - Match of Ellipsoid with Vertical Projection of Main Hull

2. Analytical Representation of the Vibration Pattern

It is also necessary in the considerations of equivalent mass to work with a pattern of vibration. For this purpose the nodal pattern at the natural frequency of 10.40 cps, as computed for the mass and stiffness reported in Reference 3 was used. This nodal pattern is plotted in Figure B-3. For convenience in the computations a deflection curve of the form $\delta = a_1\,(173.88 - x) + a_3\,(173.88 - x)^3$

$$= a_1 x' + a_3 x'^3$$

was assumed and the values of a_1 and a_3 to match the experimental curve determined by the method of Least Squares. The resulting analytical expression for the deflection is:

$$\delta = 4.11 \times 10^{-6} - 11.08 \times 10^{-12}\,(x')^3$$

This is plotted on Figure B-3 to indicate the adequacy of the match.

3. The Distribution of the Entrained Water Inertia on the Ellipsoid

We are now concerned with the water inertia of an ellipsoid of

major axis, $a_1 = 190.52'$

minor axes, b_1 and $c_1 = 17.015'$

vibrating in a longitudinal mode such that $\delta = 4.11 \times 10^{-6}\,x - 11.08 \times 10^{-12}\,x^3$

where x is measured from the center of the ellipsoid.

The velocity in the longitudinal direction is $= j\omega\delta$

Express this as:

$$u_e = Ax - Bx^3$$

Because of the poisson ratio effects there will be a radial velocity to the hull associated with the longitudinal velocity.

The longitudinal strain $= \dfrac{\partial\delta}{\partial x}$

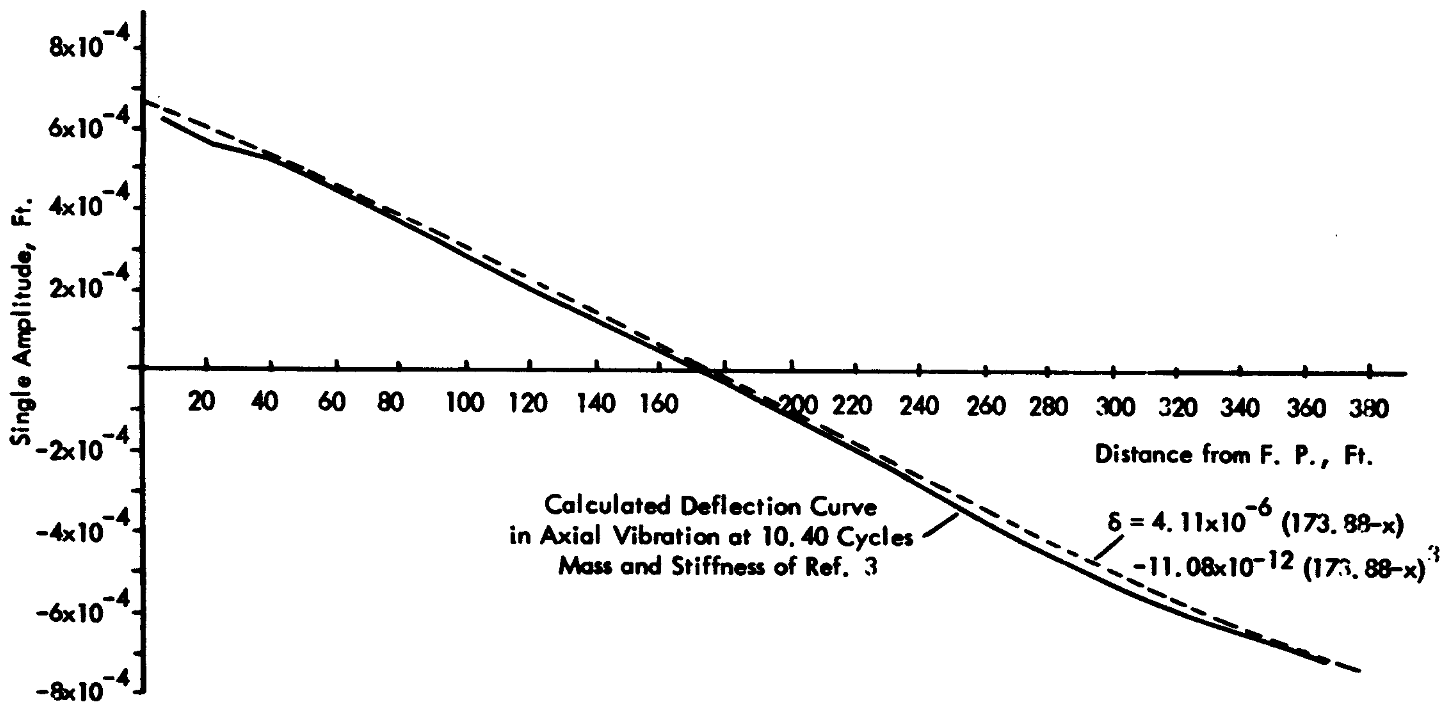


Figure B-3 - Representation of Normal Mode Deflection by a Functional Relationship

Corresponding to this longitudinal strain there will be a poisson strain $\sigma \frac{\partial \delta}{\partial x}$, where σ is a ratio that depends upon the poisson ratio of the material of the submarine and the ratio of frame area to shell area. Hence the poisson velocity $= y$ (or z) $\sigma \frac{\partial \delta}{\partial x}$ and

$$v_e = \sigma y (A - 3 Bx^2)$$

$$w_e = \sigma z (A - 3 Bx^2)$$

The methods used for determining the water inertia were originally developed by Lamb[10] and are well presented in Reference 11. The methods involve the expression of the properties of the ellipsoid and the vibration in terms of ellipsoidal coordinates. In this manner a potential function can be obtained. In most works with virtual inertia it is possible through energy considerations to obtain an equivalent mass for the water vibrating with the ellipsoid. However, in this case the effects of the entrained water are distributed over the full length of the ellipsoid and in order to find the effects of this entrained water at any section of the hull it is necessary to compute at each longitudinal location the component of the force associated with the dynamic pressures between the hull and the water that result from the modal pattern. This force, when related to the amplitude of vibration at this location, can be used to determine the equivalent mass of the water at this location.

The discussion of the geometry and the kinematics of a vibrating ellipsoid given in Reference 11 is directly applicable to this problem and therefore will not be repeated. Because the concern of this study is axial vibration rather than bending vibrations the applications of the general theory differ.

Express the velocities in ellipsoidal coordinates

$$u_e = Ax - Bx^3$$

$$= Ak\mu\zeta_o - B (k\mu\zeta_o)^3$$

$$v_e = \sigma y\,(A - 3\,Bx^2)$$

$$= \sigma k\,(1-u^2)^{1/2}\,(\zeta_o^{\,2}-1)^{1/2}\,\cos\theta\left[A - 3B\,(ku\zeta_o)^2\right]$$

$$w_e = \sigma k\,(1-u^2)^{1/2}\,(\zeta_o^{\,2}-1)^{1/2}\,\sin\theta\left[A - 3B\,(ku\zeta_o)^2\right]$$

Since these velocities are those in the surface of the ellipsoid this indicated by ζ_o, the ellipsoid representing the hull.

$$F(\mu,\theta) = \mu u_e + (1-\mu^2)^{1/2}\,\zeta_o\,(\zeta_o^{\,2}-1)^{-1/2}\,(v_e\cos\theta + w_e\sin\theta)$$

$$= \mu\left[Ak\mu\zeta_o - B\,(k\mu\zeta_o)^3\right]$$

$$+ (1-\mu^2)^{1/2}\,\zeta_o\,\sigma k\,(1-\mu^2)^{1/2}\left[A - 3B\,(k\mu\zeta_o)^2\right]$$

$$= \mu\left[Ak\mu\zeta_o - B\,(k\mu\zeta_o)^3\right] + \sigma k\zeta_o\,(1-\mu^2)\left[A - 3B\,(k\mu\zeta_o)^2\right]$$

$$= A\left[k\mu^2\zeta_o + \sigma k\zeta_o\,(1-\mu^2)\right] - B\left[(ku\zeta_o)^3 + 3\,\sigma k\zeta_o\,(1-\mu^2)\,k\mu\zeta_o^{\,2}\right]$$

$$= A\left[ku^2\zeta_o\,(1-\sigma) + \sigma k\zeta_o\right] - B\left[(k\mu\zeta_o)^3\,\mu\,(1-3\sigma) + \frac{3}{\mu}\,\sigma\,(k\mu\zeta_o)^3\right]$$

Applying the condition that $k\,F(\mu,\theta) = \frac{\partial\phi}{\partial\zeta}$, to the general solution for velocity potential in ellipsoidal coordinates, i.e.,

$$\phi = \sum_{n=0}\sum_{s=0}\left(B_n^{\,s}\sin s\theta + C_n^{\,s}\cos s\theta\right)P_n^s(\mu)\,Q_n^s(\zeta)$$

leads to

$$\sum_{n=0} \sum_{s=0}^{n} (B_n^s \sin s\theta + C_n^s \cos s\theta) P_n^s(\mu) \dot{Q}_n^s(\zeta_o) = kF(\mu, \theta)$$

Since the F (μ, θ) is independent of θ this condition is only met where $s = 0$
Therefore B_n^s falls out of the solution

C_n^s becomes C_n

P_n^s becomes P_n

$\dot{Q}_n^s$ becomes $\dot{Q}_n$ and the equation becomes,

$$\sum_{n=0}^{\infty} C_n P_n(\mu) \dot{Q}_n(\zeta_o) = kF(\mu, \theta)$$

Hence

$$C_n = \frac{2n+1}{4\pi \dot{Q}_n(\zeta_o)} \quad k \times 2\pi \int_{-1}^{1} F(\mu, \theta) P_n(\mu) d\mu$$

$$= \frac{2n+1}{2\dot{Q}_n(\zeta_o)} \; k \Big[Ak\zeta_o (1-\sigma) \int_{-1}^{1} \mu^2 P_n(\mu) du$$

$$+ A\sigma k\zeta_o \int_{-1}^{1} P_n(\mu) d\mu - Bk^3 \zeta_o^3 (1-3\sigma) \int_{-1}^{1} \mu^4 P_n(\mu) d\mu$$

$$-3B\sigma k^3 \zeta_o^3 \int_{-1}^{1} \mu^2 P_n(\mu) \cdot d\mu \Big]$$

And using Table 1 of Reference 11

$$C_o = \frac{1}{2\dot{Q}_o(\zeta_o)} \quad k\left\{\frac{Ak\zeta_o}{3}\,(2+4\sigma) - Bk^3\zeta_o{}^3\left(\frac{2}{5}+\frac{4}{5}\sigma\right)\right\}$$

$$C_1 = 0$$

$$C_2 = \frac{5}{2\dot{Q}_2(\zeta_o)} \quad k\left\{\frac{4}{15}\,Ak\zeta_o\,(1-\sigma) - Bk^3\zeta_o{}^3\left(\frac{8}{35}+\frac{4}{35}\sigma\right)\right\}$$

$$C_3 = 0$$

$$C_4 = \frac{9}{2\dot{Q}_4(\zeta_o)} \quad k\left\{-Bk^3\zeta_o{}^3\,(1-3\sigma)\,\frac{16}{315}\right\}$$

$$C_n = 0, \; n \geq 5$$

Therefore:

$$\phi = \frac{1}{\dot{Q}_o(\zeta_o)} \quad k\,(1+2\sigma)\left[\frac{Ak\zeta_o}{3} - \frac{Bk^3\zeta_o{}^3}{5}\right] P_o(\mu)\,Q_o(\zeta)$$

$$+ \frac{1}{\dot{Q}_2(\zeta_o)} \quad k\left[\frac{2}{3}\,Ak\zeta_o\,(1-\sigma) - \frac{2}{7}\,Bk^3\zeta_o{}^3\,(2+\sigma)\right] P_2(\mu)\,Q_2(\zeta)$$

$$- \frac{8}{35}\,\frac{1}{\dot{Q}_4(\zeta_o)} \quad k\,Bk^3\zeta_o{}^3\,(1-3\sigma)\;P_4(\mu)\;Q_4(\zeta)$$

When the potential function of the fluid is determined, the usual practice for obtaining the mass of the entrained water is to compute the kinetic energy of the fluid and by relating this to the amplitude of motion to determine the equivalent mass. However, for this problem the distribution of the equivalent mass along the length of the hull is desired, and the amplitude of vibration is not a constant value but a function of the longitudinal position along the hull.

To avoid these difficulties the distribution of the equivalent mass is obtained by computing the pressure on the ellipsoid generated by the vibratory motion, multiplying this pressure by the projected area normal to the longitudinal axis for each unit of axial length. The equivalent mass per unit length at a given axial position is then the force per unit of length divided by the product of the amplitude of vibration at the location and the square of the circular frequency.

The pressure associated with a velocity potential function is:

$$p = \rho\left[\frac{d\phi}{dt} + \frac{q^2}{2} + f(t)\right]$$

In this relation q^2 is the square of the fluid velocity, which for the small motions is negligible. $f(t)$ is an external pressure that does not exist.

Thus for this problem

$$p = \rho\frac{d\phi}{dt} = \rho j\omega\phi$$

since the ellipsoid is vibrating in simple harmonic motion.

Since

$$a = k\zeta_o \qquad c = (\zeta_o^{\,2} - 1)^{1/2}$$

$$\zeta_o = \frac{a}{\sqrt{a^2 - c^2}} \qquad k = \sqrt{a^2 - c^2}$$

$$\frac{p}{j\omega\rho} = \frac{1}{\dot{Q}_o(\zeta_o)} \sqrt{a^2 - c^2}\,(1+2\sigma)\left[\frac{Aa}{3} - \frac{Ba^3}{5}\right] P_o(\mu)\, a_o(\zeta_o)$$

$$+ \frac{1}{\dot{Q}_2(\zeta_o)} \sqrt{a^2 - c^2}\left[\frac{2}{3}\, Aa\,(1-a) - \frac{2}{7}\, Ba^3 (2+\sigma)\right] P_2(\mu)\, Q_2(\zeta_o)$$

$$- \frac{8}{35}\, \frac{1}{\dot{Q}_4(\zeta_o)} \sqrt{a^2 - c^2}\; Ba^3\,(1-3\sigma)\, P_4(\mu)\, Q_4(\zeta_o)$$

The axial force $= p \times \pi \left[y^2 - \left(y + \frac{dy}{dx} \times 1\right)^2 \right]$

$$= - p \times \pi \times 2\, y \frac{dy}{dx} \times 1$$

Since for an ellipsoid

$$y \frac{dy}{dx} = - \frac{xc^2}{a^2}$$

The axial force $= p \; (2\pi \frac{c^2}{a^2} x)$ lb per ft.

The entrained inertia $= \dfrac{2\pi p \, \frac{c^2}{a^2} x}{\omega^2 \delta}$

The mathematical expression for the deflection curve which was derived earlier is :

$$\delta = a_1 x - a_3 x^3$$

or in terms of velocity coefficients:

$$\delta = \frac{A}{j\omega} x - \frac{B}{j\omega} x^3$$

The entrained inertia therefore =

$$- \frac{2\pi\rho \dfrac{c^2}{ax}}{A - Bx^2} \times \sqrt{a^2 - c^2} \left\{ \frac{(1 + 2\sigma)}{\dot{Q}_o(\zeta_o)} \left[\frac{A}{3} - \frac{Ba^2}{5} \right] P_o(\mu)\, Q_o(\zeta_o) \right.$$

$$+ \frac{1}{\dot{Q}_2(\zeta_o)} \left[\frac{2}{3} A (1 - \sigma) - \frac{2}{7} Ba^2 (2 + \sigma) \right] P_2(\mu)\, Q_2(\zeta_o)$$

$$\left. - \frac{8}{35} \frac{1}{\dot{Q}_4(\zeta_o)} \quad Ba^2 (1 - 3\sigma)\, P_4(\mu)\, Q_4(\zeta_o) \right\}$$

where:

$$\mu = \frac{x}{a}, \; Bx^2 = Ba^2\mu^2$$

For the submarine SSB(N) 598 $a = 190.52'$
$c = 17.015'$

$$\zeta_o^2 = \frac{(190.52)^2}{(190.52)^2 - (17.015)^2} = \frac{36,297.87}{36,008.36} = 1.00804008$$

$$\zeta_o^3 = 1.012084$$

$$\zeta_o = 1.004012 \qquad \zeta_o^4 = 1.0161448$$

$$Q_o(\zeta) = \frac{1}{2}\log\frac{\zeta+1}{\zeta-1}$$

$$Q_2(\zeta) = \frac{1}{2}(3\zeta^2 - 1)\ Q_o(\zeta) - 3/2\,\zeta$$

$$Q_4(\zeta) = \frac{1}{8}(35\zeta^4 - 30\zeta^2 + 3)\ Q_o(\zeta) - \frac{35}{8}\zeta^3 + \frac{55}{24}\zeta$$

$$\dot{Q}_o(\zeta) = \frac{1}{2}\ \frac{\zeta-1}{\zeta+1}\ \frac{(\zeta-1)1 - (\zeta+1)\ 1}{(\zeta-1)^2}$$

$$= -\frac{1}{\zeta^2 - 1}$$

$$\dot{Q}_2(\zeta) = \frac{1}{2}(6\zeta)\ Q_o(\zeta) + \frac{1}{2}(3\zeta^2 - 1)\left(\frac{-1}{\zeta^2-1}\right) - 3/2$$

$$= 3\zeta\ Q_o(\zeta) - \frac{3\zeta^2 - 2}{\zeta^2 - 1}$$

$$\dot{Q}_4(\zeta) = \frac{1}{8}(35 \times 4\zeta^3 - 60\zeta)\ Q_o(\zeta) - \frac{1}{8}\ \frac{(35\zeta^4 - 30\zeta^2 + 3)}{\zeta^2 - 1}$$

$$-\frac{105}{8}\zeta^2 + \frac{55}{24}$$

$$\dot{Q}_4(\zeta) = \left[\frac{35}{2}\zeta^3 - \frac{15}{2}\zeta\right]\ Q_o(\zeta) - \frac{1}{8}\left[35\zeta^4 - 30\zeta^2 + 3\right.$$

$$+ 105\zeta^4 - 105\zeta^2 - \frac{55}{3}\zeta^2 + \frac{55}{3}\Big] \frac{1}{\zeta^2 - 1}$$

$$= (\frac{35}{2}\zeta^3 - \frac{15}{2}\zeta)\, Q_o(\zeta) - \frac{1}{2}\,(35\zeta^4 - \frac{115\zeta^2}{3} + \frac{16}{3})\,\frac{}{\zeta^2 - 1}$$

$$= \frac{5}{2}\,(7\zeta^3 - 3\zeta)\;Q_o(\zeta) - \frac{1}{6}\,(105\zeta^4 - 115\zeta^2 + 16)\,\frac{1}{\zeta^2 - 1}$$

$$Q_o(\zeta_o) = \frac{1}{2}\log\frac{2.00401}{0.00401} = \frac{1}{2}\Big[0.6951 - (1.3893 - 6.9078)\Big]$$

$$= \frac{1}{2}\,(6.2136) = 3.1068$$

$$Q_2(\zeta_o) = \frac{1}{2}\,(3 \times 1.00804 - 1) \times 3.1068 - 3/2\,(1.00401)$$

$$= 1.6382$$

$$Q_4(\zeta_o) = \frac{1}{8}\Big[35 \times (1.00804)^2 - 30\,(1.00804) + 3\Big]\;3.1068 - \frac{35}{8}\,(1.00804)^{3/2} + \frac{55}{24}\zeta$$

$$\begin{aligned} &= 3.23257 \\ &- 4.42787 \quad = 1.10556 \\ &+ 2.30086 \end{aligned}$$

$$\dot{Q}_o(\zeta_o) = -\frac{1}{\zeta^2 - 1} = -\frac{1}{.00804008} = -124.377$$

$$\dot{Q}_2\,(\zeta_o) = 3 \times 1.004012 \times 3.1068 - 124.377\,(3\times 1.00804-2)$$
$$= 9.35768 - 127.377$$
$$= -118.019$$

$$\dot{Q}_4\,(\zeta_o) = \frac{5}{2}\,(7\times 1.012084 - 3 \times 1.004012)\,3.1068$$
$$- \frac{1}{6}\,(105 \times 1.0161448 - 115 \times 1.00804008 + 16)\,\frac{1}{.00804008}$$
$$= 31.6315 - 140.3509$$
$$= -108.7194$$

$$b^2 = (17.015)^2 = 289.51 \qquad \sqrt{a^2 - c^2} = \sqrt{36,008.36}$$
$$= 189.76$$

σ = approx. 0.20

The entrained inertia =

$$-\frac{2\pi\rho \times 289.51 \times 189.76}{190.52}\left\{ (1.40) \times \frac{3.1068}{-124.377}\left[\frac{A}{3} - \frac{Ba^2}{5}\right] P_o\,(\mu) \right.$$
$$+ \frac{1.6382}{-118.019}\left[\frac{2(0.8)}{3}\,A - \frac{2}{7}\,(2.2)\,Ba^2\right] P_2\,(\mu)$$
$$\left. - \frac{8}{35} \times \frac{1.10556}{-108.72}\,(1-0.6)\,Ba^2\,P_4\,(\mu)\,\frac{1}{A-Ba^2\mu^2}\right\}$$

$$\rho = 1.99 \frac{\text{lb sec}^2}{\text{ft}^4}$$

$$\frac{Ba^2}{A} = \frac{11.08 \times 10^{-12}}{4.11 \times 10^{-6}} \times (190.52)^2 = 0.0976 \text{ for 2nd mode of SSB(N) 598}$$

The entrained inertia then becomes:

$$3600 \left[0.01093\, P_o(\mu) + 0.00655\, P_2(\mu) - 0.0000976\, P_4(\mu) \right]$$

$$\frac{1}{1 - 0.0976\mu^2}$$

$$= 39.4 \left[P_o(\mu) + 0.599\, P_2(\mu) - 0.00892\, P_4(\mu) \right] \frac{1}{1 - 0.0976\mu^2}$$

The values of this entrained inertia are computed in Table B-II and plotted in Figure B-4.

4. The Distribution of Entrained Water Inertia on the Actual Hull

Since the shape of the hull differs from the ellipsoid it can be expected that the inertia of the entrained water will differ from that on the idealized ellipsoid. To investigate how much difference this might be, assume that the pressure generated on the hull at any position along the longitudinal axis is the same for the actual submarine as for the ellipsoid, When this is done the distribution of water inertia is quite different as is shown in Figure B-5. It will be noted that the blunt bow and the discontinuities at the superstructure tend to give large values of water inertia and the pointed stern tends to decrease

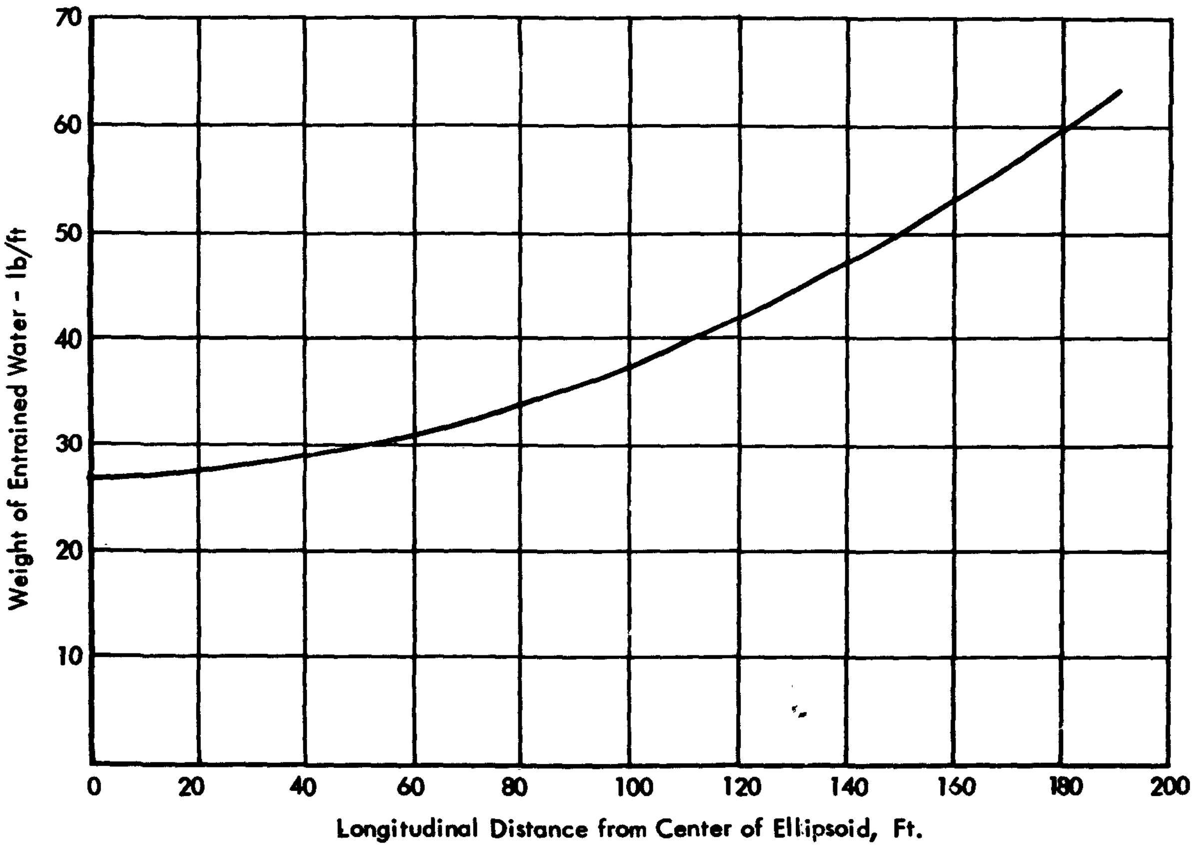


Figure B-4 - Water Inertia for an Ellipsoid representing SSB(N) 598

TABLE B-II - CALCULATION OF LOCAL VALUES OF ENTRAINED MASS

$\frac{y}{a} = \mu$	$P_2(\mu)$	$0.599\,P_2(\mu)$	$P_4(\mu)$	$-.00892P_4(\mu)$	Σ	$\frac{1}{1-.00976\mu^2}$	M
0	-.50	-.300	0.375	-.0033	0.697	1	27.43
1	1	0.599	1	-.00892	1.590	.9902	63.30
0.5	-.125	-.0749	-.28906	.0026	0.9277	.9976	36.60
0.75	.34375	.2057	-.35010	.0031	1.2088	.9945	47.9
0.25	-.40625	-.2438	.15771	-.0014	.7548	.9994	29.78

Total Entrained Water Mass

μ	Mass/ft	SM	Product
0	27.43	1	27.43
0.25	29.78	4	119.12
0.5	36.60	2	73.20
0.75	47.90	4	191.60
1.0	63.30	1	63.30
		Total	474.65

Mass $= 474.65 \times \frac{190.52}{4\times3} \times 2 = 15,380$ lbs. This is identical with the gross value of entrained mass used in the longitudinal vibration calculations in Reference 3.

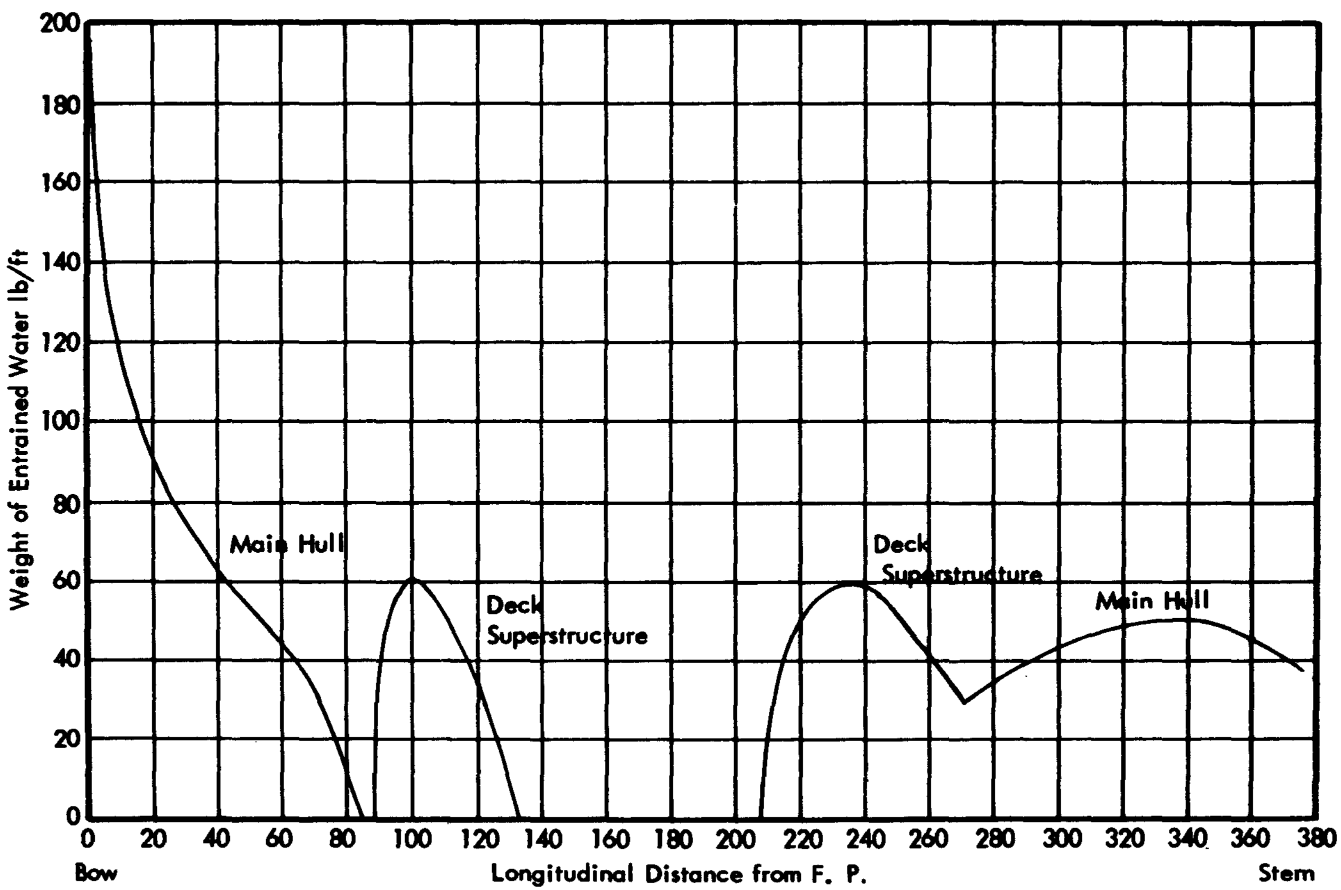


Figure B-5 - Water Inertia for SSB(N) 598

Using Pressures for an Ellipsoid and the Area Curve of the Actual Ship

the water inertia. Because of the large difference between the water inertia given by Figure B-5 and that of an idealized ellipsoid shown in Figure B-4, it would be highly desirable to develop an accurate method of computing the water inertia on an irregular body but the procedures for doing this are not developed and are outside of the scope of this project.

For the longitudinal vibration studies on the SSB(N) 598 the values of water inertia given in Figure B-5 are used. It should be noted that in the longitudinal modes the total mass of the entrained water is only 15,000 pounds (for either the idealized ellipsoid or the actual hull shape). This is about 1% of the submerged weight of the hull. The small value of this entrained mass contrasts strongly with the large values of entrained mass in the bending modes.

5. A Brief Consideration of the Effects of Fluid Elasticity upon the Entrained Mass and Energy Dissipation of a Prolate Ellipsoid Vibrating Longitudinally

When a ship is vibrating in its bending modes it has been shown that there is no significant energy dissipated in the water. This can be explained in terms of the length of a wave length in the ship at a given vibration frequency as compared with that of sound in the water at the same frequency. It can also be shown that the entrained water carried by the ship in bending vibration can be computed accurately without considering the compressibility effects of the water. However, when a submarine is vibrating axially it is known that vibratory energy is fed into the water - explained in terms of the wave length of the axial vibration being much closer to the wave length of the sound in the water. For this reason it is desirable to have a solution for the entrained mass and for the energy dissipation of an ellipsoid vibrating longitudinally.

The solution for the vibration of a prolate ellipsoid in a compressible fluid is similar to that for the incompressible fluid but somewhat more complicated. Where the fluid is incompressible the equation for fluid potential is $\nabla^2 \psi = 0$ but when the fluid is

compressible the scalar wave equation $\nabla^2 \psi = \frac{1}{C^2} \frac{\partial^2}{\partial t^2} \psi$ applies.

For simple harmonic motion

$$\psi (x, y, z, t) = \psi (x, y, z) e^{-i \omega T}$$

and the equation simplifies to the scalar Helmholtz equation

$$\nabla^2 \psi + \beta^2 \psi = 0, \quad \beta = \omega / C \qquad \text{B(4)}$$

For the submarine problem, it is desirable to express this equation in ellipsoidal coordinates (u, ζ, θ). These are related to the rectangular coordinates by:

$$\begin{aligned} x &= k \mu \zeta & 1 &\leq \zeta \leq \infty \\ y &= k (1 - \mu^2)^{1/2} (\zeta^2 - 1)^{1/2} \cos \theta & -1 &\leq \mu \leq 1 \\ z &= k (1 - \mu^2)^{1/2} (\zeta^2 - 1)^{1/2} \sin \theta & 0 &\leq \theta \leq 2\pi \end{aligned} \qquad \text{B(5)}$$

foci are at $x = \pm \frac{1}{2} k$

Then the Heimholtz equation takes the form:

$$\frac{\partial}{\partial \mu} \left[(1 - \mu^2) \frac{\partial \psi}{\partial \mu} \right] + \frac{\partial}{\partial \zeta} \left[(\zeta^2 - 1) \frac{\partial \psi}{\partial \zeta} \right] + \frac{\zeta^2 - \mu^2}{(1 - \mu^2)(\zeta^2 - 1)} \frac{\partial^2 \psi}{\partial \theta^2}$$

$$- (\zeta^2 - \mu^2) \beta^2 k^2 \psi \qquad \text{B(6)}$$

This may also be written in the form:

$$\frac{\partial}{\partial\mu}\left[(1-\mu^2)\frac{\partial\psi}{\partial\mu}\right]+\frac{\partial}{\partial\zeta}\left[(\zeta^2-1)\frac{\partial\psi}{\partial\zeta}\right]+\frac{1}{(1-\mu^2)}\frac{\partial^2\psi}{\partial\theta^2}+\frac{1}{(\zeta^2-1)}\frac{\partial^2\psi}{\partial\theta^2}$$

$$+\beta^2 k^2(\zeta^2-\mu^2)\psi = 0 \qquad \text{B(6a)}$$

When we let $\psi = \phi(\theta)\, S(\mu)\, J(\zeta)$

The equation separates to the ordinary differential equations

$$\phi'' + m^2\phi = 0 \qquad \text{B(7a)}$$

$$\frac{d}{d\zeta}\left[(\zeta^2-1)\frac{\partial J}{\partial\zeta}\right]-\left(A-h^2\zeta^2+\frac{m^2}{\zeta^2-1}\right)J=0 \qquad \text{B(7b)}$$

$$\frac{d}{d\mu}\left[(1-\mu^2)\frac{\partial S}{\partial\mu}\right]+\left(A-h^2\mu^2-\frac{m^2}{1-\mu^2}\right)S=0 \qquad \text{B(7c)}$$

where $h = \beta k$ and m and A are separation constants.

The solutions of B(7a) are $\phi(\theta) = \begin{matrix}\cos m\theta\\ \sin m\theta\end{matrix}$

Equations B(7b) and B(7c) are identical and so the angular function S and the radial function J are solutions of the same equation over different ranges of the variable. These equations can be carried to a solution through the use of spheroidal wave functions.

Studies of the radiation of noise from submarine hulls [12, 13, 14] show that for the

frequency range of propeller generated excitations, the influence of compressibility of the water is small and will tend to decrease the amount of virtual mass. Since this mass is already small, the effects of compressibility have not been explored any farther.

APPENDIX C

VIRTUAL INERTIA OF ENTRAINED WATER IN BENDING MODES

Since the studies of Lewis[15] and Taylor[16] on the effects of entrained water upon ship hull vibrations made practical the predictions of natural frequencies by calculations,[11, 17-23 incl] there have been only moderate advances in the understanding of water inertia. Because the complexity of the water inertia problem it has been difficult to include these advances in the calculation methods that are applied to ships.

The most practical method of including the water inertia in the calculation of ship bending modes still appears to be that used by Lewis and by Taylor. This consists of computing the water inertia per unit length of each section of the ship as though each were an infinitely long prism moving in parallel motion. The result of these calculations is a curve of sectional virtual masses. These sectional inertias are then reduced by a factor, generally designated by J, which represents the reduction in inertia that occurs because of the longitudinal shape of the hull and the modal pattern of vibration. The values of J are generally taken to be the same as those that can be computed for a translating and a vibrating ellipsoid as developed by Lamb[10]. Lamb[10] in his theoretical presenation includes a table of values of J for a translating and for a rotating rigid ellipsoid. Both Lewis and Taylor determined J for the bending ellipsoid. More recently, Macagno and Landweber[11] developed J values for two mode and three mode vibration. The shapes of the modal patterns were those obtained experimentally on the GOPHER MARINER[24] and were approximated by second and fourth order equations in the case of the two node vibrations and by third and fifth order equations in the case of the three node vibrations. The J values that are obtained for prolate ellipsoids of different ratios of major to minor axes that are vibrating in these several modes are plotted in Figure C-1. The surprising thing about this information is the change

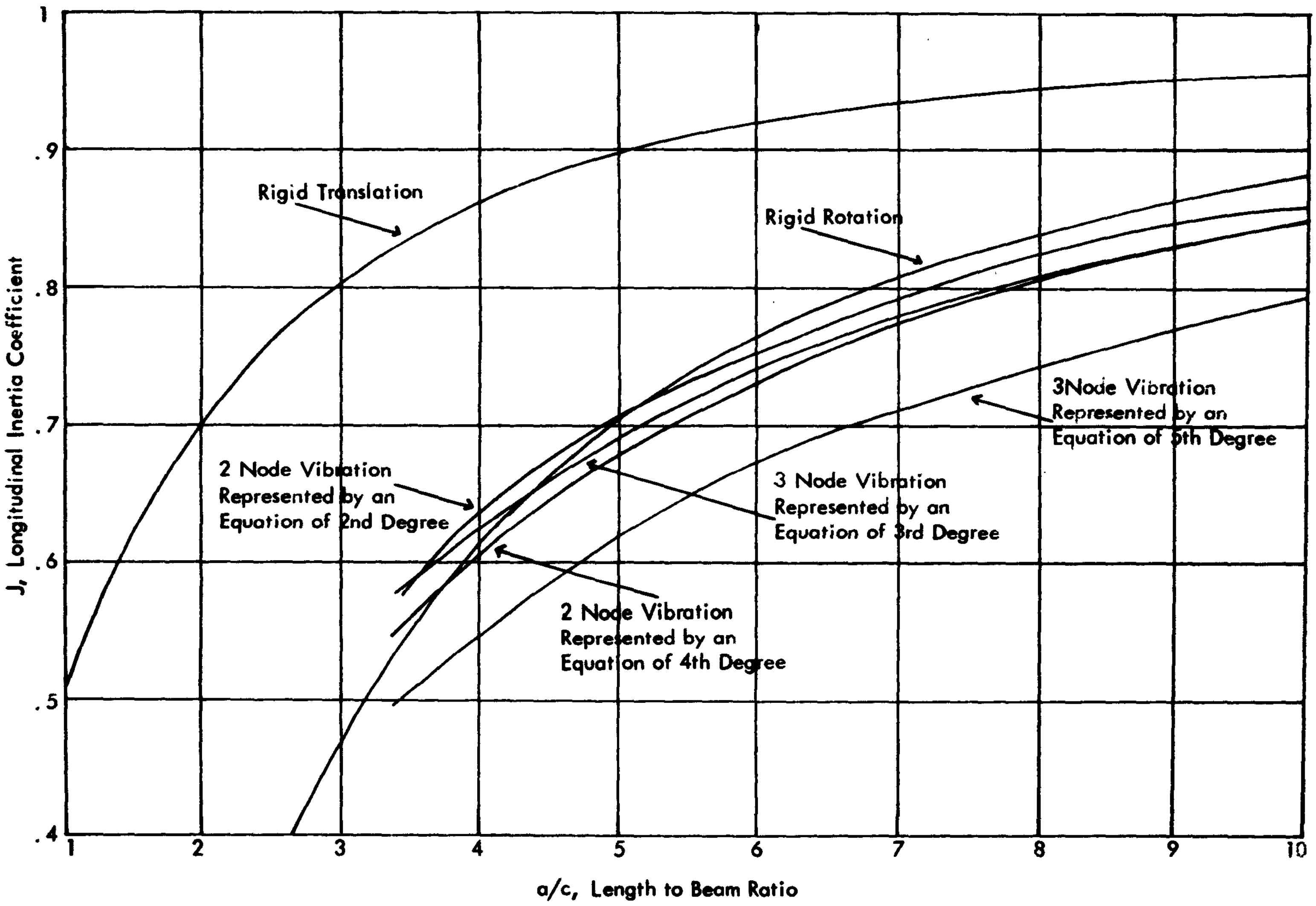


Figure C-1 - Longitudinal Inertia Coefficients for Motions of an Ellipsoid

in the value of J that occurs because of the more detailed representation of the same modal pattern as the approximating function is changed from a second to a fourth degree equation in the two node case, and by a third to a fifth degree equation in the three node case. As far as we know, there have been no determinations of J for higher modes of bending vibration than the 3 node.

The effects of multi-noded vibration patterns upon the J values are best investigated through consideration of an infinitely long circular cylinder vibrating vertically with a sinusoidal variation of amplitude along its length. This problem was first considered by Kennard[25] and is discussed in more detail in Appendix B of Reference 26. According to this development

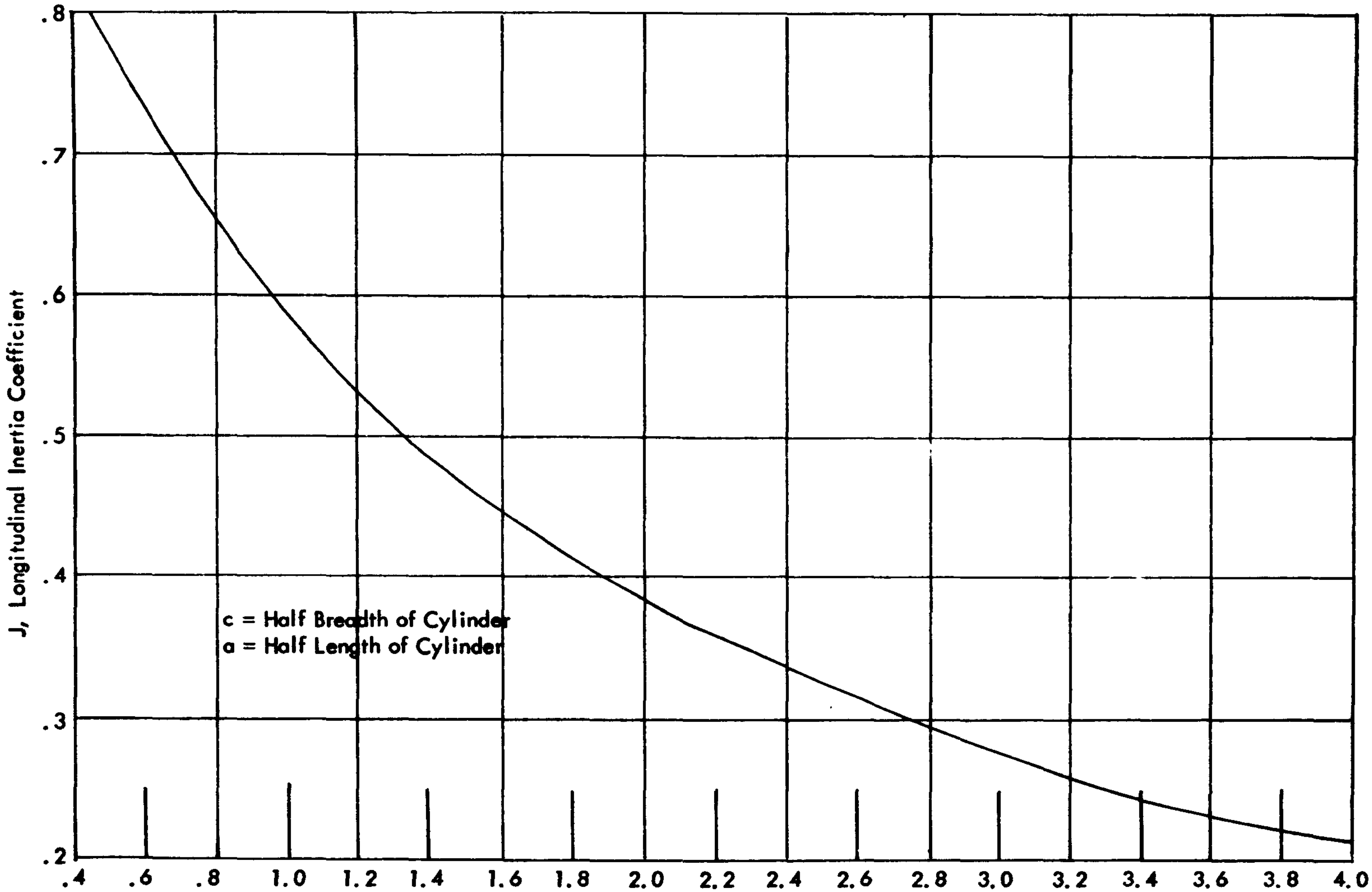
$$J = -\frac{k_1(a)}{k_1'(a)} = \frac{k_1(a)}{k_1(a) - a k_o(a)}$$

where k_o and k_1 are the first and second modified Bessel functions of the second kind, k_1' is the first derivative of k_1 and $a = \frac{\pi c}{N}$ where c is the radius of the cylinder and N is the distance between nodes. A plot of J as a function of a is given in Figure C-2. It is useful to have this function expressed as a simple power series in a. Within the limits $0.6 < a < 4$

$$J \cong 0.0609 + 0.715\frac{1}{a} - 0.19093\frac{1}{a^2} + 0.00297\frac{1}{a^3} + \cdots$$

and within the limits $0 < a < 0.6$ $\quad J \cong 1.030 - \frac{4.90}{a}$

A comparison of the values of the longitudinal inertia coefficient, J, as determined for an ellipsoid by Macagno and Landweber[11] for two noded and three noded vibrations and as determined from Kennard's relationship is given in Figure C-3. The Macagno and Landweber values are based upon a detailed study of the potential flow. It will be noted that the infinite cylinder simplification matches the theoretical value of J quite closely for the 2 node



$$\alpha = \frac{\pi c}{L} = \frac{\pi c}{2a}$$

Figure C-2 - Longitudinal Inertia Coefficients for a Long Cylinder Deflecting in Sinusoidal Waves of Length L

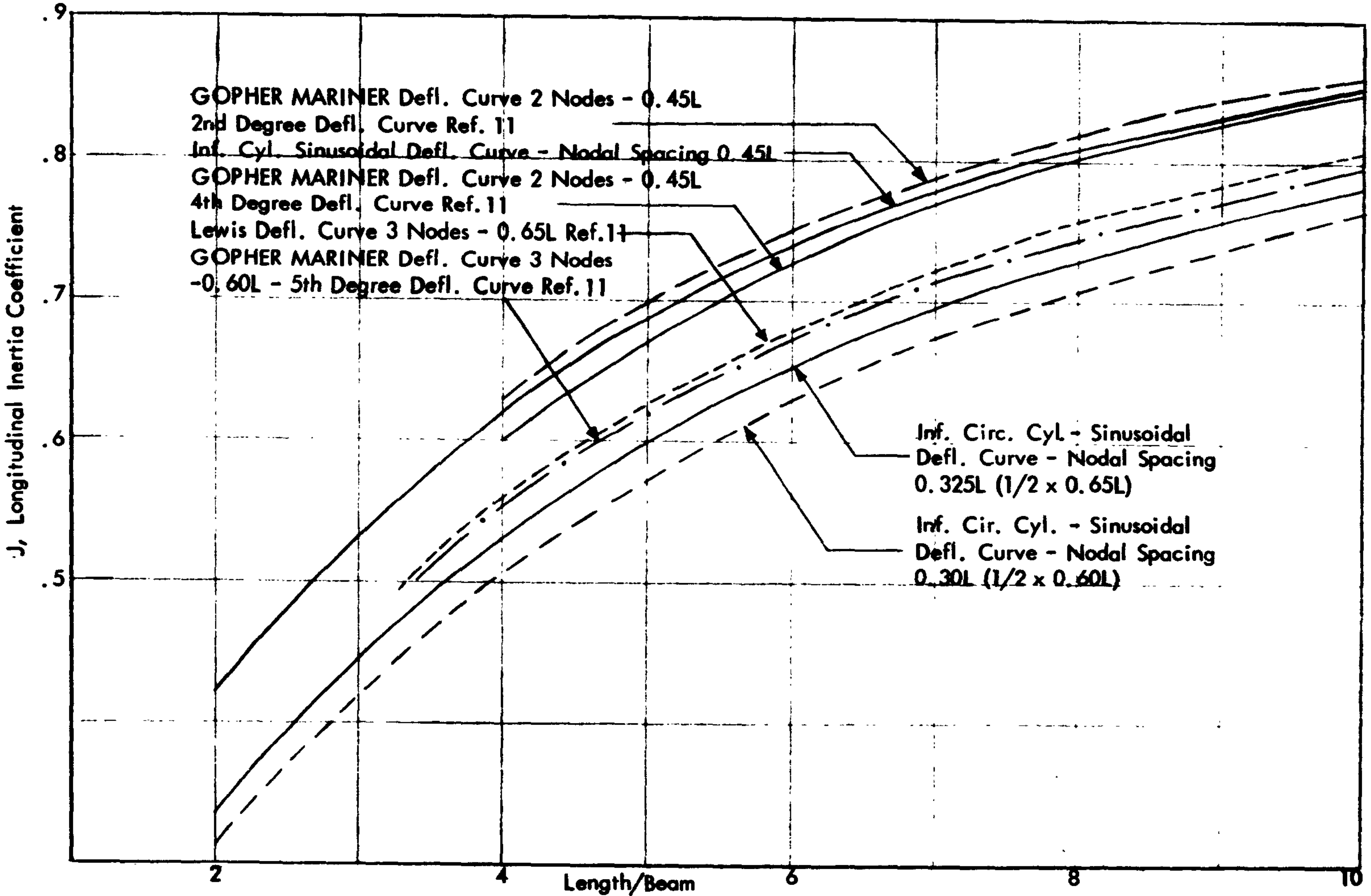


Figure C3 - Longitudinal Inertia Coefficients for a Vibrating Ellipsoid

vibration but appears to be low (i.e., to overcorrect the water inertia) for the three node vibration. Intuitively, it would be expected that the infinite cylinder simplification would match the higher modes more accurately than the lower modes, since the end effects are of less importance.

For the present hull vibration calculations, it was decided to apply a longitudinal inertia coefficient correction to all calculations. The magnitude of this correction is taken to be 0.87 for low frequencies until in the calculation it is found that the deflection curve crosses the axis at two points. For subsequent calculations the longitudinal inertia coefficient for the calculations at any calculation frequency is taken as the value computed for the next lower frequency. The computer ascertains the distance between the most remote zero crossings and divides this distance by one more than the number of intermediate crossings to find the average distance between nodes. It then computes the value of a and the value of J for a long circular cylinder using the series formula. The water inertias used in the subsequent calculation are the section water inertias (2 dimensional values) multiplied by this determined value of longitudinal inertia coefficient, J.

APPENDIX D

STIFFNESSES OF THE HULL IN THE AXIAL DIRECTION, in VERTICAL BENDING AND IN VERTICAL SHEAR

The sectional stiffnesses (stiffness per unit length) of the hull are computed for the axial direction, AE; for vertical bending, EI; and for vertical shear, KAG, and the three values are plotted in Figure 2 along the length of the submarine. These stiffness plots reflect the discontinuities that occur as the thickness of the plating changes.

1. Calculation of Axial Stiffness

The main contributor to the axial stiffness is the plating of the outer and inner shells. Since most of the sections are circular, the cross-sectional area of the steel is easy to compute. This area is corrected for the effects of slope and for the stiffening effect of the transverse framing (See Appendix E). In addition to the shell plating, longitudinal framing, keels and stringers, where continuous, contribute to the longitudinal stiffness. The contribution of such heavy longitudinal structures as pressure tanks and long heavy foundations to the longitudinal stiffness is difficult to assess. There is no adequate theory that can be used for this and so judgement is required. In Appendix F these problems are discussed and some methods developed for use in assessing the stiffness contributions of discontinuous structures.

Because of its light weight and its flexible connection to the hull, the contribution of the superstructure plating to longitudinal stiffness is neglected. Light platform decks are not included for the same reasons.

The stiffness between two stations is found by forming the summation of l/AE for the sections of constant area and finding the reciprocal of the summation.

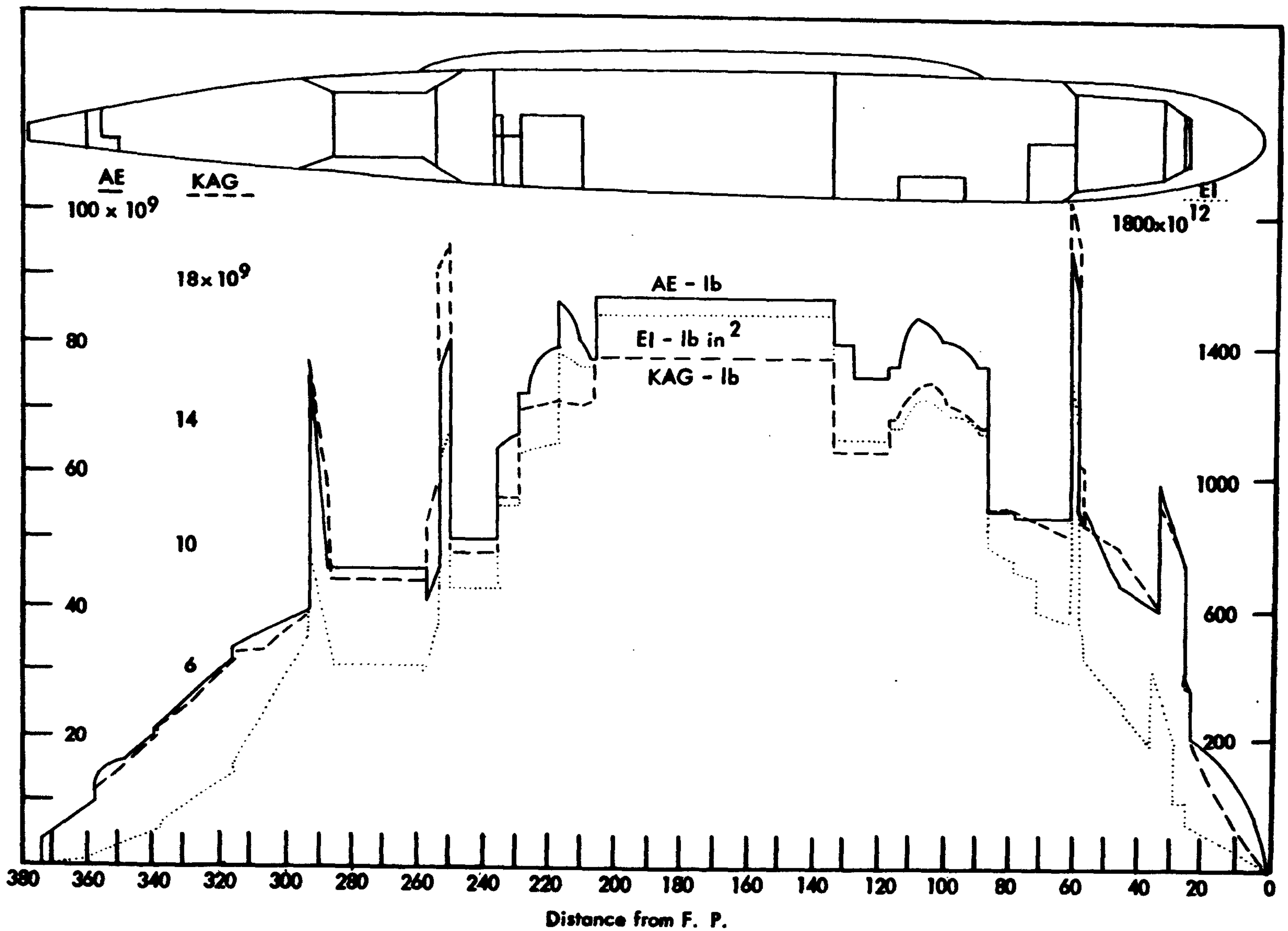


Figure 2 - SSB(N) 598 - Axial, Shear and Bending Stiffness

2. Calculation of Stiffness in Vertical Bending

As with the axial stiffness, the major resistance to bending is given by the inner and outer shell plating. The stiffening effects of the frames is included in evaluating the bending stiffness but not any correction for the conical shape of the hull. Discontinuous longitudinal members are generally of less importance in bending than in axial stiffness and are treated in the same manner. The center of gravity of the section inertia is taken at the centerline of the ship. This is not entirely proper and it would probably be advisable in future calculations to compute the location of the center of the structural section relative to the centerline of the ship and to correct the moment of inertia for the offset. Figure D-2 is a computing form developed for this purpose.

The bending stiffness between two calculation stations is the reciprocal of the summation (or integral) of l/EI for sections of constant moment of inertia over the interval between the stations.

3. Calculation of Stiffness in Vertical Shear

The stiffness in vertical shear for a unit length (KAG) is found by multiplying the components of the cross-sectional area by shear coefficients, K, and the shear modulus, G. Again the major elements in the shear resistance are the inner and outer shells. These elements, since they are circular shells, carry a shear coefficient of 1/2. Other elements that provide shear stiffness in the vertical direction are vertical sides of tanks and foundations. For these the shear coefficient is one and is applied only to vertical webs working between heavy flanges. If these webs are discontinuous the transfer of the loading into and out of them must be estimated. Horizontal areas do not contribute to the shear stiffness.

When the overall shear stiffness between two sections is required, this is obtained from the sectional stiffnesses by forming the sum of l/KAG for the individual sections of constant KAG. The reciprocal of this summation is the shear stiffness.

CALCULATION FOR LOCAL AXIAL, SHEAR AND BENDING STIFFNESS

Frame No. ______________ Distance From F. P. ______________

Structure	Dimensions	Area				C.G.	Moment	K^2	I
		Axial	Corr.*	Corr.axial	Shear				

* Corrections for (1) Slope
(2) Frame Support
(3) Discontinuous Structure

$\frac{AK^2}{I}$ ______

AE = ______

EI = ______

KAG = ______

Figure D-2 - Computing Form for Determining Sectional Stiffness Characteristics

APPENDIX E

INFLUENCE OF THE CONICAL SHAPE AND OF THE TRANSVERSE FRAMING UPON THE STIFFNESS OF A SUBMARINE HULL

A conical hull has a somewhat lower axial stiffness per unit length than that of a cylindrical hull of the same radius and hull thickness. A hull that is stiffened by frames, because it is not free to expand and contract under Poisson's ratio effects, will be somewhat stiffer than the unsupported hull. The problem of the conical shell is considered in Reference 27 and the effects of transverse framing upon the longitudinal stiffness is mentioned in Reference 28.

Consider the conical shell reinforced by circumferential frames. A sketch of the shell is shown in Figure E1.

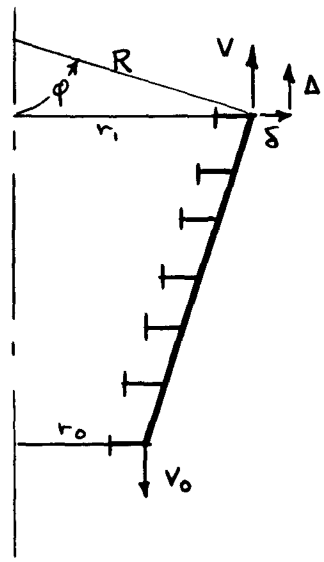


Figure E-1 - Diagram of Shell Supported by Circumferential Frames

Assume that the effects of the frames are uniformly distributed along the length of the shell, that the frames can carry a circumferential load but no axial load and that the ratio of the frame area to the shell area per unit length is β. Let the pressure normal to the axis between the frames and the shell be P_f.

The following equations of equilibrium and deflection will apply (See Ref. 27).

$$V = Vo \cdot \frac{r_o}{r} \tag{E1(a)}$$

$$N_x \text{ (the meridional force) } = \frac{V}{\sin\phi} \tag{E1(b)}$$

$$N_\theta \text{ (the hoop force) } = rP_r \tag{E1(c)}$$

$$\delta = r\varepsilon_\theta \tag{E2(a)}$$

$$\Delta = -\delta \cot\phi \Big|_{x_o}^{x} + \frac{1}{\sin\phi} \int_{x_o}^{x} \varepsilon_x \, dx \tag{E2(b)}$$

For shell: $$\varepsilon_\theta = \frac{N\theta}{Eh} - \mu \frac{Nx}{Eh} \tag{E3(a)}$$

For Frames: $$\varepsilon_\theta = \frac{1}{\beta} \frac{N\theta}{Eh} \tag{E3(b)}$$

$$\varepsilon_x = \frac{Nx}{Eh} - \mu \frac{N\theta}{Eh} \tag{E3(c)}$$

Equating E3(a) to E3(b) and solving for $N\theta$

$$N\theta = \frac{\beta\mu Nx}{1+\beta}$$

Therefore $E_x = \frac{N_x}{Eh} \left(1 - \frac{\beta\mu^2}{1+\beta}\right)$

$$\delta = r \frac{\mu N_x}{(1+\beta)Eh}$$

$$\frac{d\Delta}{dx} = \frac{\mu N_x}{(1+\beta)Eh} \frac{dr}{dx} \cot\phi + \frac{1}{\sin\phi} \frac{N_y}{Eh} \left(\frac{1-\beta\mu^2}{1+\beta}\right)$$

$$= \frac{N_x}{Eh}\left[\frac{\mu}{1+\beta} \frac{\cos^2\phi}{\sin\phi} + \frac{1}{\sin\phi} \left(1 - \frac{\beta\mu^2}{1+\beta}\right)\right]$$

$$= \frac{V}{Eh\sin^2\phi\ (1+\beta)}\left[\mu\cos^2\phi + 1 + \beta(1-\mu^2)\right]$$

The unit length stiffness for an axial load is:

$$K = \frac{2\pi r\, h\, E \sin^2\phi\, (1+\beta)}{\mu\cos^2\phi + 1 + \beta(1-\mu^2)}$$

The correction to the area stiffness is therefore

$$\frac{\sin^2\phi\,(1+\beta)}{\mu\cos^2\phi + 1 + \beta(1-\mu^2)}$$

For a circular cylinder $\phi = \frac{\pi}{2}$ and

$$\text{the correction factor} = \frac{1+\beta}{1+\beta(1-\mu^2)}$$

This agrees with Schade's value.[29]

The maximum correction factor will occur at a bulkhead where β may be considered very large and in this case will be $\frac{1}{1-\mu^2} = 1.1$ (about a 10% increase).

For normal framing β is about 0.4 and the correction factor in the cylindrical section 1.03, i.e., the stiffness is increased about 3% by the frame support.

For an unsupported conical shape the correction factor $= \dfrac{\sin^2\phi}{1+\mu\cos^2\phi}$

For the steepest conical shapes on the 598 (the ends of the double hull section) $\phi = 66.9^\circ$. In this case the stiffness factor uncorrected for framing = 0.739 (about a 25% decrease in stiffness). For the outer surface of the hull the correction is normally only a few per cent and would only be this large near the bow of the ship.

APPENDIX F

TREATMENT OF DISCONTINUOUS STRUCTURES

Any complex structure, such as a ship, has many discontinuous structural elements in it. In the submarine that was studied the more important discontinuous elements are the tops and sides of pressure proof tanks and the tanks that are integral with the foundations.

The problem can be idealized quite easily, since the hull is heavy and cylindrical bending of the hull can be neglected. The problem becomes one of the stress and strain distribution in the base plate and the stiffening plate where neither moves normal to its plane. The three examples illustrated in Figure F-1 cover all the cases necessary for the prediction of the effect of the discontinuous structure on the stiffness of the hull.

The base plate can be defined by a width w (= $2\pi r$ for the cylinder) and thickness h. The discontinuous plate can be characterized by a length, l, a width, b, and a thickness, t.

There appears to be no solution to this problem in the readily available literature and although it would be possible to develop a solution, this would require some time and is outside of the scope of this study. The results of similar studies of the effects of flange widths upon the effectiveness of beams are presented by Schade in References 29 and 30. The loading condition considered by Schade that would most closely match the problems defined above is the beam subjected to a constant moment. However, he does not develop his curves for this case. The nearest case is a beam carrying a uniform load. For a practical working procedure it was decided to utilize the curve of effective breadth ratio developed by Schade for a flange connected to double webs and carrying a uniform load. A plot of this curve (Figure 10(a) page 41 of Reference 29) is given in Figure F-2. This curve, when entered with the length of the discontinuous member, L, and the width, B, equal to the plate width when fastened on two sides or twice the plate width when fastened on one side, gives the ratio between the effective plate width and the actual plate width at the center of the plate. An application will make the process

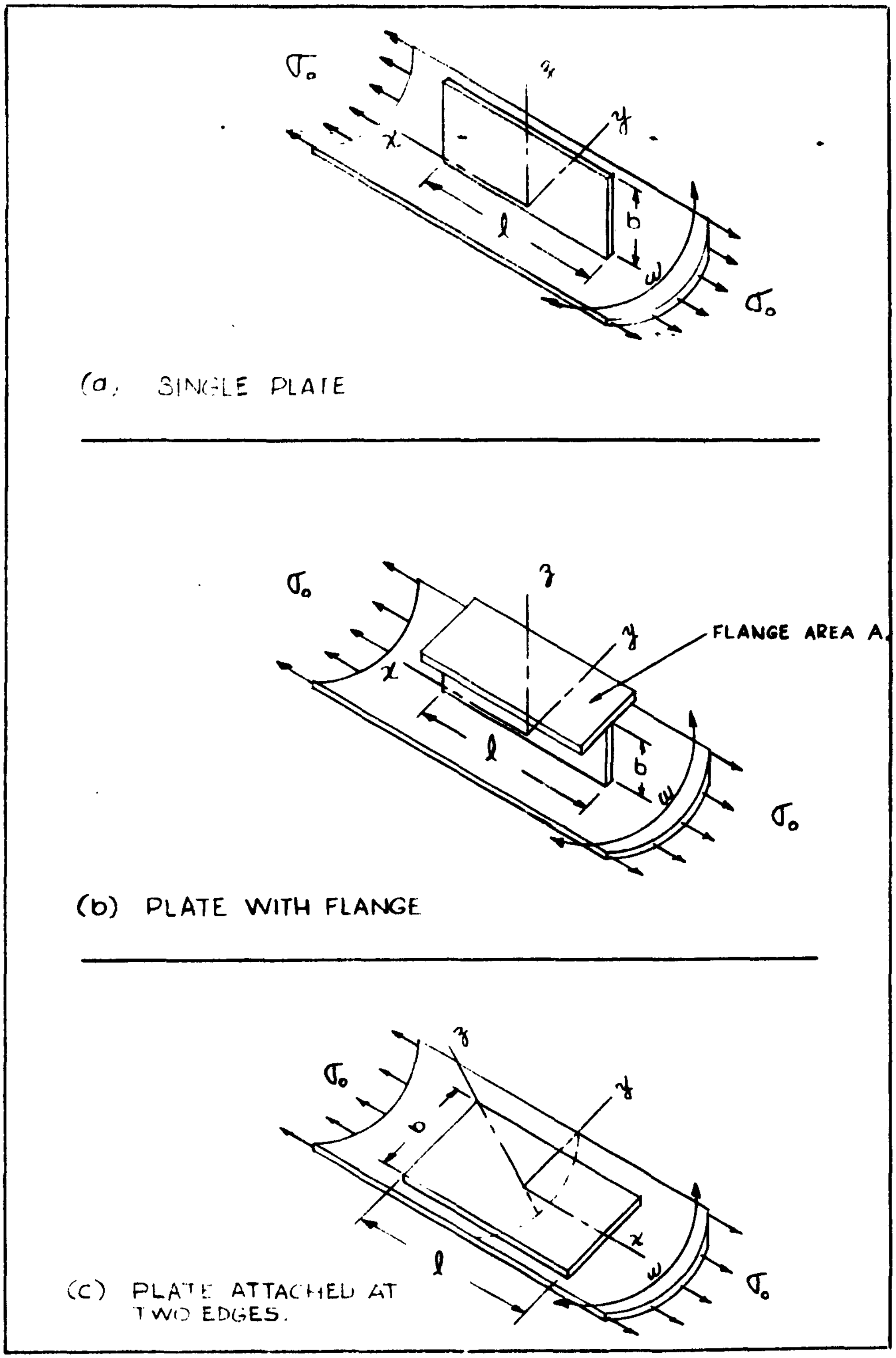
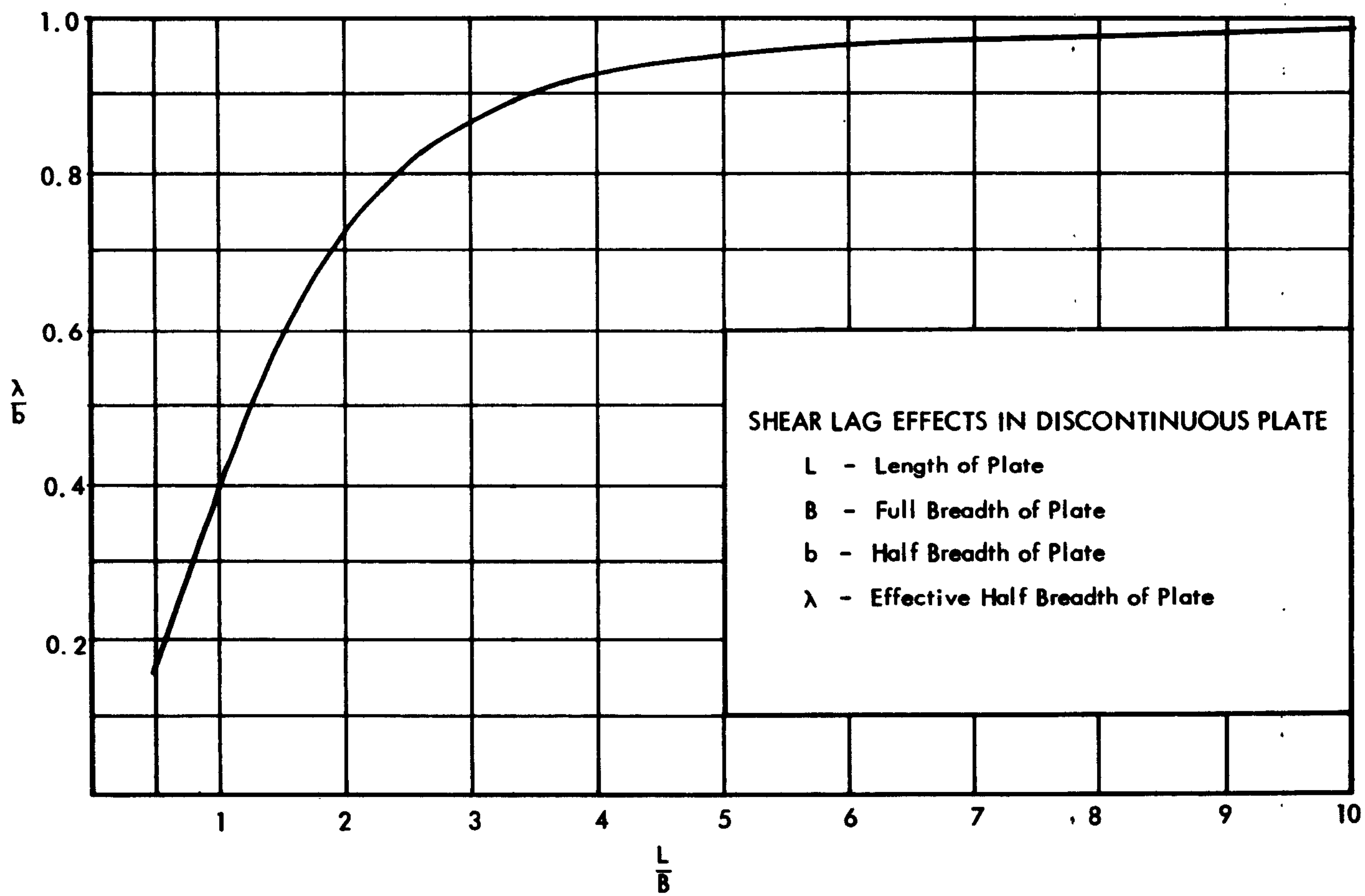


Fig. F1 Discontinuous plate attached to an Axially Stressed Plate that is restrained from bending.



<u>Figure F-2</u> - Effective Breadth Ratio for Plate Attached at Both Sides

clear. One of the ballast tank tops in the GEORGE WASHINGTON is 20.3 feet long and 22.6 feet wide and is attached to the hull at both sides. The ratio of length to breadth is 0.90. Entering the curve for effective breadth ratio, the value of λ/b is found to be 0.35. Thus the effective breadth is taken as 0.35 x 22.6 x 12 or 95" and the stiffening effect of the tank top at its center would be that of an additional steel area of the hull whose area is 95" times the plate thickness. It will be noted that this stiffening effect is that at the center. As the ends of the tank are approached the stiffening effects will fall off until they become zero at the ends. The rate of fall off is assumed to be parabolic. Thus at the quarter length of the tank top the assumed equivalent breadth would be 3/4 of 95" or 72".

APPENDIX G

PROCEDURES USED IN DEFINING THE SPRUNG MASSES IN THE SHIP

One of the primary difficulties in determining the response of a ship is the definition of the structure in terms of mass and elastic constants. Normally in the past, the ship has been defined as an elastic beam to which masses representing the weights of the ship structure, cargo, and entrained water are considered to be rigidly attached. More recently, attention has been given to representing weights that are not rigidly connected to the main hull girder as masses connected to the girder through a simple spring. In applying this procedure, it is necessary to define both the main hull girder and what constitutes a rigid connection, since obviously all connections are only relatively rigid. For the purposes of the vibration calculations consider that vibrations of the ship are those of the main shear members (usually sides but sometimes also bulkheads connecting the upper and lower flanges, the deck and the bottom) and that the rigidity of the connection of all weights to these shear members, called the main hull girder, is to be investigated in determining whether a mass is sprung or not. Also for the purposes of vibration calculations any mass whose connection through the ship structure to the main hull girder is sufficiently rigid to give a natural frequency if held rigidly at the hull equal to 1.5 or more times the hull frequency that is being investigated can be considered as rigidly connected, but otherwise should be considered to be flexibly connected. Flexibly connected members can be concentrated weights (shock mounted equipment are of this category) whole portions of the ship structure or cargo and in some cases the ship bottom with its water inertia. Because of the importance of the propeller and shafting system as a flexibly mounted system, particular attention has been paid to it (See Appendix H) and a procedure developed for finding the response of elastic systems (the hull and the propulsion system) elastically connected (through the bearings).

For the submarine, fortunately, most of the structure is rigid and the only masses that are sprung are those that are shock mounted. These consist of definite machine components

generally assembled on a common bed plate to form a package comprising a machine assembly on a common foundation that is connected to the hull through members that are highly elastic, the shock mounts. The characteristics of these mounts - both the stiffness in several directions and the damping have been accurately determined. It is thus possible to characterize these sprung-masses quite simply and accurately. In order to set a reasonable limit upon the number of masses that are treated only those masses whose weight exceeds 2000 pounds are considered. The items considered, their weights, stiffnesses and damping and location are given in Table I.

Because of the large mass of entrained water associated with them and because they are located at the stern of the ship, where the effects of mass are pronounced, the most important sprung masses are the control surfaces (stabilizers and stern diving planes) in vertical bending. The dynamic responses of these structures to a harmonic vertical motion was presented in Reference 2. For the purposes of these calculations, these surfaces are considered to be adequately represented by the sum of a rigidly attached mass and a simply sprung mass of such size that together they represent the predicted response of these structures which was presented in Reference 2. In future studies they should be incorporated in the calculations as subassemblies.

TABLE I - SSB(N) 598 GEORGE WASHINGTON SPRUNG WEIGHTS

Flexibly-Mounted Element	Distance from F.P. (feet)	Weight (lbs)	Longitudinal		Vertical	
			Stiffness lb/in	Damping * lb sec/in	Stiffness lb/in	Damping * lb sec/in
Each of two Fair-water diving planes	98	Long. 12,771 Vert. 7,750 37,300	Rigid		Rigid 122,200	60n
Gyro. Hyd. System	125	6,900	12,400	24.6	31,440	29.3
L.P. Blower	128	3,145	5,720	11.5	13,600	12.6
O2 Gen. Plant	213	15,200	11,600	27.8	50,000	60.7
Air Cond. Set	217	15,200	11,200	19.3	51,600	41.6
400 Cycle M.G. sets	224	12,261	8,100	15.2	38,700	33.3
H.P. Air Comp.	228	3,450	6,160	25.25	14,400	14.5
Main Coolant Pumps	251.5	51,893	307,000	192	Rigid	
Trim and Drain Pumps	268	5,300	9,000	15.9	16,000	16.4
300 KW M.G. Sets	276	32,920	20,640	107	46,000	104
H.P. Air Comp.	292	3,991	7,150	14.2	16,700	16.7
Air Cond. Sets each of 2	294	27,470	36,120	81.5	80,500	79.1
H.P. Air Comp.	298	3,991	7,150	14.2	16,700	16.7
Hydraulic Plant	346	4,300	6,400	13.6	16,800	19.8
Each of 2 stabilizers + Stern Planes	360	Long. 36,495 Vert. 68,100 125,000	Rigid		Rigid 1.820×10^6	110n

* Damping is located between mass and foundation except for fairwater and stern planes which are to ground. Fairwater and stern plane damping constant is proportional to ship speed. This proportionality is represented by n, the frequency of oscillation, cps, for the case of a 5-bladed propeller.

APPENDIX H

PROCEDURES USED IN DEFINING THE MASS-ELASTIC CHARACTERISTICS OF THE PROPULSION SYSTEM

1. In Longitudinal Vibration

Although the propulsion system is broken by a dental coupling, the longitudinal forces that can be carried by the coupling without causing slippage are generally larger than the longitudinal vibratory forces. The coupling therefore does not isolate the propulsion system from the propeller and shafting in longitudinal vibration. The same reasoning applies to the dental couplings in the reduction gear and the turbines. To check this, consider first the Zurn coupling located forward of the emergency propulsion motor. At 155 rpm this coupling transmits 241,000 lb ft of torque. The pitch circle of the coupling teeth is 23.50" diameter so that the total force on the teeth is 246,000 pounds. This load is carried by 94 teeth each 2-7/8" wide and therefore corresponds to a load of 910 pounds per inch on each tooth. For this type of load with turbine oil lubrication, a minimum coefficient of friction of 0.10 is reasonable. Thus the coupling could carry 25,000 pounds before slipping would occur. This is well in excess of the harmonic force across the coupling.

With the couplings considered as elastic and not isolating members the whole rotating system constitutes the vibrating system. This comprises the propeller; the thrust bearing, which is the connection to the hull; the propulsion motor, the Zurn coupling, the reduction gear, the turbines and all the connecting shafting. When the weight and longitudinal stiffness of these elements are studied they fall naturally into a number of discrete weights connected by spring elements that have low weights. The arrangement is shown in Figure 5.

The weights at the discrete points are generally formed by the summation of manufacturer's weights for the parts but in some cases, for example, the reduction bull gear, it is necessary to compute the weights of components to form another grouping. The weight

of the propeller is increased by a water inertia for axial motion with no rotation as given by Lewis and Auslaender.[31] The propeller is also assumed to be a source of concentrated damping whose value is computed by the methods of Reference 31.

Proceeding from the propeller, half of the weight of the shafting between the propeller and the thrust bearing is assigned to the propeller and half to the thrust bearing mass. Also included with the thrust shaft mass is the weight of the rotor of the emergency propulsion motor which is closely connected to it. Because of the longitudinal flexibility of the web of the low speed reduction gear (the bull gear) the next natural weight division includes the bull gear shaft and hub and to this is added the Zurn disconnect coupling. The next weight item consists of the bull gear rim, the four pinions geared to it and the couplings and shafting associated with the pinions. The succeeding weight group includes the high speed gear hubs and shafts together with their couplings and shafting. The rims of the high speed gears with their mating pinions and couplings form the next weight grouping and the turbine rotors the final grouping. The weight breakdown of these items is given in Table H1.

The axial stiffnesses of the connections between the weights are generally computed from the lengths and diameters of the shafting. Where the shaft is pressed into the propeller hub and into couplings, it is assumed that the equivalent elastic length of the junction is one-third of a shaft diameter. A typical shaft stiffness calculation covering the section between the propeller and the thrust bearing is given in Table H2. A very flexible element in the longitudinal vibration system is the web connecting the hub with the rim of the low speed gear. The web consists of three plates 1" thick that connect a hub that is about 31" diameter to a rim that has an inside diameter of about 113". The web plates are connected together by 6 tubes each 19" in diameter and on a 72" diameter base circle that have a 3/8" wall thickness. The webs are bored out with 18" diameter holes in many of these tubes. They also incorporate "sound stoppers" in the form of partially flattened circumferential tubes of about 4" O.D. and 1/2" wall. Obviously the accurate calculation of the stiffness of this web would be difficult. As a guide to choosing the stiffness, we compute the stiffness

TABLE H-1 - Weight Distribution in Propulsion System in Longitudinal Vibration

Station A:		
Propeller		23, 550
Entrained water		17, 730
1/2 Shafting between Propeller and Thrust Shaft		10, 400
	Total	51, 680
Station B:		
1/2 Shafting between Propeller and Thrust Shaft		10, 400
Thrust Shaft		5, 230
Emergency Propulsion Motor and Shaft		16, 270
	Total	31, 900
Station C:		
Zuen Coupling		3, 500
Second Reduction Gear Hub and Shaft		10, 157
	Total	13, 657
Station D:		
Second Reduction Gear Rim and Deadening Rings		11, 010
Four pinions and Associated intermediate Shafting		4, 096
	Total	15, 106
Station E:		
First reduction gear hub, coupling and shaft 4 at 1000 pounds each		4, 000

TABLE H-1 (continued)

Station F:	
First reduction gear rim, web and deadening rings 4 at 828 pounds each	3,312
High speed pinions, coupling and distance 2 at 592 pounds each	1,184
Total	4,496
Station G:	
Turbine rotor, coupling and distance piece 2 at 3670 pounds each	7,340

TABLE H-2 - Typical Calculation for the Axial Stiffness between the Propeller and the Thrust Bearing

Based on EB drawing 9871-00142 Bu Ships No. SSG(N) 598-203-1891801

Main Shafting Details

Section	Length	OD	ID	Area	ℓ/A
Prop. to Sleeve	6" +6-1/4"*	16.00	4.5	185.2	0.066
Sleeve Section	75"	16.00	4.5	185.2	.405 } .396
Sleeve (bronze)	75"	17.373	16.00	9.5	15.8+ }
Between sleeves	177-1/8"	16.00	9.00	137.5	1.289
Stuffing bx. sleeve sect.	61"	16.065	9.00	138.1	.441 } .429
Stuffing bx. sleeve (bz)	61"	17.373	16.065	8.9	13.72+ }
Shaft	91 $\frac{13}{16}$"	16.003	9.000	137.6	0.667
Enlargement	9-5/16" + 5.7"*	17.25	9.00	170.1	0.088
Coupling	16-1/2"	28	9	552	0.030
Thrust flange	3"	28		616	0.005
Thrust shaft	23"	15-3/4		194.8	0.118
					3.088

* 1/3 of diameter for fit

+ℓ/A multiplied by two because of ratio of Elastic Moduli of Bronze and Steel

The shaft and its sleeve are in parallel so $\left(\frac{1}{\alpha}\right)_{comb} = \frac{1}{\alpha_{shaft}} + \frac{1}{\alpha_{sleeve}}$

The longitudinal stiffness between the propeller and the thrust collar is

$$\frac{AE}{\ell} = \frac{30 \times 10^6}{3.088} \approx 9.7 \times 10^6 \text{ lb/in.}$$

This value has been taken as 10×10^6 in subsequent calculations.

of a set of three webs loaded by a unit force on the rim and restrained at the center. The deflection would be that of a circular plate rigidly restrained at the inner and outer radii which is loaded at the center by a moment. The deflection of such a plate is given by Timoshenko as $\phi = \dfrac{M}{a_3 E h^3}$. Taking an inner radius of 15.5" and an outer radius of 56.5" the ratio becomes 0.275 and the corresponding value of $a_3 = 2.7$. Since the moment in each plate for a force F is $\frac{F}{3} \times \frac{118}{2}$, the deflection under load is $\frac{118}{2} \times \phi$ and the plate thickness 1", the deflection for a unit load is 14.3×10^{-6} inches. The corresponding stiffness $= 0.07 \times 10^6$ pounds per inch. Because of the stiffening effects of the pipes, etc., this stiffness was arbitrarily increased to 0.10×10^6 pounds per inch. It is not anticipated that this stiffness will be of much influence on the hull vibration and so no great efforts were expended in trying to improve its accuracy.

A very important stiffness in the response of the hull to the propeller forces is that of the connection between the shaft and the hull - i.e., the thrust bearing. For this reason the stiffness of this element was studied in considerable detail. The calculation for this stiffness presented here is based upon summing the flexibilities of the several components. The plans that are used are the following:

Kingsbury Machine Works, Inc., Philadelphia, Pennsylvania
Kingsbury Dwg. No. 464275 - Bu Ships Dwg. No. F 3, 297, 494A
37" Kingsbury Thrust and Journal Bearing Assembly

Kingsbury Dwg. No. 464276 - Bu Ships Dwg. No. F 3, 304, 231A - 37" (6x6) Kingsbury Thrust Bearing, Housing Details

Kingsbury Dwg. No. 464277 - Bu Ships Dwg. No. F 3, 304, 232A - 37" Kingsbury Thrust Bearing, Bearing Shell, Oil Catcher, etc. Details

Kingsbury Dwg. No. 464278, - Bu Ships Dwg. No. F 3, 303, 179A - 37" (6x6) Kingsbury Thrust Bearing, Shoe, Base Ring, Leveling Plates, etc., Details

1. Deflection of thrust collar:

The bending deflection is negligible

The shear deflection is given by:

$$\delta_s = \frac{3}{4} \frac{P}{\pi h G} \quad \mathrm{Log} \frac{a}{r}$$

a = 1/2 loading diameter $= \frac{1}{2} \times 27 \frac{13''}{16}$

r = 1/2 root diameter = 1/2 x 15-3/4"

$\frac{a}{r} = 1.77$

h = plate thickness = 6.375"

G = shear modulus = 11.8×10^6

$\delta_s = 0.001812 \times 10^{-6}$ in/lb

2. Deflection of Spherical Surfaces under Shoes: 12" spherical radius against a flat surface
   6 shoes

By the Hertz formula (Page 376 of Ref. 33)

$$\delta = 1.23 \sqrt[3]{\frac{p^2}{E^2 R_2}} \qquad p = \frac{F}{6}$$

$$\frac{\phi\delta}{\phi p} = 1.23 \sqrt[3]{\frac{1}{E^2 R_2}} \times \frac{2}{3} p^{-1/3}$$

$$= 0.82 \sqrt[3]{\frac{1}{E^2 R_2 p}}$$

assume F = 120,000 lb p = 20,000 R_2 = 12" E = 30 x 10^6

$$\text{Then } \frac{\phi\delta}{\phi p} = 0.82\sqrt[3]{\frac{1}{(30)^2 \times 10^{12} \times 12 \times 20,000}}$$

$$= 0.137 \times 10^{-6} \text{ in/lb}$$

$$\frac{\phi\delta}{\phi F} = 0.0228 \times 10^{-6} \text{ in/lb}$$

3. Deflection of upper and lower leveling plate

Consider this deflection to be the sum of the deflections of two cantilevers. The first 3.8" long. 4.5" wide and 3" deep and the second 1" long. 4.5" wide and $\frac{23''}{16}$ deep. Since there are six leveling plates which carry the load on two sides the force on each cantilever is F/12.

Therefore

$$\frac{\delta}{F} \text{ upper leveling plates} = \frac{1}{12} \frac{(3.8)^3}{3 \times 30 \times 10^6 \times \frac{4.5 \times 3^3}{12}}$$

$$\times \frac{1^3}{3 \times 30 \times 10^6 \times \frac{4.5 \times (\frac{23}{16})^3}{12}} = 0.00\,583 \times 10^{-6} \text{ in/lb}$$

The lower leveling plate would have the same deflection.

4. Deflection in the thrust bearing housing:

Consider as direct shear between the center of the shaft and the flanges. The

thickness of the housing is 1-3/4", the length of the housing carrying the shear is about 30 inches on each side and the length of the section across which the shear is transmitted is about 20 inches.

The deflection, δ, $= \frac{F \times 20}{1.75 \times 60 \times 11.8 \times 10^6} = 0.0161 \times 10^{-6}$ in/lb. Because of bending in the housing and the flanges this is probably small. Consider the housing deflection to be $0.03" \times 10^{-6}$.

5. Deflection of the foundation:

The deflection of the thrust bearing foundation is computed through the use of the following plans:

EB Div. Dwg. 9841-057, Bu Ships Dwg. SSB(N) 598-112-1890616K - foundation - Propulsion Plant

EB Div. Dwg. 9832-020, Bu Ships Dwg. SSB(N) 598-101-1890537F
Frames 83, 84, 85, 86, 87, 88, 89, 90, 91 and Tank Flat

EB Div. Dwg. 9833-006, Bu Ships Dwg. SSB(N) 598-114-189 0564C
BHD and Framing 92, 93, 94

The thrust bearing sits on a double beam about 5 frames, i.e., 130" long. Each beam consists of upper and lower flanges about 20" wide and 3/4" thick separated by two webs each 1/2" thick. The spacing between the flanges is about 22". Therefore, the moment of inertia of each beam is

$$I = 2 \times 15 \times 11^2 \times \frac{1 \times (22)^3}{12} = 4520 \text{ in}^4$$

Assuming that these beams are pinned at the ends and are subjected to an end moment the rotation will be $\frac{ml}{3EI}$. Since the center of the beam is about 36" below the ℄ of the shaft

the deflection at the shaft centerline for a unit load will be:

$$\frac{\delta}{P} = \frac{F \times 36 \times 130 \times 36}{3 \times 4520 \times 2 \times 30 \times 10^6} = 0.208 \times 10^{-6} \text{ in/lb}$$

The whole foundation structure is quite loosely tied except for the after end attachment to the tank flat. The deflection of this foundation structure in the longitudinal direction under a unit load can be obtained by computing the deflection of the tank top to which it is attached in shear and in bending and adding to the resulting stiffness, the additional stiffnesses of the longitudinal members.

The deflection of the tank top between frames 89 and 92 in shear is (since there are two parallel shear elements)

$$\frac{\delta}{P} = \frac{1}{2}\frac{\ell}{AG} = \frac{1/2 \times 50''}{3/8'' \times 78'' \times 11.8 \times 10^6} = 0.0725 \times 10^{-6} \text{ in/lb}$$

The deflection of the tank top between frames 89 and 92 in bending is determined as follows:

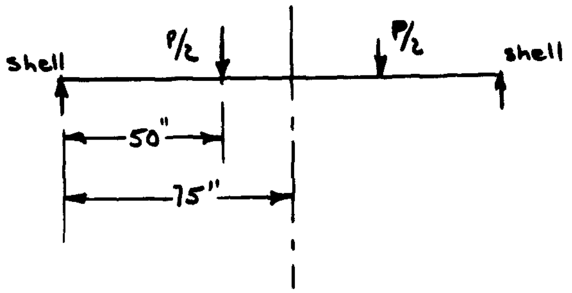
Because of shear lag the equivalent breadth of the tank closure on Frame 89 is obtained from Figure F-2.

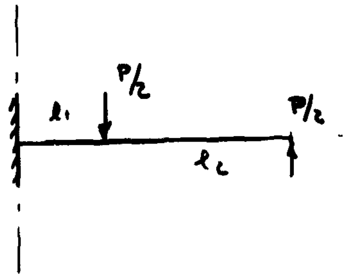
$$L = 165'', \; B = 54'' \quad \frac{L}{B} = 3. \quad \frac{x}{b} = 0.86$$

The central moment of inertia determined as follows:

Section	Dim.	Area	C. G.	M_{fr}	$I_{fr\ 92}$
Bhkd. Fra. 92	1-1/8" x 120"	135	0	0	
Tank Top web.	1/2 x 78"	39	39	1520	19,800
Tank closure Fr. 89	1-1/8 x 54"	61	78"	4760	371,000
		235	26.8	6280	390,800
					- 169,000
					= 221,800

The beam is loaded as follows:





If a uniform beam

$$\delta = \frac{P}{2} \times \ell_2 \times \frac{\ell_1}{EI} \times \ell_2 + \frac{P}{2}\,\frac{\ell_2^{\,3}}{3EI}$$

$$= \frac{\frac{P}{2}}{30 \times 10^6 \times 222{,}000}\left[50^2 \times 25 + \frac{50^3}{3}\right]$$

$$\frac{\delta}{P} = 0.0157 \times 10^{-6}$$

The flexibility of the tank [illegible] 32×10^{-6} in/lb

The shear flexibility of the longitudinal members are as follows:

For members 25" off ℄ $\frac{\delta}{P'}$, $= \dfrac{60}{60 \times \frac{1}{2} \times 11.8 \times 10^6}$

$= 0.17 \times 10^{-6}$

For members 19" off ℄ $\frac{\delta}{P'}$, $= \dfrac{60}{40 \times \frac{1}{2} \times 11.8 \times 10^6}$

$= 0.25 \times 10^{-6}$

Since the tank top, the two longitudinal members 25" off the ℄ of the ship and the two longitudinal members 19" off the ℄ of the ship are in parallel, the overall flexibility of the foundation connection to the ship is the reciprocal of the sum of the reciprocals of the parts.

$$\frac{1}{\alpha_{total}} = \frac{1}{0.0882 \times 10^{-6}} + \frac{2}{0.17 \times 10^{-6}} + \frac{2}{0.25 \times 10^{-6}}$$

$$= 11.22 \times 10^{6} \times 11.8 \times 10^{6} \times 8 \times 10^{-6}$$

$$\alpha_{TOT} = 0.0323 \times 10^{-6} \text{ in/lb}$$

When all of the flexibilities are added the total flexibility of the connection between the thrust shaft and the hull is obtained.

Element		Flexibility in/lb
Thrust Collar		0.0018×10^{-6}
Spherical Surfaces under shoes		0.0228×10^{-6}
Upper loading plates		0.0058×10^{-6}
Lower leveling plates		0.0055×10^{-6}
Thrust bearing housing		0.0300×10^{-6}
Rotation of found. beams		0.2080×10^{-6}
Axial defl. of foundation		0.0323×10^{-6}
	Total	0.3065×10^{-6}

The corresponding stiffness between the thrust shaft and the hull is 3.28×10^{6}, say 3.3×10^{6} lb/in.

2. In Vertical Vibration

A sketch of the parts of propulsion system that participate in the vertical vibration as a separate system sprung from the hull is shown in Figure H1. It will be noted that this

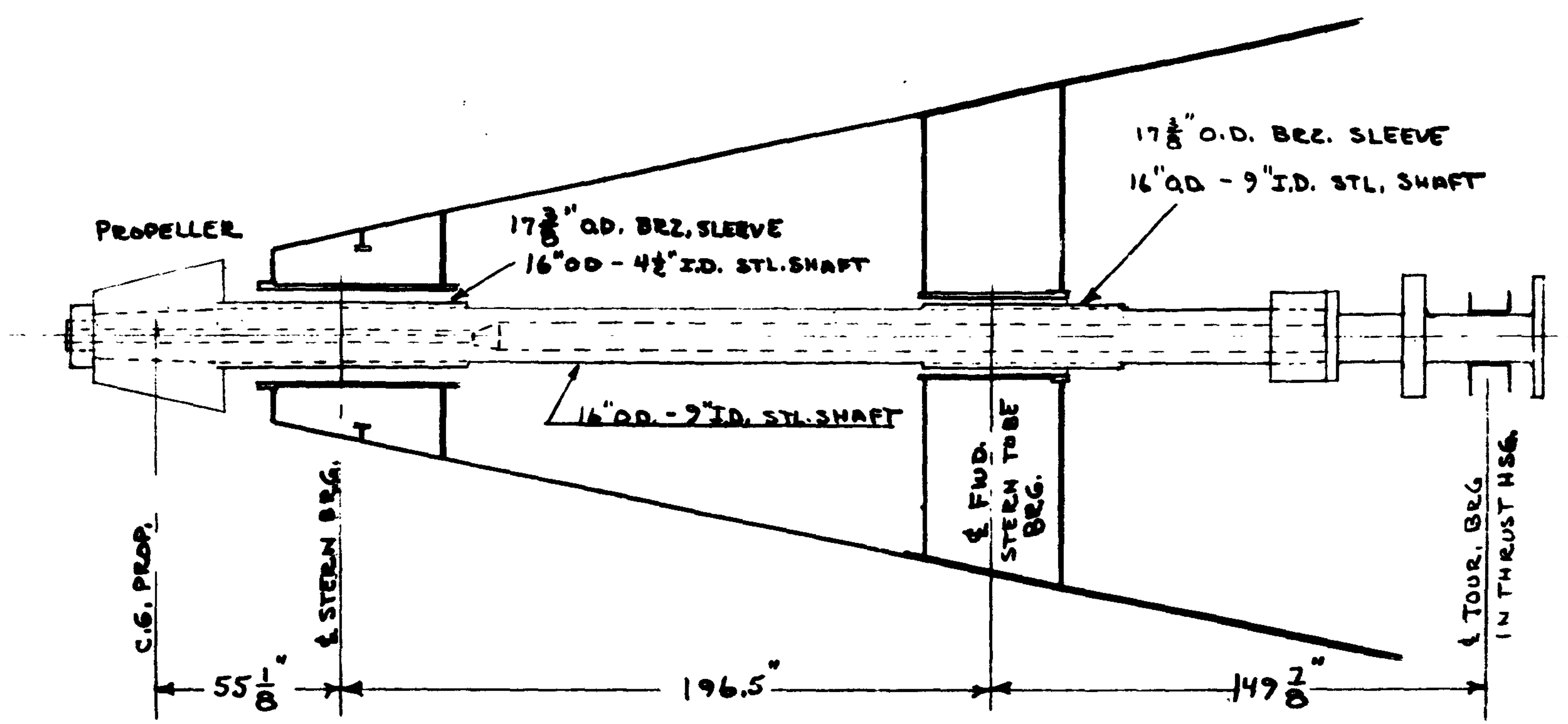


Figure H1- The Propulsion System in Vertical Vibration

system consists of the propeller and shafting up to the thrust bearing and that the thrust bearing and the elements of the propulsion system forward of the thrust bearing are considered to be rigidly connected to the hull.

The weights at the several stations on the shaft are those of the propulsion shafting between half way to the station forward and half way to the one aft. The $\Delta x/EI$ is formed in the same manner. The propeller weight consists of 23,550 pounds for the propeller plus 4350 pounds for the entrained water. Lewis and Auslaender[31] are indefinite as to the proper value to use for entrained mass in the transverse direction. The value that is shown is obtained by multiplying the value for entrained mass in the longitudinal direction with the propeller restrained from rotation, an entrained mass of 17,730 pounds, by the square of the ratio of the transverse projection of one blade at 0.7R to the axial projection. This ratio = pitch/0.7πD and has the value 0.495. Thus the assumed value of entrained mass in the transverse direction is 4350 pounds and even this value is probably high.

The moment of inertia of the entrained water about the transverse axis is formed by applying Lewis and Auslaender's[31] suggestion of multiplying the value for longitudinal water inertia by $D^2/16$ and is found to be 40.9×10^6 pounds inches squared. The moment of inertia of the propeller itself about the transverse axis is taken a half of the polar moment of inertia and is therefore 18.09×10^6 pounds inches squared. The sum of these is taken as 59×10^6 pounds inches squared. The computer program does not treat the moment of inertia as concentrated with the mass but rather spread out between the concentrated masses. In accordance with this the 59×10^6 value is considered to be distributed between the stem tube bearing and the propeller. A more precise representation of this inertia would be obtained by treating the propeller as two masses a short distance apart with the rotary inertia distributed between them.

The damping on the propeller in the transverse direction was obtained from Lewis and Auslaender[31] value for torsional vibration damping by the following procedures. b_θ by Lewis and Auslaender's procedures was found to be 45,300 ft lb sec. It is assumed that the damping torque (=45,300$\omega\theta$) acts at 0.7 radius ($0.7 \times \frac{16}{2} = 5.6$ ft). Thus the damping

force at 0.7 radius = $\frac{45,300}{5.6}$ can be related to the linear harmonic velocity at 0.7R, $U_{0.7}$ (= 5.6ωθ) to give the net damping force at 0.7 radius $= \frac{45,300}{(5.6)^2} U_{0.7R} = 1440\ U \frac{\text{lb ft.}}{\text{sec.}}$

The force corresponding to a transverse harmonic velocity rather than a radial harmonic velocity is assumed to be half of the later, or 720 U. When this value is changed to inch units and multiplied by an H factor of 0.60 the assumed value of vertical damping coefficient = 40 lb in/sec is obtained.

It remains now to determine the stiffness of the coupling between the shafting system and the hull. For the journal bearing that is incorporated in the thrust bearing housing and for all the bearings forward of this since they are oil lubricated bearings and are mounted on solid structure, the weights are assumed to be integral with the hull. The elasticities of the connections occur in the stuffing box and stern tube bearings and in bending of the shafting. The flexibilities of the bearings arise from local deformation of the hull structure, deformations of the bearing support structure and deformation in the bearing staves. When these are evaluated it is found that the local deformation of the hull is negligible, the deformation of the bearing support structure, although designed to allow some accommodation of the bearing, is small but that the deformation in the bearing staves is significant. The stuffing box is assumed to give no support to the shaft.

The flexibility of the rubber staves in the stern tube bearing is determined as follows:

The shaft diameter in the bearing is 17-3/8"

The length of the stern bearing is 60"

The static load on the stern tube bearing, taken as the propeller weight + 2/3 of the shaft weight, = 37,400 lbs

The projected area of the bearing = 1042 in^2

The load per unit of projected area = 35.9 lb/in^2

From Figure H2 the slope of the deflection curve for a No. 10 stave on a 14-5/8" diameter shaft at a load of 35.9 lb/in^2 is 0.001623 $in/lb/in^2$. Thus the overall flexibility

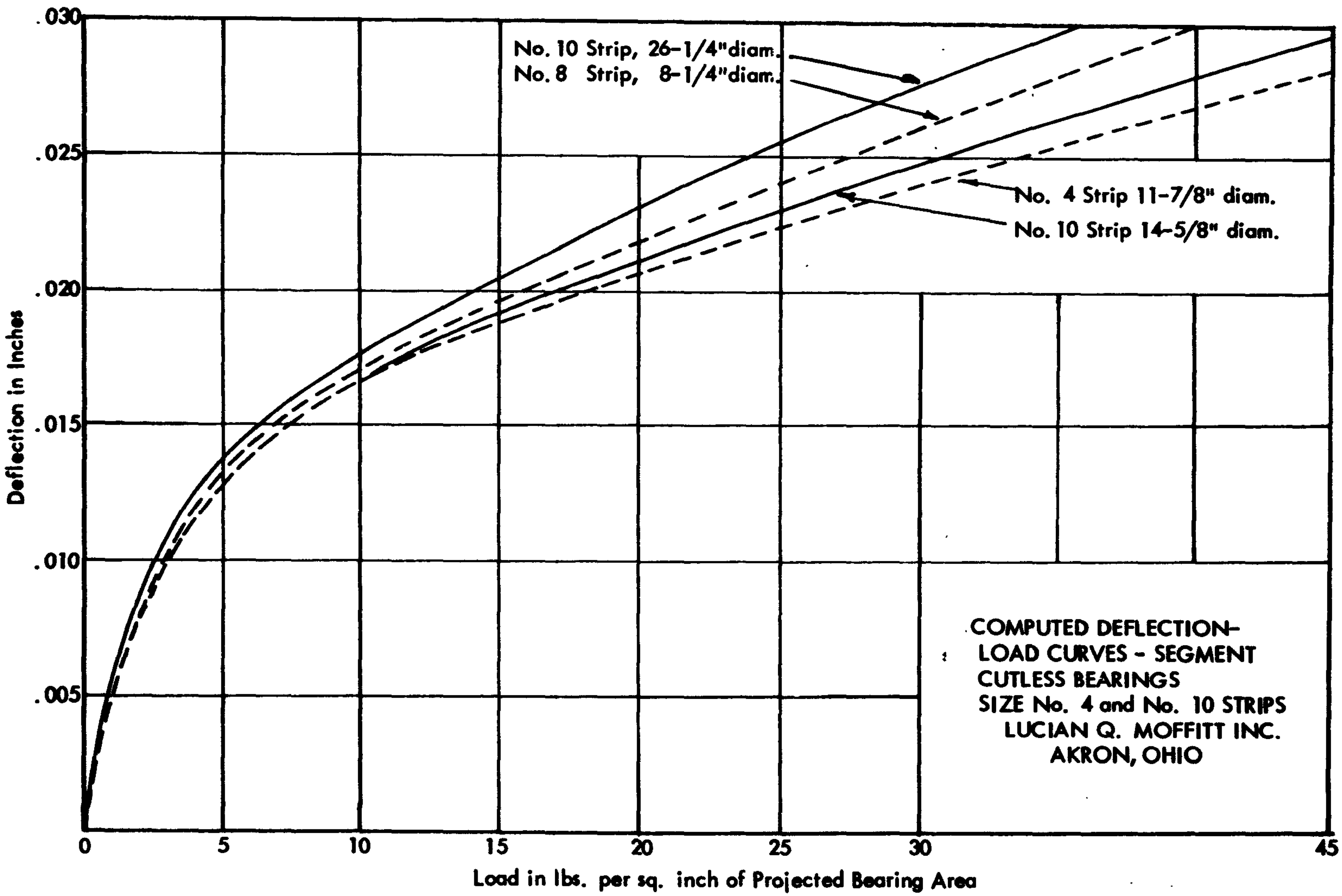


Figure H3- DEFLECTION CURVES FOR SEGMENTED CUTLESS BEARING

of the bearing is $0.001623 \div 1042 = 1.558 \times 10^{-6}$ in/lb. When the flexibility of the bearing support tubes, and the shaft tubes are computed they are found to be 0.30×10^{-6} in/lb and the corresponding stiffness of the stern tube bearing connection between shafting and hull is 0.538×10^{6} lb/in.

For the forward, stuffing box, bearing the projected area is 521 sq. in. and the load is assumed to be 7000 lb giving a pressure per square inch of projected area of 13.5 lb/in^2.

From Figure H2 the slope of the deflection curve for a No. 10 stave with a 14-5/8" diameter shaft and a load of 13.5 lb/in^2 is 0.00225 in/lb/in^2. Thus the overall bearing flexibility is $0.00225/521 = 4.31 \times 10^{-6}$ in/lb.

From detailed calculations the flexibility of the forward tube and the bulkhead support is 0.01×10^{-6} in/lb. Thus the overall flexibility of the forward bearing is 4.32×10^{-6} in/lb. and the corresponding stiffness of the connection between the shafting system and the hull at this point is 0.232×10^{-6} lb/in.

APPENDIX I

GENERALIZED BENDING RESPONSE CODE (GBRC 1)

The generalized bending response code (GBRC 1) can be used to calculate the response of beam-spring systems to specified simple harmonic driving forces and/or moments. For example, a ship hull connected elastically to other elastic systems, say a propulsion system, as well as to sprung masses, can be treated. The program is a FORTRAN II program which has been run on the IBM 7090. It is now in regular use at David Taylor Model Basin, and is referred to by number 1-840-277-01.

In this appendix, the difference equations used in GBRC 1 to represent a non-uniform beam (hull, shafting, etc.) in bending are derived. Then, these equations are modified to include more general systems such as systems of beams connected by springs. By permitting complex scaling factors for various input parameters, a variety of damping options are included, for example, Rayleigh damping and hysteresis damping. Damping coefficients for connections between sections can be given explicitly.

Longitudinal or torsional vibration problems can be solved using the program. For example, if each beam is made up of sections with spring connections only, the equations reduce to those for a mass-spring system. These equations are those used to approximate beam systems connected by springs in longitudinal or torsional vibration problems.

The program contains a special option which permits the effective mass for each section to be calculated as a function of frequency.

A detailed description of the input preparation required for GBRC 1 is included here. In Appendix J actual input sheets for the particular vertical bending calculations included in this report are given.

The output generated by GBRC 1 includes a tabulation of the input used to describe the problem, an optional summary output which lists amplitudes for deflections and moments versus frequency for a selected set of sections, and finally for each selected frequency, the

real and imaginary parts of the deflection and moments as well as their amplitude and phase angle for each section.

Method of Analysis

The system of differential equations to be satisfied by the hull, shafting system, etc. in bending is given by

(3.1) $$\frac{\partial V(x,t)}{\partial x} = -\mu(x)\frac{\partial^2 y(x,t)}{\partial t^2} - c(x)\frac{\partial y}{\partial t} + P(x,t)$$

(3.2) $$\frac{\partial M(x,t)}{\partial x} = V(x,t) + I_{\mu z}(x)\frac{\partial^2 \gamma(x,t)}{\partial t^2} + Q(x,t)$$

(3.3) $$\frac{\partial y(x,t)}{\partial x} = \gamma(x,t) - \frac{V(x,t)}{KAG(x)}$$

(3.4) $$\frac{\partial \gamma(x,t)}{\partial x} = \frac{M(x,t)}{EI(x)}$$

with the following notation:

- x: distance in the longitudinal direction measured from the origin of coordinates
- t: time variable
- y: displacement normal to x in the plane of bending
- γ: angular displacement relative to the z-axis
- V: shearing force in the direction of flexural vibration (y-direction)
- M: bending moment
- μ: effective mass per unit length

$I_{\mu z}$: effective rotary inertia per unit length

KAG: shear rigidity

EI: bending rigidity

P: external forcing function, force in y-direction

c: damping coefficient

Q: external forcing moment

If the forcing functions are assumed to be a simple harmonic function of time so that

$$P(x,t) = e^{i\omega t}P(x)$$

$$Q(x,t) = e^{i\omega t}Q(x)$$

Then, also,

$$y(x,t) = e^{i\omega t}y(x)$$

$$\gamma(x,t) = e^{i\omega t}\gamma(x)$$

$$V(x,t) = e^{i\omega t}V(x)$$

$$M(x,t) = e^{i\omega t}M(x)$$

and equations (1) to (4) become

(3.5) $$\frac{dV(x)}{dx} = \mu(x)\,\omega^2 y(x) - ic(x)\omega y(x) + P(x)$$

(3.6) $$\frac{dM(x)}{dx} = V(x) - I_{\mu z}(x)\,\omega^2\,\gamma(x) + Q(x)$$

(3.7) $$\frac{dy(x)}{dx} = \gamma(x) - \frac{V(x)}{KAG(x)}$$

(3.8) $$\frac{d\gamma(x)}{dx} = M(x)/EI(x)$$

Solving (3.6) and (3.7) for V(x) and $\gamma(x)$ we obtain

$$\frac{1}{KAG(x)} \frac{dM(x)}{dx} + \frac{dy(x)}{dx} = \left(1 - \frac{I_{\mu z}(x)\,\omega^2}{KAG(x)}\right)\gamma(x) + \frac{Q(x)}{KAG(x)}$$

or

(3.9) $$\gamma(x) = \left(\frac{1 - I_{\mu z}(x)\,\omega^2}{KAG(x)}\right)^{-1} \left(\frac{1}{KAG(x)} \frac{dM(x)}{dx} - Q(x) + \frac{dy(x)}{dx}\right)$$

and

$$\frac{dM(x)}{dx} + I_{\mu z}(x)\,\omega^2 \frac{dy(x)}{dx} = V(x) \left(1 - \frac{I_{\mu z}(x)\,\omega^2}{KAG(x)}\right)$$

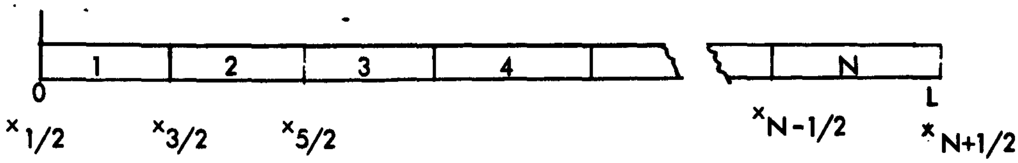
or

(3.10) $$V(x) = \left(1 - \frac{I_{\mu z}(x)\,\omega^2}{KAG(x)}\right)^{-1} \left(\frac{dM(x)}{dx} + I_{\mu z}(x)\omega^2 \frac{dy}{dx}\right)$$

We now subdivide the interval from x = 0 to x = L by the abscissas

$$x_{1/2} = o < x_{3/2} < x_{5/2} < \dots < x_{N+1/2} = L$$

as shown below.



Then if (3.5) is ingegrated from $x_{n-1/2}$ to $x_{n+1/2}$, for n = 1, 2, 3, .., N , we have

$$V_{n+1/2} - V_{n-1/2} = \int_{x_{n-1/2}}^{x_{n+1/2}} \left(\mu(x)\,\omega^2 - ic(x)\omega\right) y\,dx + \int_{x_{n-1/2}}^{x_{n+1/2}} P(x)dx$$

which is approximated by

$$\text{(3.11)} \quad V_{n+1/2} - V_{n-1/2} = \left[(\mu\,\Delta x)_n\,\omega^2 - i(c\,\Delta x)_n\,\omega\right] y_n + (P\Delta x)_n$$

where

$$(\mu\Delta x)_n = \int_{x_{n-1/2}}^{x_{n+1/2}} \mu(x)\,dx$$

$$(c\,\Delta x)_n = \int_{x_{n-1/2}}^{x_{n+1/2}} c(x)dx$$

$$(P \Delta x)_n = \int_{x_{n-1/2}}^{x_{n+1/2}} P(x)dx$$

Now, from (3.10) we can approximate $V_{n+1/2}$ for n = 1, ..., N-1 by

$$(3.12) \quad V_{n+1/2} = \frac{1}{(\Delta x)_{n,n+1}} \left(1 - \left(\frac{I_{\mu z}}{KAG}\right)_{n,n+1} \omega^2\right)^{-1} \left(M_{n+1} - M_n - Q_{n,n+1} + \left(I_{\mu z}\right)_{n,n+1} \omega^2 \left(y_{n+1} - y_n\right)\right)$$

where

$$Q_{n,n+1} = \int_{x_n}^{x_{n+1}} Q(x)dx$$

$$(I_{\mu z} \Delta x)_{n,n+1} = \int_{x_n}^{x_{n+1}} I_{\mu z}(x)dx$$

$$\left(\frac{\Delta x}{KAG}\right)_{n,n+1} = \int_{x_n}^{x_{n+1}} \frac{dx}{KAG(x)}$$

and $$x_n = 1/2\,(x_{n+1/2} + x_{n-1/2}) \;.$$

The end conditions to be imposed determine $V_{1/2}$ and $V_{n+1/2}$. In particular,

(3.13) for $V(O) = 0$ use $V_{1/2} = 0$

for $V(L) = 0$ use $V_{N+1/2} = 0$

Substituting (3.12) into (3.11) and taking (3.13) into account gives for n = 1, 2, ..., N:

(3.14) $$\alpha_{n,n+1}(M_{n+1} - M_n) + \beta_{n,n+1}(y_{n+1} - y_n)$$

$$- \alpha_{n-1,n}(M_n - M_{n-1}) - \beta_{n-1,n}(y_n - y_{n-1}) = \delta_n y_n + P_n + \alpha_{n,n+1} Q_{n,n+1}$$
$$- \alpha_{n-1,n} Q_{n-1,n}$$

where

$$\alpha_{n,n+1} = \begin{cases} \left[1 - \left(\dfrac{I_{\mu z}\,\omega^2}{KAG}\right)_{n,n+1}\right]^{-1} \dfrac{1}{(\Delta x)_{n,n+1}} & \text{for } n = 1, 2, 3, \ldots, N-1 \\ 0 \text{ otherwise} \end{cases}$$

$$\beta_{n,n+1} = \begin{cases} (I_{\mu z})_{n,n+1}\,\omega^2\,\alpha_{n,n+1} & \text{for } n = 1, 2, 3, \ldots, N-1 \\ 0 \text{ otherwise} \end{cases}$$

$$\delta_n = (\mu \Delta x)_n \omega^2 - i(c \Delta x)_n \omega \quad \text{for } n = 1, 2, \ldots, N$$

Now (3.8) can be integrated to give for n = 1, 2, ..., N

$$\gamma_{n+1/2} - \gamma_{n-1/2} = \int_{x_{n-1/2}}^{x_{n+1/2}} \frac{M(x)}{EI(x)} \, dx$$

which can be approximated by

$$(3.15) \quad \gamma_{n+1/2} - \gamma_{n-1/2} = \left(\frac{\Delta x}{EI}\right)_n M_n$$

where

$$\left(\frac{\Delta x}{EI}\right)_n = \int_{x_{n-1/2}}^{x_{n+1/2}} \frac{dx}{EI}$$

From (3.9), $\gamma_{n+1/2}$ can be approximated for n = 1, 2, ..., N-1

$$(3.16) \quad \gamma_{n+1/2} = \left[1 - \left(\frac{I_{\mu z}}{KAG}\right)_{n,n+1} \omega^2\right]^{-1} \left[\frac{M_{n+1} - M_n}{(KAG\,\Delta x)_{n,n+1}} + \frac{y_{n+1} - y_n}{(\Delta x)_{n,n+1}}\right]$$

while $\gamma_{1/2}$ and $\gamma_{N+1/2}$ are determined by end conditions.

In particular since M(O) = 0, and V(O) = 0, we will use the end condition:

$$M_1 = 0$$

Similarly for the other end we will use

$$M_n = 0$$

Substituting (3.16) into (3.15) taking account of end conditions, we obtain for n = 2, 3, .., N-1

$$(3.17) \quad \epsilon_{n,n+1} (M_{n+1} - M_n) - \alpha_{n,n+1} (y_{n+1} - y_n)$$

$$- \epsilon_{n-1,n} (M_n - M_{n-1}) - \alpha_{n-1,n} (y_n - y_{n-1}) = \zeta_n M_n$$

where

$$\epsilon_{n,n+1} = \frac{1}{(KAG)_{n,n+1}} \alpha_{n,n+1} \quad \text{for } n = 1, 2, \ldots, N-1$$

$$\zeta_n = \left(\frac{\Delta x}{EI}\right)_n \quad \text{for } n = 2, 3, \ldots, N-2$$

For convenience (3. 14) and (3. 17) can be rewritten as follows:

(3. 18) $$- \beta_{n-1,\, n}\, y_{n-1} - \alpha_{n-1,\, n} M_{n-1} + (\beta_{n,\, n+1} + \delta_n + \beta_{n-1,\, n})\, y_n$$

$$+ (\alpha_{n,\, n+1} + \alpha_{n-1,\, n})\, M_n - \beta_{n,\, n+1}\, y_{n+1} - \alpha_{n,\, n+1} M_{n+1} = -P_n + \alpha_{n,\, n+1}\, Q_{n,\, n+1} - \alpha_{n-1,\, n}\, Q_{n-1,\, n}$$

(3. 19) $$- \alpha_{n-1,\, n}\, y_{n-1} - \varepsilon_{n-1,\, n} M_{n-1} + (\alpha_{n,\, n-1} + \alpha_{n-1,\, n})\, y_n$$

$$+ (\varepsilon_{n,\, n+1} + \varepsilon_{n-1,\, n} + \zeta_n)\, M_n - \alpha_{n,\, n+1}\, y_{n+1} - \varepsilon_{n,\, n+1} M_{n+1} = 0$$

For $n = 1$ and $n = N$, the equations will incorporate the end conditions specified. In summary, equations (3. 18) and (3. 19) for $n = 1, 2, \ldots, N-1$ can be rewritten in matrix notation

(3. 20) $$A \vec{z} = \vec{P}$$

where

$$(3.21) \qquad A = \begin{vmatrix} A_{11} & A_{12} & 0 & \cdots & & & 0 \\ A_{21} & A_{22} & A_{23} & 0 & & & \vdots \\ 0 & A_{32} & A_{33} & A_{34} & \ddots & & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & A_{N-1,\,N} & \\ 0 & \cdots & & 0 & A_{N,\,N+1} & A_{N,\,N} & \end{vmatrix}$$

$$(3.22) \qquad \vec{z} = \begin{vmatrix} \vec{z}_1 \\ \vec{z}_2 \\ 1 \\ 1 \\ 1 \\ 1 \\ \vec{z}_N \end{vmatrix} , \qquad \vec{P} = \begin{vmatrix} \vec{P}_1 \\ \vec{P}_2 \\ 1 \\ 1 \\ 1 \\ 1 \\ \vec{P}_N \end{vmatrix}$$

where

$$(3.24) \qquad \vec{z}_n = \begin{vmatrix} y_n \\ \\ M_n \end{vmatrix} \qquad \vec{P}_n = \begin{vmatrix} \tilde{P}_n \\ \\ 0 \end{vmatrix}$$

with $\widetilde{P}_n = P_n - \alpha_{n-1,n}\, Q_{n-1,n} + \alpha_{n,n+1}\, Q_{n,n+1}$,

and for $n = 2, 3, \ldots, N-1$

$$A_{n,n} = \begin{vmatrix} \delta_n + \beta_{n-1,n} + \beta_{n,n+1} & \alpha_{n,n+1} + \alpha_{n-1,n} \\ \alpha_{n,n+1} + \alpha_{n-1,n} & \zeta_n + \epsilon_{n-1,n} + \epsilon_{n,n+1} \end{vmatrix}$$

$$A_{n,n+1} = \begin{vmatrix} -\beta_{n,n+1} & -\alpha_{n,n+1} \\ -\alpha_{n,n+1} & -\epsilon_{n,n+1} \end{vmatrix}$$

$$A_{n,n-1} = \begin{vmatrix} -\beta_{n-1,n} & -\alpha_{n-1,n} \\ -\alpha_{n-1,n} & -\epsilon_{n-1,n} \end{vmatrix} = A_{n-1,n}$$

and finally

$$A_{1,1} = \begin{vmatrix} \delta_1 + \beta_{1,2} & \alpha_{1,2} \\ \alpha_{1,2} & \zeta_1 + \epsilon_{1,2} \end{vmatrix}$$

$$A_{NN} = \begin{vmatrix} \delta_N + \beta_{N-1,N} & \alpha_{N-1,N} \\ \alpha_{N-1,N} & \zeta_N + \epsilon_{N-1,N} \end{vmatrix}$$

where ζ_1 and ζ_n are chosen large enough so that M_1 and M_N will vanish for practical purposes.

The above formulation can be generalized to apply to systems of beams with special connections. In this case each beam can be subdivided and the sections numbered as for example in Figure I-1. Then a system of matrix equations of the form

$$A\vec{z} = \vec{P} \tag{3.23}$$

can set up where in general

$$A = \begin{vmatrix} A_{11} & A_{12} & A_{13} & \cdots & A_{1N} \\ A_{21} & A_{22} & A_{23} & \cdots & A_{2N} \\ & & & & \\ A_{N1} & A_{N2} & A_{N3} & \cdots & A_{NN} \end{vmatrix}$$

Here the $A_{n,m}$ are 2x2 submatrices of A and $\vec{z}$ and $\vec{P}$ are defined as in (3.22). In general, $A_{n,m}$ will contain non-zero elements whenever the section numbered n is connected to the section numbered m. In particular, if n and m are adjacent beam sections,

$$A_{n,m} = \begin{vmatrix} -\beta_{n,m} & -\alpha_{n,m} \\ -\alpha_{n,m} & -\varepsilon_{n,m} \end{vmatrix}$$

If sections n and m are connected by a spring

$$A_{n,m} = \begin{vmatrix} +k_{n,m} + i\omega c_{n,m} & 0 \\ 0 & 0 \end{vmatrix}$$

where $k_{n,m}$ is the spring constant for the connection and $c_{n,m}$ is the damping coefficient for the connection. In general

$$A_{n,n} = - \sum_{\substack{m=1 \\ m \neq n}} A_{n,m} + D_n$$

where

$$D_n = \begin{vmatrix} \delta_n & 0 \\ 0 & \zeta_n \end{vmatrix}$$

In addition, if section n is connected to ground, $A_{n,n}$ must be augmented by

$$\begin{vmatrix} -k_n - ic_n\omega & 0 \\ 0 & 0 \end{vmatrix}$$

where k_n is the spring constant for the ground connection and c_n is the damping coefficient.

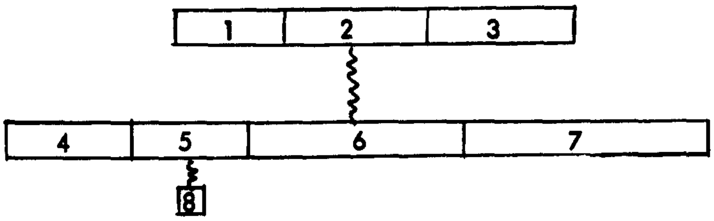


Figure I-1 - Typical Connected Beam Elements

When the above formulation is applied to the set of beams with special connections illustrated in Figure I-1 and a unit force is applied at Station 6, equation (25) takes the form

$$A\vec{z} = \vec{P}$$

where

$$A = \begin{vmatrix} A_{11} & A_{12} & 0 & 0 & 0 & 0 & 0 & 0 \\ A_{21} & A_{22} & A_{23} & 0 & 0 & A_{26} & 0 & 0 \\ 0 & A_{32} & A_{33} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & A_{44} & A_{45} & 0 & 0 & 0 \\ 0 & 0 & 0 & A_{54} & A_{55} & A_{56} & 0 & A_{58} \\ 0 & A_{62} & 0 & 0 & A_{65} & A_{66} & A_{67} & 0 \\ 0 & 0 & 0 & 0 & 0 & A_{76} & A_{77} & 0 \\ 0 & 0 & 0 & 0 & 0 & A_{85} & 0 & A_{88} \end{vmatrix}$$

and

$$P = \begin{vmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ J \\ 0 \\ 0 \end{vmatrix} \quad , \quad J = \begin{vmatrix} 1 \\ 0 \end{vmatrix}$$

The Effective Mass

The effective mass $(\mu \Delta x)_n$ for section n, as it is introduced in equation (3.11) and later used in the definition of δ_n in equation (3.14), is made up of two parts. The first is the mass of the ship section. The second is a frequency dependent virtual mass which takes into account the inertial effect of the surrounding water. The method used to calculate the virtual mass is that discussed in Appendix C of this report which is based on that considered by E. K. Kennard (26).

This code provides for the automatic calculation of the virtual mass coefficient J as a function of frequency for each section n of a system and then the effective mass for section n is calculated using

$$(\mu \Delta x)_n = m_n + J(\omega)\, \overline{m}_n$$

where

m_n is the specified mass of section n

$\overline{m}_n$ is the specified water inertia associated with section n

$J(\omega)$ is the virtual mass coefficient

For each system, the displacement of each section is calculated for a set of frequencies. Until the frequency is such that the real part of the displacement for a section has at least two zeros, an initial specified value of J will be used for that system. As soon as the displacement has at least two zeros, then let x_a represent the position of the first of them, and x_b the position of the last. An average distance between the zeros of the real part of the displacement can be calculated using

$$d = \frac{x_b - x_a}{n - 1}$$

where n is the number of zeros. Then

$$J = \frac{k_1(\alpha)}{k_1(\alpha) - \alpha k_0(\alpha)}$$

where

$$\alpha = \frac{\pi c}{d}$$

c is the system radius, and k_0 and k_1 are the first and second modified Bessel functions of the second kind. J is approximated for $.6 < \alpha < 4$ by

$$.0609 + \frac{.715}{\alpha} - \frac{.19093}{\alpha^2} + \frac{.00297}{\alpha^3}$$

and for $o < a < .6$ by

$$1.030 - 1.56 \frac{d}{c}$$

Input Preparation

The input data required for the GBRC-1 program is that required for setting up the system of difference equations described earlier. In particular, the following kinds of data can be included for each case:

1) The total number of beam sections. This must be less than 80.

2) The range of frequency values and the frequency interval for which response curves are required.

3) The system to which each beam section belongs, i.e., whether to the hull, shaft, or some other system.

4) For the hull and/or any other system, for which the J-factor is required for the added or virtual mass calculation, the following data is required:

a) An initial value for the J-factor

b) The radius

c) A water inertia term for each section

5) The mass of each section

6) The integral over each section of the reciprocal of the bending rigidity, i.e., $(\Delta x/EI)_n$.

7) The integral from the mid-point of each section to the mid-point of the next of the reciprocal of the shear rigidity, i.e., $(\Delta x/KAG)_{n,n+1}$ or $(\Delta_x/KAG)_{n,m}$.

8) The integral of the mass polar moment of inertia from the mid-point of each section to the mid-point of the next, i.e., $(I_{\mu z}\Delta x)_{n,n+1}$ or $(I_{\mu z}\Delta x)_{n,m}$.

9) The spring constant and damping coefficient associated with each spring connection.

10) For each set of parameters, for each system a complex scaling factor can be supplied. This allows damping to be introduced in various ways. In particular, a complex

scaling factor for the ship mass corresponds to the introduction of a Reyleigh damping coefficient.

For each section and connection a system number between 1 and 80 inclusive must be specified. This number is used to select the scaling factors to be applied to the parameters for that section or connection. It is also used in order that the program can select the set of sections and connections which make up a given system so that the zeros of the displacement of that system can be determined for the calculation of the virtual mass coefficient J. Section numbers for sections which are connected to each other should differ by less than 15.

The input data cards for GBRC-1 are laid out as shown in Figures I-2 to I-6. The contents of these cards are described in detail below. In general, when a series of problems is being run, only those data cards which contain changes from the previous case are required. The numbers punched in columns 3 and 4 of each card identify the kind of information that card contains. An input deck should begin with a title card, and the set of data for each problem should begin with a data control card (columns 3 and 4 contain 90) which gives, for each type of data card included, the number of such cards. The data cards for the problem should follow in the same sequence as they are given on the data control card. The last card of the input deck for a set of problems should contain 99 in columns 3 and 4 if no additional problems are to be run, and 98 in columns 3 and 4 if a complete data set for another unrelated problem follows.

Detailed formats for each type of card are given below.

1) Run Title Card

Columns	Contents
7 to 12	(62H△△
13 to 72	These contain whatever title is to be associated with this run.

TITLE ____ PROGRAMMER ____ DATE ____

PROBLEM NO. ____ PHASE ____ LABEL ☐☐☐☐ ____ SHEET ____ OF ____

RUN TITLE CARD

000000(62 HΔΔ

DATA CONTROL CARD

NO TYPE NO TYPE etc.

0090

5 7 9 11 13 15 17 19 21 23 25 27 29 31 33 35 37 39 41 43 45 47 49 51 53 55 57 59 61 63 65 67 69 71

CASE TITLE CARD

001000(62 HΔΔ

OPTION CONTROL CARD

	OP1	OP2	OP3	OP4	OP5	OP6	OP7	OP8	OP9	OP10
00200000	9	13	17	21	25	29	33	37	41	45

EDIT CONTROL CARD

00210000	9	13	17	21	25

29

GENERAL DATA CARD

	NO. SECTIONS	ω_1 (CPS)	ω_2 (CPS)	$\Delta\omega$ (CPS)
0030	5	9	17	25

SYSTEMS DATA CARDS

	SYSTEMS	RADIUS	INITIAL J
0031			
0031			

Figure I-2 - Data Input Card (1)

TITLE ______________________ PROGRAMMER ______________ DATE ________

PROBLEM NO. ________________ PHASE ______ LABEL [| | |] ______ SHEET ____ OF ____

SECTION PARAMETER VALUES - UNSCALED

	5	9	13	17	25	33	41	49	57	65	73
	SECTION NO	END CONDN.	SYSTEM	MASS	WATER INERTIA	$(\Delta x / EI)_n$	$(\Delta x)_{n,n+1}$	$(\Delta x / KAG)_{n,n+1}$	$(I_{\mu z} \Delta x)_{n,n+1}$	P_n	
0043											
0043											
0043											
0043											
0043											
0043											
0043											
0043											
0043											
0043											
0043											
0043											
0043											
0043											
0043											
0043											
0043											
0043											
0043											
0043											
0043											
0043											
0043											
0043											

Figure I-3 – Data Input Card (2)

TITLE ______ PROGRAMMER ______ DATE ______

PROBLEM NO. ______ PHASE ______ LABEL ______ SHEET ______ OF ______

SECTION DATA CARDS

	5	9	13	17	25	33	41	49	57	65	73
	SECTION NO.	END CONDN.	SYSTEM	MASS	WATER INERTIA	$(\Delta x/EI)_n$	$(\Delta x)_{n,n+1}$	$(\Delta x/kAG)_{n,n+1}$	$(I\mu z\,\Delta x)_{n,n+1}$	P_n	
REAL PART OF SCALING FACTORS											
00410000											
00410000											
00410000											
IMAGINARY PART OF SCALING FACTORS											
00420000											
00420000											
00420000											
PARAMETER VALUES - UNSCALED											
0043											
0043											
0043											
0043											
0043											
0043											
0043											
0043											
0043											
0043											
0043											
0043											
0043											
0043											
0043											

Figure I-4 - Data Input Card (3)

TITLE ______ PROGRAMMER ______ DATE ______

PROBLEM NO. ______ PHASE ______ LABEL ☐☐☐☐ ______ SHEET ______ OF ______

	5	9	13	17	25	33	41	40	57	65	73
SPECIAL CONNECTION CARDS											
	n	m	SYSTEM	$K_{n,m}$	$C_{n,m}$	$C_{n,m}/\omega$	$\Delta X_{n,m}$	$(\Delta X/KAG)_{n,m}$	$(I_{\mu z}\Delta x)_{n,m}$		
REAL PART OF SCALING FACTORS											
0051											
0051											
0051											
IMAGINARY PART OF SCALING FACTORS											
0052											
0052											
0052											
PARAMETER VALUES - UNSCALED											
0053											
0053											
0053											
0053											
0053											
0053											
0053											
0053											
0053											
0053											

Figure I-5 - Data Input Card (4)

TITLE ______ PROGRAMMER ______ DATE ______

PROBLEM NO. ______ PHASE ______ LABEL - ______ SHEET ______ OF ______

SPECIAL CONNECTIONS - PARAMETER VALUES

	5	9	13	17	25	33	41	49	57	65	73
	n	m	SYSTEM	$K_{n,m}$	$C_{n,m}$	$C_{n,m}/\omega$	$(\Delta x)_{n,m}$	$(\Delta x/KAG)_{n,m}$	$(I_{\mu z}\,\Delta x)_{n,m}$		
0053											
0053											
0053											
0053											
0053											
0053											
0053											
0053											
0053											
0053											
0053											
0053											
0053											
0053											
0053											
0053											
0053											
0053											
0053											
0053											
0053											
0053											
0053											
0053											

Figure I-6 - Data Input Card (5)

2) Data Control Card

Columns	Contents
3 to 4	90 (Type of data)
5 to 6 7 to 8	Columns 5, 6 contain number of data cards of type given by columns 7, 8 which follow for this case.
9 to 10 11 to 12 etc.	Similar to previous four columns for next data cards for this case.

3) Case Title Card

Columns	Contents
3 to 4	10
7 to 12	(62H△△
13 to 72	Title for this case

4) Option Control Card

3 to 4	20
9 to 12	Op1: Added mass option selector 0 if added mass as specified in input is used, 1 if J water inertia as specified in input is to be used (See Section 4)
13 to 16	Op 2: Selector for A-matrix and P vector setup option. 0 for present.
17 to 20	Op 3: Selector for edit routine to be used at each frequency. 0 for present.
21 to 24	Op 4: Selector for final edit routine. 0 for present.

Columns	Contents
25 to 28	Op 5: Selector for A-matrix routine. 0 if A-matrix is not printed; 1 if A-matrix is to be printed.

5) Edit Control Card

Columns	Contents
3 to 4	21
9 to 12 13 to 16 17 to 20 21 to 24 25 to 28	Section numbers for those sections for which displacements and moments are to be tabulated vs frequency.

6) General Data Card

Columns	Contents
3 to 4	30
7 to 8	Number of sections; this can be at most 80.
9 to 16	Starting frequency in CPS
17 to 24	Upper limit for the frequency in CPS
25 to 32	Frequency interval to be used in CPS

7) System Data Cards for Added Mass Calculation

Columns	Contents
3 to 4	31
7 to 8	System number. The hull sections, shafting sections, etc., can be distinguished from each other by assigning a system number to each set of sections. The range of these numbers can be from 1 to 80.
9 to 16	Radius associated with system in the units in which the unscaled distances between adjacent section centers are given.
17 to 24	Initial J-value for use with this system.

8) Section Parameter Cards

Columns	Contents
3 to 4	43 or 44
7 to 8	Section number
12	End condition - normally 0. 1 for $V(o) = M(o) = 0$ 2 for $V(L) = M(L) = 0$
15 to 16	System number
17 to 24	Ship mass
25 to 32	Water inertia
33 to 40	$(\Delta x/EI)_n$
41 to 48	$(\Delta x)_{n, n+1}$
49 to 56	$(\Delta x/KAG)_{n, n+1}$
57 to 64	$(1_{\mu z}\Delta x)_{n, n+1}$
65 to 72	P_n if columns 3 and 4 contain 43, $Q_{n, n+1}$ if those columns contain 44.

9) Scaling Factor Cards for Section Parameters

Columns	Contents
3 to 4	41 if this card contains real parts of scaling factor. 42 if this card contains the imaginary parts of the scaling factors.
15 to 16	System number for which scaling factors on this card are to be applied.
17 to 72	Scaling factor for each parameter are given in those columns containing that parameter value on the section parameter cards.

10) Special Connection Parameter Cards

3 to 4	53
7 to 8) 11 to 12)	The section numbers for the connection - n and m if m = 0, this will represent a connection from section n to ground.
15 to 16	System number with which connection is to be associated
17 to 24	$k_{n,m}$ Spring constant for connection
25 to 32	$C_{n,m}$ Constant damping coefficient for connection
33 to 40	$C_{n,m}/\omega$ Coefficient for frequency dependent damping for connection
41 to 48	$(\Delta x)_{n,m}$
49 to 56	$(\Delta x/KAG)_{n,m}$
57 to 63	$(I_{\mu z}\Delta x)_{n,m}$

Note that in general either $k_{n,m}$, $C_{n,m}$ and/or $\frac{C_{n,m}}{\omega}$ or $(\Delta x)_{n,m}$, $\frac{\Delta x}{KAG}_{n,m}$

and $(I_{\mu z}\,\Delta x)_{n,m}$ will be supplied depending on whether sections n and m are connected by a spring or are two adjacent sections of a beam. It is important that the system numbers in these two cases be distinct, if J factors for the virtual mass calculation are to be calculated for the beam.

11) Scaling factor cards for parameters in special connections

Columns	Contents
3 to 4	51: This card contains real parts of scaling factors
	52: This card contains imaginary parts of scaling factors.

Columns	Contents
13 to 16	System number with which these scaling factors are to be associated
17 to 56	Scaling factors for parameters in corresponding locations on special connection Parameter Cards.

12) End of Data

Columns	Contents
3 to 4	99

APPENDIX J

INPUT DATA FOR VERTICAL BENDING CALCULATIONS

TITLE CBRC 1 PROGRAMMER ______ DATE 10/10.

PROBLEM NO. Case 12 Conesco PHASE ______ LABEL 8 4 0 277 SHEET 1 OF 3

RUN TITLE CARD

000000(62 HΔΔ Conesco Problem Series - Project 4-962-Vertical Bending-

DATA CONTROL CARD

NO TYPE NO TYPE etc.

0090	01	10	01	21	01	30	01	41	25	43
	5	7	9		13	15	17	19	21	23

25 27 29 31 33 35 37 39 41 43 45 47 49 51 53 55 57 59 61 63 65 67 69 71

CASE TITLE CARD

001000(62 HΔΔ Case 12 SSBN 598 Submerged Sept 16, 1962

OPTION CONTROL CARD

	OP1	OP2	OP3	OP4	OP5	OP6	OP7	OP8	OP9	OP10
00200000										
	9	13	17	21	25	29	33	37	41	45

EDIT CONTROL CARD

00210000	0001	0012	0025		
	9	13	17	21	25 29

GENERAL DATA CARD

	NO. SECTIONS	ω_1 (CPS)	ω_2 (CPS)	$\Delta\omega$ (CPS)
0030	0025	1.0	26.0	0.5
	5	9	17	25

SYSTEMS DATA CARDS

	SYSTEMS	RADIUS	INITIAL J
0031			
0031			

TITLE ______ PROGRAMMER ______ DATE 10/10

PROBLEM NO. Case 12 4-962 PHASE ______ LABEL [8][4][D] 277 SHEET 2 OF 3

SECTION DATA CARDS

5	9	13	17	25	33	41	49	57	65	73
	SECTION NO.	END CONDN.	SYSTEM	MASS	WATER INERTIA	$(\Delta x/EI)_n$	$(\Delta x)_{n,n+1}$	$(\Delta x/KAG)_{n,n+1}$	$(I_{MZ}\Delta x)_{n,n+1}$	P_n
REAL PART OF SCALING FACTORS										
0041	0000		0001	1.0	1.0	1.0E-8	1.0	1.0E-5	1.	1.
0041	0000									
0041	0000									
IMAGINARY PART OF SCALING FACTORS										
0042	0000									
0042	0000									
0042	0000									
PARAMETER VALUES - UNSCALED										
0043	0001	-001	0001	2.3520	5.4795	12.8732	15	2.1319	62.44	
0043	0002		0001	2.5605	4.4091	8.0585	15	.9504		
0043	0003		0001	2.0820	2.8665	3.7922	15	.6671		
0043	0004		0001	4.8915	4.8925	2.0146	15	.5414		
0043	0005		0001	4.8300	5.4045	1.4015	15	.4642		
0043	0006		0001	5.0580	6.4470	1.1115	15	.3459		
0043	0007		0001	8.8680	7.3140	1.0233	15	.3754		
0043	0008		0001	8.9085	8.0610	0.9343	15	.3188		
0043	0009		0001	11.7810	8.5635	0.5655	15	.3324		
0043	0010		0001	12.4485	8.9340	0.4327	15	.2494		
0043	0011		0001	15.8820	9.4170	0.3663	15	.2084		
0043	0012		0001	11.7195	9.7815	0.3358	15	.2005		
0043	0013		0001	11.0325	9.8055	0.3358	15	.2005		
0043	0014		0001	11.7615	9.8055	0.3358	15	.2005		
0043	0015		0001	11.8020	9.8055	0.3358	15	.2005		

TITLE ______ PROGRAMMER ______ DATE 1/10

PROBLEM NO. Case 12 Conesco 4-962 PHASE ______ LABEL B40 277 SHEET 3 OF 3

SECTION PARAMETER VALUES - UNSCALED

	5	9	13	17	25	33	41	49	57	65	73
	SECTION NO	END CONDN.	SYSTEM	MASS	WATER INERTIA	$(\Delta x/EI)_n$	$(\Delta x)_{n,n+1}$	$(\Delta x/KAG)_{n,n+1}$	$(I_{\mu z}\Delta x)_{n,n+1}$	P_n	
0043	0016		0001	11.4690	9.8055	0.3358	15	0.2068			
0043	0017		0001	7.8270	9.6630	0.3476	15	.2221			
0043	0018		0001	9.8040	9.4170	0.3663	15	.2250			
0043	0019		0001	6.7230	11.3145	0.3884	15	.2508	1787.95		
0043	0020		0001	8.4300	8.4340	0.4284	15	.3324			
0043	0021		0001	4.8090	8.4645	0.5328	15	.2843			
0043	0022		0001	9.3255	7.5720	0.7584	15	.3705			
0043	0023		0001	8.6385	6.0120	1.3157	15	.4880			
0043	0024		0001	6.7440	3.8805	2.9304	15	1.6863			
0043	0025	0002	0001	2.9145	1.4655	9.2098	15				
00[illegible]											
0043											
0043											
0043											
0043											
0043											
0043											
0043											
0043											
0043											
0043											
0043											
0043											
0043											

TITLE CBRC1

PROBLEM NO. Conesco Case 13 4-962 PROGRAMMER ______ DATE ______

PHASE ______ LABEL 840 277 SHEET 1 OF 8

RUN TITLE CARD

000000(62HΔΔ SSB(N) 598 OCT 20, 1962 Conesco Vertical Bending

DATA CONTROL CARD

NO TYPE NO TYPE etc.

00020	01	10	01	21	01	30	02	41	01	42	42	43	02	51	01	52	28	53

5 7 9 11 13 15 17 19 21 23 25 27 29 31 33 35 37 39 41 43 45 47 49 51 53 55 57 59 61 63 65 67 69 71

CASE TITLE CARD

001000(62HΔΔ Case 13 25 Masses in Hull J = .87 For 1 to 3.5 cps

OPTION CONTROL CARD

	OP1	OP2	OP3	OP4	OP5	OP6	OP7	OP8	OP9	OP10
00200000										

9 13 17 21 25 29 33 37 41 45

EDIT CONTROL CARD

00210000	0001	0041			

9 13 17 21 25 29

GENERAL DATA CARD

	NO. SECTIONS	ω_1 (CPS)	ω_2 (CPS)	$\Delta\omega$ (CPS)
0030	42	1.0	3.5	0.5

5 9 17 25

SYSTEMS DATA CARDS

	SYSTEMS	RADIUS	INITIAL J
0031			
0031			

TITLE ____ PROGRAMMER ____ DATE ____

PROBLEM NO. Conesco Case 13 PHASE ____ LABEL 840 277 SHEET 2 OF 8

SECTION DATA CARDS

5	9 SECTION NO.	11 END CONDN.	13 SYSTEM	17 MASS	25 WATER INERTIA	33 $(\Delta x/EI)_n$	41 $(\Delta x)_{n,n+1}$	49 $(\Delta x/kAG)_{n,n+1}$	57 $(I_{MZ}\Delta x)_{n,n+1}$	65 P_n	73
REAL PART OF SCALING FACTORS											
00410000			0001	.0023591	.0023350	0.990E-12	12	0.990E-4		1.0	
00410000			2	.0023591							
00410000											
IMAGINARY PART OF SCALING FACTORS											
00420000						-.994E-12		-.099E-9			
00420000											
00420000											
PARAMETER VALUES - UNSCALED											
0043	0001	0001	0001	200 000	133 200	10.00	15	3.2			
0043	2		1	519 600	456 000	1.157	15	24.0			
0043	3		1	660 300	572 000	0.345	12	19.1			
0043	4		1	921 800	631 000	0.243	12	13.2			
0043	5		1	654 652	704 000	0.164	15	11.03			
0043	6		1	449 000	744 000	0.176	15	13.33			
0043	7		1	810 400	840 000	0.128	0.	0.			
0043	0010		1	765 000	902 000	0.127	15	12.6			
0043	11		1	473 100	925 000	0.128	0.	0.			
0043	14		1	443 700	929 000	0.107	15	11.25			
0043	15		1	492 600	930 000	0.107	12	11.50			
0043	16		1	445 000	930 000	0.107	12	11.52			
0043	17		1	478 000	930 000	0.107	12	11.50			
0043	18		1	921 800	929 000	0.110	12	12.30			
0043	19		1	1 072 200	900 000	0.130	0.	0.			

TITLE ______ PROGRAMMER ______ DATE ______

PROBLEM NO. Conesco Case 13 PHASE ______ LABEL 840 277 SHEET 3 OF 8

SECTION PARAMETER VALUES - UNSCALED

	SECTION NO (5)	END CONDN. (9)	SYSTEM (13)	MASS (17)	WATER INERTIA (25)	$(\Delta x/EI)_n$ (33)	$(\Delta x)_{n,n+1}$ (41)	$(\Delta x/KAG)_{n,n+1}$ (49)	$(I_{\mu z}\Delta x)_{n,n+1}$ (57)	P_n (65)
0043	23		0001	1 156 200	845 000	0.175	0.	0.		
0043	25		1	779 043	794 000	0.211	15	16.92		
0043	26		1	630 900	694 000	0.343	0.	0.		
0043	24		1	559 300	612 000	0.352	15	18.43		
0043	30		1	390 850	557 000	0.334	0.	0.		
0043	35		1	354 000	452 000	0.454	15	28.63		
0043	36		1	280 100	354 000	0.825	15	36.3		
0043	37		1	214 300	249 000	1.440	15	52.4		
0043	38		1	209 700	135 200	4.046				
0043	41	0002	1	191 700	48 900	10				1.0
0043	8		2	37 300						
0043	9		2	37 300						
0043	12		2	6 900						
0043	13		2	3 145						
0043	20		2	15 200						
0043	21		2	15 200						
0043	22		2	12 261						
0043	24		2	3 450						
0043	27		2	5 300						
0043	28		2	32 920						
0043	31		2	3 991						
0043	32		2	27 470						
0043	33		2	27 470						
0043	34		2	3 991						

TITLE ____

PROBLEM NO. Conesco Case 13

PROGRAMMER ____ DATE ____

PHASE ____ LABEL 890 277 SHEET 4 OF 8

SECTION PARAMETER VALUES - UNSCALED

	5	9	13	17	25	33	41	49	57	65
	SECTION NO	END CONDN.	SYSTEM	MASS	WATER INERTIA	$(\Delta x/EI)_n$	$(\Delta x)_{n,n+1}$	$(\Delta x/\kappa AG)_{n,n+1}$	$(I_{\mu z}\Delta x)_{n,n+1}$	P_n
0043	0039		0002	4300						
0043	40		2	125000						
0043	42		2	125000						
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										

TITLE ______ PROGRAMMER ______ DATE ______

PROBLEM NO. Conesco Case 13 PHASE ______ LABEL 840 277 SHEET 5 OF 8

5	9	13	17	25	33	41	49	57	65	73

SPECIAL CONNECTION CARDS

	n	m	SYSTEM	$K_{n,m}$	$C_{n,m}$	$C_{n,m}/\omega$	$\Delta X_{n,m}$	$(\Delta X/KAG)_{n,m}$	$(I_{\mu z}\Delta x)_{n,m}$	
REAL PART OF SCALING FACTORS										
0051			0001				12	0.990E-9		
0051			2	1.0	1.0	1.0				
0051										
IMAGINARY PART OF SCALING FACTORS										
0052			0001					-0.099E-9		
0052										
0052										
PARAMETER VALUES - UNSCALED										
0053	0007	0008	0002	122 200	0.					
0053	7	9	2	122 200	0.					
0053	0011	0012	2	31 440	24.3					
0053	11	13	2	13 600	12.6					
0053	19	20	2	50 000	60.7					
0053	19	21	2	51 600	41.6					
0053	19	22	2	38 700	33.[illegible]					
0053	23	24	2	14 400	14.5					
0053	26	27	2	16 000	16.4					
0053	26	28	2	46 000	1[illegible]4					

TITLE ____ PROGRAMMER ____ DATE ____

PROBLEM NO. CASE 13 PHASE ____ LABEL 840 277 SHEET 4 OF 8

SPECIAL CONNECTIONS - PARAMETER VALUES

	n	m	SYSTEM	$K_{n,m}$	$C_{n,m}$	$C_{n,m}/\omega$	$(\Delta x)_{n,m}$	$(\Delta x/KAG)_{n,m}$	$(I_{\mu z}\Delta x)_{n,m}$		
	5	9	13	17	25	33	41	49	57	65	73
0053	0030	0031	0002	16 700	16.7						
0053	30	32	2	80 500	79.1						
0053	36	33	2	80 500	79.1						
0053	36	34	2	16 700	16.7						
0053	35	39	2	16 800	19.8						
0053	13	40	2	1820 000							
0053	41	42	2	1820 000							
0053	7	11	1				15	12.23			
0053	11	14	1				15	12.38			
0053	14	23	1				15	13.65			
0053	23	22	1				15	18.34			
0053	26	29	1				15	20.75			
0053	30	32	1				15	25.22			
0053	38	41	1				15	108.5			
0053	5		2			60.					
0053	9		2			60.					
0053	40		2			110.					
0053	42		2			110.					
0053											
0053											
0053											
0053											
0053											
0053											

TITLE
PROBLEM NO. *Change Cards To Case 13*
ω = 3.5 To 25

PROGRAMMER
PHASE LABEL 840 277

DATE
SHEET 7 OF 8

RUN TITLE CARD
000000(62HΔΔ

DATA CONTROL CARD

	NO TYPE	NO TYPE etc.				
0090	01	10	01	30	01	41

CASE TITLE CARD
001000(62HΔΔ *Case 13 J = .82 For 3.5 To 25.0 cps*

OPTION CONTROL CARD

	OP1	OP2	OP3	OP4	OP5	OP6	OP7	OP8	OP9	OP10
00200000										

EDIT CONTROL CARD

00210000					

GENERAL DATA CARD

	NO. SECTIONS	ω_1 (CPS)	ω_2 (CPS)	$\Delta\omega$ (CPS)
0030	0042	3.5	25.0	0.5

SYSTEMS DATA CARDS

	SYSTEMS	RADIUS	INITIAL J
0031			
0031			

TITLE ______ PROGRAMMER ______ DATE ______

PROBLEM NO. Conesco Case 13 PHASE ______ LABEL 840 277 SHEET 8 OF 8

SECTION DATA CARDS

Section No. (5)	End Condn. (9)	System (13)	Mass (17)	Water Inertia (25)	$(\Delta x/EI)_n$ (33)	$(\Delta x)_{n,n+1}$ (41)	$(\Delta x/kAG)_{n,n+1}$ (49)	$(I_{\mu z}\Delta x)_{n,n+1}$ (57)	P_n (65)	(73)
REAL PART OF SCALING FACTORS										
00410000		0001	.002591	.002125	0.990E-12	12	0.990E-9			
00410000										
00410000										
IMAGINARY PART OF SCALING FACTORS										
00420000										
00420000										
00420000										
PARAMETER VALUES - UNSCALED										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										

TITLE GBRC1: DATA FORMAT 1 PROGRAMMER ______ DATE ______

PROBLEM NO. Case 14 Follows Case 13 PHASE ______ LABEL 840 277 SHEET 1 OF 5

RUN TITLE CARD

000000(62HΔΔ

DATA CONTROL CARD

NO TYPE NO TYPE etc.

0090	01	10	01	21	01	30	02	41	01	42	03	43	01	51	01	52	04	53
	5	7	9		13	15	17	19	21	23	25	27	29	31	33	35	37	39

41 43 45 47 49 51 53 55 57 59 61 63 65 67 69 71

CASE TITLE CARD

001000(62HΔΔ Case 14 Case 13 WITH Propulsion System,

OPTION CONTROL CARD

	OP1	OP2	OP3	OP4	OP5	OP6	OP7	OP8	OP9	OP10
00200000										
	9	13	17	21	25	29	33	37	41	45

EDIT CONTROL CARD

00210000	0001	0037	0041	0045	
	9	13	17	21	25 29

GENERAL DATA CARD

	NO. SECTIONS	ω_1 (CPS)	ω_2 (CPS)	$\Delta\omega$ (CPS)
0030	0045	1.0	3.5	0.2
5	9		17	25

SYSTEMS DATA CARDS

	SYSTEMS	RADIUS	INITIAL J
0031			
0031			

TITLE GARC I: DATA FORMAT 2 PROGRAMMER DATE

PROBLEM NO. Case 14 PHASE LABEL 840 277 SHEET 2 OF 5

SECTION DATA CARDS

5 9 13 17 25 33 41 49 57 65 73

SECTION NO.	END CONDN.	SYSTEM	MASS	WATER INERTIA	$(\Delta x/EI)_n$	$(\Delta x)_{n,n+1}$	$(\Delta x/kAG)_{n,n+1}$	$(I_{MZ}\Delta x)_{n,n+1}$	P_n	
REAL PART OF SCALING FACTORS										
00410000		0001	.002591	.002254	.990 E-12	12.0	0.990E -9		1.0	
00410000		0003	.002591		0.990E-12	1.0	0.990E -9	2591.0	1.0	
00410000										
IMAGINARY PART OF SCALING FACTORS										
00420000		0003			-.099 E-12		-0.099E -9			
00420000										
00420000										
PARAMETER VALUES - UNSCALED										
0043 0043		0003	13600.		1214	196.5				
0043 0044		0003	8220.		641.5	55.125		59.0		
0043 0045		0003	17900.		75.5					
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										

TITLE GBRC1: DATA FORMAT 3 PROGRAMMER ______ DATE ______

PROBLEM NO. Case 14 PHASE ______ LABEL 840 277 SHEET 3 OF 5

	5	9	13	17	25	33	41	49	57	65

SPECIAL CONNECTION CARDS

	n	m	SYSTEM	$K_{n,m}$	$C_{n,m}$	$C_{n,m}/\omega$	$\Delta X_{n,m}$	$(\Delta X/KAG)_{n,m}$	$(I_{\mu z}\Delta x)_{n,m}$
REAL PART OF SCALING FACTORS									
0051			0003	1.			1.0	0.990 E-9	
0051									
0051									
IMAGINARY PART OF SCALING FACTORS									
0052			0003					-.099 E-9	
0052									
0052									
PARAMETER VALUES - UNSCALED									
0053	37	43	0003				149.875		
0053	38	43	0002	232000.					
0053	41	44	0002	538000.					
0053	45		0002		40.				
0053									
0053									
0053									
0053									
0053									
0053									

TITLE GBRC 1: DATA FORMAT 1

PROBLEM NO. Case 14

PROGRAMMER

PHASE

LABEL 840 277

DATE

SHEET 4 OF 5

RUN TITLE CARD

000000(62 HΔΔ

DATA CONTROL CARD

NO TYPE NO TYPE etc.

0090	01	10	01	30	01	41
	5	7	9	11	13	15

CASE TITLE CARD

001000(62 HΔΔ Case 14 J=.82 For 3.5 To 25.0 cps

OPTION CONTROL CARD

	OP1	OP2	OP3	OP4	OP5	OP6	OP7	OP8	OP9	OP10
00200000										
	9	13	17	21	25	29	33	37	41	45

EDIT CONTROL CARD

00210000					
	9	13	17	21	25

GENERAL DATA CARD

	NO. SECTIONS	ω_1 (CPS)	ω_2 (CPS)	$\Delta\omega$ (CPS)
0030	0045	3.5	25.	0.5
	5	9	17	25

SYSTEMS DATA CARDS

	SYSTEMS	RADIUS	INITIAL J
0031			
0031			

TITLE *GBRC1: DATA FORMAT 2* PROGRAMMER ______ DATE ______

PROBLEM NO. *Case 14* PHASE ______ LABEL 8 4 D *277* SHEET *5* OF *5*

SECTION DATA CARDS

5 9 13 17 25 33 41 49 57 65 73

SECTION NO.	END CONDN.	SYSTEM	MASS	WATER INERTIA	$(\Delta x/EI)_n$	$(\Delta x)_{n,n+1}$	$(\Delta x/kAG)_{n,n+1}$	$(I\mu z\,\Delta x)_{n,n+1}$	P_n	
REAL PART OF SCALING FACTORS										
00410000		0001	.002591	.002125	9.90E-13	12.	0.990 E-9		1.4	
00410000										
00410000										
IMAGINARY PART OF SCALING FACTORS										
00420000										
00420000										
00420000										
PARAMETER VALUES - UNSCALED										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										

TITLE GBRC I: DATA FORMAT I PROGRAMMER ______ DATE ______

PROBLEM NO. Case 15 Follows Case 14 PHASE ______ LABEL 840 277 SHEET 1 OF 4

RUN TITLE CARD

000000(62 HΔΔ

DATA CONTROL CARD

NO TYPE NO TYPE etc.

00000 01 10 02 45

5 7 9 13 15 17 19 21 23 25 27 29 31 33 35 37 39 41 43 45 47 49 51 53 55 57 59 61 63 65 67 69 71

CASE TITLE CARD

001000(62 HΔΔ Case 15 Similiar To Case 14 Except That Force is at Propeller

OPTION CONTROL CARD

OP	OP2	OP3	OP4	OP5	OP6	OP7	OP8	OP9	OP10

00200000 9 13 17 21 25 29 33 37 41 45

EDIT CONTROL CARD

00210000 9 13 17 21 25 29

GENERAL DATA CARD

NO. SYSTEMS	ω_1 (CPS)	ω_2 (CPS)	$\Delta\omega$ (CPS)

0030 1 5 9 17 25

SYSTEMS DATA CARDS

	SYSTEMS	RADIUS	INITIAL J
0031			
0031			

TITLE GBRC1: DATA FORMAT 2 PROGRAMMER ______ DATE ______

PROBLEM NO. Case 15 PHASE ______ LABEL 840 277 SHEET 2 OF 4

SECTION DATA CARDS

1	5	9	13	17	25	33	41	49	57	65	73
	SECTION NO.	END CONDN.	SYSTEM	MASS	WATER INERTIA	$(\Delta x/EI)_n$	$(\Delta x)_{n,n+1}$	$(\Delta x/kAG)_{n,n+1}$	$(I\mu z\,\Delta x)_{n,n+1}$	P_n	
REAL PART OF SCALING FACTORS											
00410000											
00410000											
00410000											
IMAGINARY PART OF SCALING FACTORS											
00420000											
00420000											
00420000											
PARAMETER VALUES - UNSCALED											
0043	0041	0002	0001	191 700.	48 900						
0043	0045		0003	27 900.						1.0	
0043											
0043											
0043											
0043											
0043											
0043											
0043											
0043											
0043											
0043											
0043											
0043											
0043											

TITLE GBRCI DATA FORMAT 1 PROGRAMMER ______ DATE ______

PROBLEM NO Case 15 PHASE ______ LABEL 540 277 SHEET 3 OF 4

RUN TITLE CARD

000000(62 HΔΔ

DATA CONTROL CARD

NO TYPE NO TYPE etc.

0000	01	10	01	30	01	41

1 5 7 9 13 15 17 19 21 23 25 27 29 31 33 35 37 39 41 43 45 47 49 51 53 55 57 59 61 63 65 67 69 71

CASE TITLE CARD

001000(62 HΔΔ Case 15 J=.87 For 1.0 To 3.5 cps

OPTION CONTROL CARD

	OP1	OP2	OP3	OP4	OP5	OP6	OP7	OP8	OP9	OP10
00200000										

9 13 17 21 25 29 33 37 41 45

EDIT CONTROL CARD

00210000					

9 13 17 21 25 29

GENERAL DATA CARD

	NO. SYSTEMS	ω_1 (CPS)	ω_2 (CPS)	$\Delta\omega$ (CPS)
0030	.0045	1.0	3.5	.2

5 9 17 25

SYSTEMS DATA CARDS

	SYSTEMS	RADIUS	INITIAL J
0031			
0031			

TITLE GBRC1: DATA FORMAT 2 PROGRAMMER ______ DATE ______

PROBLEM NO. Case 15 PHASE ______ LABEL. 840 277 SHEET 4 OF 4

SECTION DATA CARDS

5 9 13 17 25 33 41 49 57 65 73

SECTION NO.	END CONDN.	SYSTEM	MASS	WATER INERTIA	$(\Delta x/EI)_n$	$(\Delta x)_{n,n+1}$	$(\Delta x/kAG)_{n,n+1}$	$(I_{\mu z}\Delta x)_{n,n+1}$	P_n	
REAL PART OF SCALING FACTORS										
00410000		0001	.002591	.002254		12.0	0.990 E-9		1.0	
00410000										
00410000										
IMAGINARY PART OF SCALING FACTORS										
00420000										
00420000										
00420000										
PARAMETER VALUES - UNSCALED										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										

TITLE ______ PROGRAMMER ______ DATE ______

PROBLEM NO. *Conesco Case 16* PHASE ______ LABEL [840] *277* SHEET *1* OF *2*

RUN TITLE CARD

000000(62 HΔΔ *SSB(N)598* *OCT. 26, 1962* *Conesco* *Vertical Bending*

DATA CONTROL CARD

NO TYPE NO TYPE etc.

0090	01	10	01	21	01	30	03	41	02	42	36	43	03	51	02	52	31	53
	5	7	9		13	15	17	19	21	23	25	27	29	31	33	35	37	39

CASE TITLE CARD

001000(62 HΔΔ *Case 16* *16 Sections + Prop. System* *J = 0.87 For 1.0 To 3.5 cps*

OPTION CONTROL CARD

	OP1	OP2	OP3	OP4	OP5	OP6	OP7	OP8	OP9	OP10
00200000										
	9	13	17	21	25	29	33	37	41	45

EDIT CONTROL CARD

00210000	0001	0028	0033	0036	
	9	13	17	21	25 29

GENERAL DATA CARD

	NO. SECTIONS	ω_1 (CPS)	ω_2 (CPS)	$\Delta\omega$ (CPS)
0030	0036	1.0	3.5	0.2
	5	9	17	25

SYSTEMS DATA CARDS

	SYSTEMS	RADIUS	INITIAL J
0031			
0031			

TITLE GBRC1: DATA FORMAT 2 PROGRAMMER ______ DATE ______

PROBLEM NO. Case 16 PHASE ______ LABEL 840 277 SHEET 2 OF 7

SECTION DATA CARDS

5 9 13 17 25 33 41 49 57 65 73

	SECTION NO.	END CONDN.	SYSTEM	MASS	WATER INERTIA	$(\Delta x/EI)_n$	$(\Delta x)_{n,n+1}$	$(\Delta x/kAG)_{n,n+1}$	$(I\mu z\,\Delta x)_{n,n+1}$	P_n	
REAL PART OF SCALING FACTORS											
00410000			0001	-.002591	.002254	.990 E-12	12.	0.990E-9	1.0 E+6	1.0	
00410000			2	.002591		0.					
00410000			3	.002591		.990 E-12	1.0	0.990E-9	2591.0	1.0	
IMAGINARY PART OF SCALING FACTORS											
00420000			0001			-.099E-12		-.099E-9			
00420000			0003			-.99 E-13		-.099E-9			
00420000											
PARAMETER VALUES - UNSCALED											
0043	0001	0001	0001	7500.	6350.	10.6	10.5	167.9			
0043	2		1	238500.	367500.	2.76	33.	81.7			
0043	3		1	1578225.	1207000.	0.672	24.	27.38			
0043	4		1	1027700.	1298000.	0.330	29.	28.95			
0043	5		1	1342000.	1575000.	0.260					
0043	8		1	1642000.	1253000.	0.236					
0043	11		1	2357950.	2266000.	0.206	30.	23.6			
0043	12		1	916000.	867000.	0.196	31.	19.5			
0043	13		1	3311400.	2880000.	0.271					
0043	18		1	841943.	824000.	0.317	9.	10.6			
0043	19		1	288600.	326000.	0.341	21.	29.12			
0043	20		1	1423600.	1686000.	0.780					
0043	27		1	661156.	808000.	1.659	22.	63.5			
0043	28		1	200250.	209300.	2.706					
0043	30		1	225770.	42800	5.99					

TITLE GBRC1 DATA FORMAT 2A PROGRAMMER ______ DATE ______

PROBLEM NO. Case 16 PHASE ______ LABEL 840 277 SHEET 3 OF 7

SECTION PARAMETER VALUES - UNSCALED

	SECTION NO (5)	END CONDN. (9)	SYSTEM (13)	MASS (17)	WATER INERTIA (25)	$(\Delta x/EI)_n$ (33)	$(\Delta x)_{n,n+1}$ (41)	$(\Delta x/KAG)_{n,n+1}$ (49)	$(I_{\mu z}\Delta x)_{n,n+1}$ (57)	P_n (65)
0043	33	0002	0001	33420.	11200.	20.0				
0043	6		2	37300.						
0043	7		2	37300.						
0043	9		2	6900.						
0043	10		2	3145.						
0043	14		2	15200.						
0043	15		2	15200.						
0043	16		2	12261.						
0043	17		2	3450.						
0043	21		2	5300.						
0043	22		2	32920.						
0043	23		2	3991.						
0043	24		2	27471.						
0043	25		2	27470.						
0043	26		2	3991.						
0043	29		2	4300.						
0043	31		2	12500.						
0043	32		2	12500.						
0043	34		3	13600.		1214	196.5			
0043	35		3	8220.		641.5	55.125		59.0	
0043	36		3	27900.		95.5				1.0
0043										
0043										
0043										

TITLE GBRC1: DATA FORMAT 3 PROGRAMMER ______ DATE ______

PROBLEM NO. Case 16 PHASE ______ LABEL 8 4 0 277 SHEET 4 OF 7

	5	9	13 · 17	25	33	41	40	57	65	73
SPECIAL CONNECTION CARDS										
	n	m	SYSTEM	$K_{n,m}$	$C_{n,m}$	$C_{n,m}/\omega$	$\Delta X_{n,m}$	$(\Delta X/KAG)_{n,m}$	$(I_{\mu z}\Delta X)_{n,m}$	
REAL PART OF SCALING FACTORS										
0051			0001	0.0	0.0		12.	0.990E-9		
0051			2	1.	1.	1.				
0051			3				1.	0.990E-9		
IMAGINARY PART OF SCALING FACTORS										
0052			0001					-0.099E-9		
0052			0003					-0.099E-9		
0052										
PARAMETER VALUES - UNSCALED										
0053	0005	0008	0001				29.	23.55		
0053	8	0011	1				28.	22.19		
0053	0013	0018	1				40	41.4		
0053	20	27	1				46	74.6		
0053	28	30	1				12	58.5		
0053	30	33	1				12	120.0		
0053	28	34	3				149.875			
0053	30	34	2	232000.						
0053	33	35	2	538000.						
0053	36		2		40.					

TITLE ______ PROGRAMMER ______ DATE ______

PROBLEM NO. ______ PHASE ______ LABEL - 840 277 SHEET 5 OF 7

SPECIAL CONNECTIONS - PARAMETER VALUES

	n	m	SYSTEM	$K_{n,m}$	$C_{n,m}$	$C_{n,m}/\omega$	$(\Delta x)_{n,m}$	$(\Delta x/KAG)_{n,m}$	$(I_{yz}\Delta x)_{n,m}$	
0053	0005	0006	0002	122 200						
0053	6		2							
0053	5	7	2	122 200.						
0053	7		2							
0053	8	9	2	31 440.	29.3					
0053	8	10	2	13 400.	12.6					
0053	13	14	2	50 000.	60.7					
0053	13	15	2	51 600.	41.6					
0053	13	16	2	38 700.	33.3					
0053	13	17	2	14 400.	14.5					
0053	20	21	2	16 000	16.4					
0053	20	22	2	46 000	104.					
0053	20	23	2	16 700	16.7					
0053	20	24	2	80500	79.1					
0053	20	25	2	80500	79.1					
0053	20	26	2	16700	16.7					
0053	28	29	2	16800	19.8					
0053	30	31	2	182 0000						
0053	30	32	2	1820000						
0053	31		2			110				
0053	32		2			110				
0053										
0053										
0053										

TITLE GBRC1 DATA FORMAT 1 PROGRAMMER DATE

PROBLEM NO. Case 16 PHASE LABEL 840 277 SHEET 6 OF 7

RUN TITLE CARD

000000(62 HΔΔ

DATA CONTROL CARD

NO TYPE NO TYPE etc.

0090	01	10	01	30	01	71
	5	7	9	13	15	

CASE TITLE CARD

001000(62 HΔΔ Case 16 16 SECTIONS + PROP SYSTEM S=.82 For 3.5 To 25 cps

OPTION CONTROL CARD

	OP1	OP2	OP3	OP4	OP5	OP6	OP7	OP8	OP9	OP10
00200000										
	9	13	17	21	25	29	33	37	41	45

EDIT CONTROL CARD

00210000					
	9	13	17	21	25

GENERAL DATA CARD

	NO. SECTIONS	ω_1(CPS)	ω_2(CPS)	$\Delta\omega$ (CPS)
0030	0036	3.5	25.	.5
	5	9	17	25

ω = 3.5 cps To 10.0 cps $\Delta\omega$ = 0.2 cps

ω = 10.0 cps To 25.0 cps $\Delta\omega$ = 0.5 cps

SYSTEMS DATA CARDS

	SYSTEMS	RADIUS	INITIAL J
0031			
0031			

TITLE GBRC I: DATA FORMAT 2 PROGRAMMER ________ DATE ________

PROBLEM NO. Case 16 PHASE ______ LABEL. 8 4 C 277 SHEET 7 OF 7

SECTION DATA CARDS

5 SECTION NO.	9 END CONDN.	13 SYSTEM	17 MASS	25 WATER INERTIA	33 $(\Delta x/EI)_{n'}$	41 $(\Delta x)_{n,n+1}$	49 $(\Delta x/kAG)_{n,n+1}$	57 $(I_{\mu z}\Delta x)_{n,n+1}$	65 P_n	73
REAL PART OF SCALING FACTORS										
00410000		0001	.002591	.002125	.990E-12	12.	0.990E-9		1.0	
00410000										
00410000										
IMAGINARY PART OF SCALING FACTORS										
00420000										
00420000										
00420000										
PARAMETER VALUES - UNSCALED										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										

TITLE GBRC 1: DATA FORMAT 1 PROGRAMMER ______ DATE ______

PROBLEM NO. Case 17 PHASE ______ LABEL 840 .277 SHEET 1 OF 8

RUN TITLE CARD

000000(62HΔΔ SSB(N) 598 OCT. 26, 1962 CONESCO VERTICAL BENDING

DATA CONTROL CARD

NO TYPE NO TYPE etc.

0090	01	10	01	21	01	30	03	41	02	42	56	43	03	51	02	52	36	53
	5	7	9		13	15	17	19	21	23	25	27	29	31	33	35	37	39

CASE TITLE CARD

001000(62HΔΔ Case 17 35 Sections + Prop System J = .87 For 1.0 To 3.5 cps

OPTION CONTROL CARD

	OP1	OP2	OP3	OP4	OP5	OP6	OP7	OP8	OP9	OP10
00200000										
	9	13	17	21	25	29	33	37	41	45

EDIT CONTROL CARD

00210000	1	46	54	56	
	9	13	17	21	25

GENERAL DATA CARD

	NO. SECTIONS	ω_1 (CPS)	ω_2 (CPS)	$\Delta\omega$ (CPS)
0030	0056	1.0	3.5	.2
	5	9	17	25

SYSTEMS DATA CARDS

	SYSTEMS	RADIUS	INITIAL J
0031			
0031			

TITLE GBRC1 DATA FORMAT 2 PROGRAMMER DATE

PROBLEM NO. Case 17 PHASE LABEL. 840 277 SHEET 2 OF 8

SECTION DATA CARDS

5	9	13	17		25	33	41	49	57	65	73
	SECTION NO.	END CONDN.	SYSTEM	MASS	WATER INERTIA	$(\Delta x/EI)_n$	$(\Delta x)_{n,n+1}$	$(\Delta x/kAG)_{n,n+1}$	$(I_{MZ}\Delta x)_{n,n+1}$	P_n	
REAL PART OF SCALING FACTORS											
00410000			0001	.002591	.002224	0.990E-12	12.	0.990E-9	0.	1.0	
00410000			0002	.002591	0.	0.	1.	0.	0.	1.0	
00410000			0003	.002591	0.	.990E-12	1.	0.990E-9	2591.	1.0	
IMAGINARY PART OF SCALING FACTORS											
00420000			0001	0.	0.	-0.099E-12		-.099E-9			
00420000			0003			-0.099E-12		-.099E-9			
00420000											
PARAMETER VALUES - UNSCALED											
0043	0001	0001	0001	7500.	6350.	10	4.5	114.			
0043	2		1	69700.	47800.	1.26	6.	33.9			
0043	3		1	157750.	115200.	1.38	9.	40.8			
0043	4		1	355100.	206000.	1.363	9.	21.6			
0043	5		1	292750.	367000.	0.317	15.	19.3			
0043	6		1	360475.	475000	0.198	16	14.3			
0043	7		1	485000.	365000	0.161	7.	6.73			
0043	8		1	341250.	355000	0.095	7.	8.33			
0043	9		1	337000.	427000	0.133	11.	12.5			
0043	0010		1	299450.	376000	0.134	10.	10.0			
0043	11		1	316800.	471000	0.0343	8.	6.42			
0043	12		1	507450.	451000	0.072					
0043	13		1	517750.	563000	0.0198	16	8.13			
0043	16		1	583250.	607000	0.0854	16.	8.10			
0043	17		1	534000.	585000	0.0511					

TITLE GBRC1: DATA FORMAT 2A PROGRAMMER DATE

PROBLEM NO. Case 17 PHASE LABEL 840 277 SHEET 3 OF 8

SECTION PARAMETER VALUES - UNSCALED

	SECTION NO (5)	END CONDN. (9)	SYSTEM (13)	MASS (17)	WATER INERTIA (25)	$(\Delta x/EI)_n$ (33)	$(\Delta x)_{n,n+1}$ (41)	$(\Delta x/KAG)_{n,n+1}$ (49)	$(I_{\mu z}\Delta x)_{n,n+1}$ (57)	P_n (65)
0043	0020		1	524750.	560000.	0.0706	9.	6.90		
0043	21		1	598500.	591000.	0.0679	10.	7.64		
0043	22		1	813450.	745000.	0.0857	14.	10.8		
0043	23		1	945000.	930000.	0.1070	16.	12.2		
0043	24		1	914000.	868000.	0.1000	12.	9.3		
0043	25		1	786000.	777000.	0.0994	13.	10.2		
0043	26		1	1009900.	953000.	0.1217				
0043	29		1	1515500.	1150000.	0.218				
0043	32		1	541943.	824000.	0.216	9.	10.6		
0043	33		1	286600.	336000	0.153	5.	6.92		
0043	34		1	484100.	456000.	0.247				
0043	36		1	562000.	623000.	0.353				
0043	38		1	377500.	607000.	0.422				
0043	43		1	334500.	434000.	0.561	11.	23.2		
0043	44		1	226500.	223000.	0.619	9.	22.0		
0043	45		1	116750.	151000.	0.632	8.	23.9		
0043	46		1	101050.	91800.	0.772	5.	17.6		
0043	47		1	55000.	68900.	1.063				
0043	49		1	44200.	48600.	1.794				
0043	51		1	225770.	42800.	4.773				
0043	54	0002	1	33420.	11200.	15-				
0043	83		2	37300.						
0043	14		2	37300.						
0043										

TITLE GBRC1 DATA FORMAT 2A PROGRAMMER ______ DATE ______

PROBLEM NO. Case 17 PHASE ______ LABEL 840 277 SHEET 4 OF 8

SECTION PARAMETER VALUES - UNSCALED

5 9 13 17 25 33 41 49 57 65 73

	SECTION NO	END CONDN.	SYSTEM	MASS	WATER INERTIA	$(\Delta x/EI)_n$	$(\Delta x)_{n,n+1}$	$(\Delta x/KAG)_{n,n+1}$	$(I_{\mu z}\Delta x)_{n,n+1}$	P_n
0043	0018		0002	6900.						
0043	19		2	3145.						
0043	27		2	15200.						
0043	28		2	15200.						
0043	30		2	12261.						
0043	31		2	2450.						
0043	35		2	5300.						
0043	37		2	32420.						
0043	39		2	3991.						
0043	40		2	27470.						
0043	41		2	27470.						
0043	42		2	3991.						
0043	45		2	4300.						
0043	52		2	125000.						
0043	53		2	125000.						
0043	54		3	11600.		1214				
0043	55		3	8230.		641.5	55.125		59.	
0043	56		3	27900.		75.5				1.0
0043										
0043										
0043										
0043										
0043										
0043										

TITLE GBRC 1: DATA FORMAT 3 PROGRAMMER DATE

PROBLEM NO. Case 17 PHASE LABEL 8 1 0 277 SHEET 5 OF 8

5 9 13 17 25 33 41 49 57 65 73

SPECIAL CONNECTION CARDS

	n	m	SYSTEM	$K_{n,m}$	$C_{n,m}$	$C_{n,m}/\omega$	$\Delta X_{n,m}$	$(\Delta X/KAG)_{n,m}$	$(I_{\mu z}\Delta X)_{n,m}$
REAL PART OF SCALING FACTORS									
0051			0001	1.0	1.0	1.0	12.	0.990 E-9	
0051			2	1.0	1.0	1.0	0.	0.	
0051			3	1.0	1.0	1.0	1.0	0.990 E-9	
IMAGINARY PART OF SCALING FACTORS									
0052			1					-.099 E-9	
0052			3					-.099 E-9	
0052									
PARAMETER VALUES - UNSCALED									
0053	0012	0013	0002	122200.					
0053	12	14	2	122200.					
0053	17	18	2	31440.	34.3				
0053	17	19	2	13600.	12.6				
0053	26	27	2	50000.	60.7				
0053	26	28	2	21600.	41.6				
0053	29	30	2	38700.	33.3				
0053	29	31	2	14400.	14.5				
0053	34	35	2	16000.	16.4				
0053	36	37	2	46000.	104.				

TITLE GBRC1 DATA FORMAT 3A PROGRAMMER DATE

PROBLEM NO. Case 17 PHASE LABEL - 340 277 SHEET 6 OF 8

SPECIAL CONNECTIONS · PARAMETER VALUES

5	9	13	17	25	33	41	49	57	65	73
	n	m	SYSTEM	$K_{n,m}$	$C_{n,m}$	$C_{n,m}/\omega$	$(\Delta x)_{n,m}$	$(\Delta x/KAG)_{n,m}$	$(I_{\mu z}\Delta x)_{n,m}$	
0053	0038	0037	0002	16700.	16.7					
0053	38	40	0002	80500.	79.1					
0053	35	41	0002	80500.	79.1					
0053	35	42	0002	16700.	16.7					
0053	42	46	0002	16800.	19.8					
0053	51	52	0002	1820000.						
0053	51	53	0002	1520000.						
0053	12	0015	0001				9.	7.36		
0053	17	20	0001				9.	7.65		
0053	26	29	0001				18.	15.2		
0053	29	32	0001				22.	26.2		
0053	34	36	0001				16.	22.2		
0053	36	38	0001				15.	18.6		
0053	38	43	0001				20.	32.2		
0053	47	49	0001				7.42	30.1		
0053	49	51	0001				4.58	28.4		
0053	50	54	0001				12.	120		
0053	49	50	0002	232000						
0053	54	55	0002	538000						
0053	50	55	0003				196.5			
0053	18		0004			26.				
0053	19		0002			64.				
0053	52		0002			110.				
0053	53		0002			111.				
0053	56		0002		40					
0053	46	50	3				149.875			

TITLE GBRC1: DATA FORMAT 1 PROGRAMMER DATE
PROBLEM NO. Case 17 PHASE LABEL 840 277 SHEET 7 OF 8

RUN TITLE CARD

000000(62HΔΔ

DATA CONTROL CARD

	NO TYPE	NO TYPE	etc.			
0090	01	10	01	30	01	41
	5	7	9	11	13	15

CASE TITLE CARD

001000(62HΔΔ Case 17 35 Sections + Prop System J = .82 For 3.5 To 25.0 cps

OPTION CONTROL CARD

	OP1	OP2	OP3	OP4	OP5	OP6	OP7	OP8	OP9	OP10
00200000										
	9	13	17	21	25	29	33	37	41	45

EDIT CONTROL CARD

00210000					
	9	13	17	21	25

GENERAL DATA CARD

	NO. SECTIONS	ω_1 (CPS)	ω_2 (CPS)	$\Delta\omega$ (CPS)
0030	0035	3.5	25	~~.2~~ .5
	5	9	17	25

3.5 cps To 10.0 cps $\Delta\omega$ = .2 cps
10.0 cps To 25.0 cps $\Delta\omega$ = .5 cps

SYSTEMS DATA CARDS

	SYSTEMS	RADIUS	INITIAL J
0031			
0031			

TITLE GBRC1: DATA FORMAT 2 PROGRAMMER DATE

PROBLEM NO. Case 17 PHASE LABEL. 840 277 SHEET 8 OF 8

SECTION DATA CARDS

5 SECTION NO.	9 END CONDN.	13 SYSTEM	17 MASS	25 WATER INERTIA	33 $(\Delta x/EI)_n$	41 $(\Delta x)_{n,n+1}$	49 $(\Delta x/KAG)_{n,n+1}$	57 $(I_{HZ}\Delta x)_{n,n+1}$	65 P_n	73
REAL PART OF SCALING FACTORS										
00410000		[illegible]	[illegible]	[illegible]	.990 E -12	[illegible]	0.990E -9		1.0	
00410000										
00410000										
IMAGINARY PART OF SCALING FACTORS										
00420000										
00420000										
00420000										
PARAMETER VALUES - UNSCALED										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										

TITLE GBRC 1: DATA FORMAT 1 PROGRAMMER ____ DATE ____

PROBLEM NO. Case 18 PHASE ____ LABEL 840 277 SHEET 9 OF ____

RUN TITLE CARD

000000(62HΔΔ

DATA CONTROL CARD

NO TYPE VO TYPE etc.

0090	01	10	01	30	01	31	01	41	0	1	2	0
	5	7	9		13	15	17	19	21	23	25	27

CASE TITLE CARD

001000(62HΔΔ Case 18 35 Sections + Prop System J Determined For Sine Mode Shape

OPTION CONTROL CARD

	OP1	OP2	OP3	OP4	OP5	OP6	OP7	OP8	OP9	OP10
00200000	0001									
	9 (?)	13	17	21	25	29	33	37	41	45

EDIT CONTROL CARD

00210000					
	9	13	17	21	25 29

GENERAL DATA CARD

	NO. SECTIONS	ω_1 (CPS)	ω_2 (CPS)	$\Delta\omega$ (CPS)
0030	0056	1.0	~~20.0~~	~~.5~~
	5	9	17	25

1.0 cps To 10.0 cps $\Delta\omega$ = .2 cps

10.0 cps To 25.0 cps $\Delta\omega$ = .5 cps

SYSTEMS DATA CARDS

	SYSTEMS	RADIUS	INITIAL J
0031	0001	16.0 (ft)	.87
0031			

TITLE GBRC 1: DATA FORMAT 2 PROGRAMMER ____ DATE ____

PROBLEM NO. Case 18 PHASE ____ LABEL 8 4 0 277 SHEET 10 OF ____

SECTION DATA CARDS

5 SECTION NO.	9 END CONDN.	13 SYSTEM	17 MASS	25 WATER INERTIA	33 $(\Delta x/EI)_{n}$	41 $(\Delta x)_{n,n+1}$	49 $(\Delta x/KAG)_{n,n+1}$	57 $(I_{MZ}\Delta x)_{n,n+1}$	65 P_n	73
REAL PART OF SCALING FACTORS										
00410000		0001	.0022591	.0022591	9.90E-13	12.	0.990E-9		1.0	
00410000										
00410000										
IMAGINARY PART OF SCALING FACTORS										
00420000										
00420000										
00420000										
PARAMETER VALUES - UNSCALED										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										

TITLE *GBRC DATA FORMAT 1* PROGRAMMER ________ DATE ________

PROBLEM NO. *Case 19* PHASE ________ LABEL 840 *277* SHEET *11* OF ____

RUN TITLE CARD

000000(62HΔΔ

DATA CONTROL CARD

NO TYPE NO TYPE etc.

0090	01	10	02	41	02	42	02	57	02 52

5 7 9 13 15 17 19 21 23 25 27 29 31 33 35 37 39 41 43 45 47 49 51 53 55 57 59 61 63 65 67 69 71

CASE TITLE CARD

001000(62HΔΔ *Case 19 35 Sections + Prop System - Sine Mode J - No Hull Damping*

OPTION CONTROL CARD

	OP1	OP2	OP3	OP4	OP5	OP6	OP7	OP8	OP9	OP10
00200000										
	9	13	17	21	25	29	33	37	41	45

EDIT CONTROL CARD

00210000					
	9	13	17	21	25 29

GENERAL DATA CARD

	NO. SECTIONS	ω_1 (CPS)	ω_2 (CPS)	$\Delta\omega$ (CPS)
0030				
	5	9	17	25

SYSTEMS DATA CARDS

	SYSTEMS	RADIUS	INITIAL J
0031			
0031			

TITLE *GBRC: DATA FORMAT 2* PROGRAMMER ______ DATE ______

PROBLEM NO. *Case 19* PHASE ______ LABEL 840 *277* SHEET *12* OF ___

SECTION DATA CARDS

	5 SECTION NO.	9 END CONDN.	13 SYSTEM	17 MASS	25 WATER INERTIA	33 $(\Delta x/EI)_{n'}$	41 $(\Delta x)_{n,n+1}$	49 $(\Delta x/kAG)_{n,n+1}$	57 $(I_{HZ}\Delta x)_{n,n+1}$	65 P_n	73
REAL PART OF SCALING FACTORS											
00410000			0001	.002591	.002591	1.000 E-12	12.	1.000 E-4		1.0	
00410000			0002	.002591	0.	1.000 E-12	1.	0.	0.	1.0	
			0003	.002591	0.	1.000 E-12	1.	1.000 E-9	2591.0	1.0	
00410000											
IMAGINARY PART OF SCALING FACTORS											
00420000			0001			0		0			
00420000			0003			0		0			
00420000											
PARAMETER VALUES - UNSCALED											
0043											
0043											
0043											
0043											
0043											
0043											
0043											
0043											
0043											
0043											
0043											
0043											
0043											
0043											
0043											

TITLE GBRC: DATA FORMAT 3 PROGRAMMER ____ DATE ____

PROBLEM NO. Case 19 PHASE ____ LABEL 840 277 SHEET 19 OF ____

5 9 13 17 25 33 41 49 57 65 73

SPECIAL CONNECTION CARDS

	n	m	SYSTEM	$K_{n,m}$	$C_{n,m}$	$C_{n,m}/\omega$	$\Delta X_{n,m}$	$(\Delta X/KAG)_{n,m}$	$(I_{\mu z}\Delta x)_{n,m}$	
REAL PART OF SCALING FACTORS										
0051			1	1.0	1.0	1.0	12.	1.000 E-9		
0051			3	1.	1.	1.	1.	1.000 E-9		
0051										
IMAGINARY PART OF SCALING FACTORS										
0052			1					0		
0052			3					0		
0052										
PARAMETER VALUES - UNSCALED										
0053										
0053										
0053										
0053										
0053										
0053										
0053										
0053										
0053										
0053										

TITLE GBRC: DATA FORMAT 1 PROGRAMMER DATE

PROBLEM NO. Case 20 PHASE LABEL 8 4 C 277 SHEET 14 OF

RUN TITLE CARD

000000(62 H$\Delta\Delta$

DATA CONTROL CARD

NO TYPE NO TYPE etc.

0090	01	10	02	41	02	42	02	51	02	52
	5	7	9	13	15	17	19	21	23	25

27 29 31 33 35 37 39 41 43 45 47 49 51 53 55 57 59 61 63 65 67 69 71

CASE TITLE CARD

001000(62 H$\Delta\Delta$ Case 20 Similiar To Case 19 with Special Damping - (1+.04J)

OPTION CONTROL CARD

	OP1	OP2	OP3	OP4	OP5	OP6	OP7	OP8	OP9	OP10
00200000										
	9	13	17	21	25	29	33	37	41	45

EDIT CONTROL CARD

00210000					
	9	13	17	21	25

29

GENERAL DATA CARD

	NO. SECTIONS	ω_1 (CPS)	ω_2 (CPS)	$\Delta\omega$ (CPS)
0030				
	5	9	17	25

SYSTEMS DATA CARDS

	SYSTEMS	RADIUS	INITIAL J
0031			
0031			

TITLE GBRC: DATA FORMAT 2 PROGRAMMER DATE

PROBLEM NO. Case 20 PHASE LABEL 840 277 SHEET 15 OF

SECTION DATA CARDS

5	9	13	17	25	33	41	49	57	65	73
SECTION NO.	END CONDN.	SYSTEM	MASS	WATER INERTIA	$(\Delta x/EI)_n$	$(\Delta x)_{n,n+1}$	$(\Delta x/kAG)_{n,n+1}$	$(I_{\mu z}\Delta x)_{n,n+1}$	P_n	
REAL PART OF SCALING FACTORS										
00410000		1	.002591	.002591	.998 E-12	12.	.9984 E-9		1.0	
00410000		3	.002591	0.	.998 E-12	1.	.9984 E-9	2591.	1.0	
00410000										
IMAGINARY PART OF SCALING FACTORS										
00420000					-.040 E-12		-.040 E-9			
00420000					-.040 E-12		-.040 E-9			
00420000										
PARAMETER VALUES - UNSCALED										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										

TITLE GBRC DATA FORMAT 3 PROGRAMMER ______ DATE ______

PROBLEM NO. Case 20 PHASE ______ LABEL 840 277 SHEET 16 OF ___

5 9 13 17 25 33 41 49 57 65 73

SPECIAL CONNECTION CARDS

	n	m	SYSTEM	$K_{n,m}$	$C_{n,m}$	$C_{n,m}/\omega$	$\Delta X_{n,m}$	$(\Delta X/KAG)_{n,m}$	$(I_{\mu z}\Delta x)_{n,m}$	
REAL PART OF SCALING FACTORS										
0051			1	1.	1.	1.	12	.9984 E-9		
0051			3	1.	1.	1.	1.	.9984 E-9		
0051										
IMAGINARY PART OF SCALING FACTORS										
0052			1					-.040 E-9		
0052			3					-.040 E-9		
0052										
PARAMETER VALUES - UNSCALED										
0053										
0053										
0053										
0053										
0053										
0053										
0053										
0053										
0053										
0053										

TITLE GBRC: DATA FORMAT 1 PROGRAMMER ______ DATE ______

PROBLEM NO. Case 21 PHASE ______ LABEL B40 277 SHEET 12 OF ______

RUN TITLE CARD

000000(62 HΔΔ

DATA CONTROL CARD

NO TYPE NO TYPE etc.

0090	01	10	02	41	02	42	02	51	02	52
	5	7	9	13	15	17	19	21	23	25

CASE TITLE CARD

001000(62 HΔΔ Case 21 Similier To Case 19 With Special Damping - (1+.25J)

OPTION CONTROL CARD

	OP1	OP2	OP3	OP4	OP5	OP6	OP7	OP8	OP9	OP10
00200000										
	9	13	17	21	25	29	33	37	41	45

EDIT CONTROL CARD

00210000					
	9	13	17	21	25

GENERAL DATA CARD

	NO. SECTIONS	ω_1 (CPS)	ω_2 (CPS)	$\Delta\omega$ (CPS)
0030				
	5	9	17	25

SYSTEMS DATA CARDS

	SYSTEMS	RADIUS	INITIAL J
0031			
0031			

TITLE GBRC: DATA FORMAT 2 PROGRAMMER ______ DATE ______

PROBLEM NO. Case 21 PHASE ______ LABEL 8 4 C 277 SHEET 18 OF ______

SECTION DATA CARDS

5 SECTION NO.	9 END CONDN.	13 SYSTEM	17 MASS	25 WATER INERTIA	33 $(\Delta x/EI)_n$	41 $(\Delta x)_{n,n+1}$	49 $(\Delta x/kAG)_{n,n+1}$	57 $(I_{HZ}\Delta x)_{n,n+1}$	65 P_n	73
REAL PART OF SCALING FACTORS										
00410000		0001	.002591	.002591	.9414 E-12	12.	.9414 E-9		1.0	
00410000		3	.002591		.9414 E-12	1.	.9414 E-9		1.0	
00410000										
IMAGINARY PART OF SCALING FACTORS										
00420000		0001			-.235 E-12		-.235 E-9			
00420000		3			-.235 E-12		-.235 E-9			
00420000										
PARAMETER VALUES - UNSCALED										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										

TITLE GBRC DATA FORMAT 3 PROGRAMMER ______ DATE ______

PROBLEM NO. Case 21 PHASE ______ LABEL 840 277 SHEET 19 OF ___

	5	9	13	17	25	33	41	49	57	65

SPECIAL CONNECTION CARDS

	n	m	SYSTEM	$K_{n,m}$	$C_{n,m}$	$C_{n,m}/\omega$	$\Delta X_{n,m}$	$(\Delta X/KAG)_{n,m}$	$(I_{\mu z}\Delta x)_{n,m}$	
REAL PART OF SCALING FACTORS										
0051			0001				12.	.9414 E-9		
0051			0003				1.	.9414 E-9		
0051										
IMAGINARY PART OF SCALING FACTORS										
0052			0001					-.235 E-9		
0052			0003					-.235 E-9		
0052										
PARAMETER VALUES - UNSCALED										
0053										
0053										
0053										
0053										
0053										
0053										
0053										
0053										
0053										
0053										

TITLE GBRC DATA FORMAT 1 PROGRAMMER ______ DATE ______

PROBLEM NO. Case 22 PHASE ______ LABEL 840 377 SHEET 20 OF ____

RUN TITLE CARD

000000(62 HΔΔ

DATA CONTROL CARD

NO TYPE NO TYPE etc.

00000 01 10 02 41 02 42 02 51 02 52 02 53

5 7 9 13 15 17 19 21 23 25 27 29 31 33 35 37 39 41 43 45 47 49 51 53 55 57 59 61 63 65 67 69 71

CASE TITLE CARD

001000(62 HΔΔ Case 22 35 Masses & Prop System increased stiffness between systems 1 and 3

OPTION CONTROL CARD

OP1 OP2 OP3 OP4 OP5 OP6 OP7 OP8 OP9 OP10

00200000

9 13 17 21 25 29 33 37 41 45

EDIT CONTROL CARD

00210000

9 13 17 21 25 29

GENERAL DATA CARD

NO. SECTIONS ω_1(CPS) ω_2(CPS) $\Delta\omega$ (CPS)

0030

5 9 17 25

SYSTEMS DATA CARDS

	SYSTEMS	RADIUS	INITIAL J
0031			
0031			

TITLE GBRC DATA FORMAT 2 PROGRAMMER ______ DATE ______

PROBLEM NO. Case 22 PHASE ______ LABEL [8|4|0] 277 SHEET 21 OF ______

SECTION DATA CARDS

5	SECTION NO. (9)	END CONDN. (13)	SYSTEM (17)	MASS (25)	WATER INERTIA (33)	$(\Delta x/EI)_n$ (41)	$(\Delta x)_{n,n+1}$ (49)	$(\Delta x/kAG)_{n,n+1}$ (57)	$(I_{\mu z}\Delta x)_{n,n+1}$ (65)	P_n (73)	
REAL PART OF SCALING FACTORS											
00410000			0001	.002591	.002591	0.990 E-12	12.	0.990 E-9		1.0	
00410000			3	.002591		0.990 E-12	1.	0.990 E-9		1.0	
00410000											
IMAGINARY PART OF SCALING FACTORS											
00420000			0001			-.099 E-12		-.099 E-9			
00420000			3			-.099 E-12		-.099 E-9			
00420000											
PARAMETER VALUES - UNSCALED											
0043											
0043											
0043											
0043											
0043											
0043											
0043											
0043											
0043											
0043											
0043											
0043											
0043											
0043											
0043											

TITLE GBRC DATA FORMAT 3 PROGRAMMER ______ DATE ______

PROBLEM NO. Case 22 PHASE ______ LABEL 84b 277 SHEET 22 OF ___

	5	9	13	17	25	33	41	49	57	65	73

SPECIAL CONNECTION CARDS

	n	m	SYSTEM	$K_{n,m}$	$C_{n,m}$	$C_{n,m}/\omega$	$\Delta X_{n,m}$	$(\Delta X/KAG)_{n,m}$	$(I_{\mu z}\Delta X)_{n,m}$	
REAL PART OF SCALING FACTORS										
0051		0	0001	1.	1.	1.	12.	0.990E-9	0.	
0051			0003				1.	0.990E-9	0.	
0051										
IMAGINARY PART OF SCALING FACTORS										
0052			0001					-.099E-9		
0052			0003					-.099E-9		
0052										
PARAMETER VALUES - UNSCALED										
0053	0049	0050	0002	700.000.						
0053	54	55	2	1200000.						
0053										
0053										
0053										
0053										
0053										
0053										
0053										
0053										

TITLE GBRC I DATA FORMAT I PROGRAMMER ______ DATE ______

PROBLEM NO. Case 23 PHASE ______ LABEL 840 277 SHEET 23 OF ____

RUN TITLE CARD

000000(62HΔΔ

DATA CONTROL CARD

NO TYPE NO TYPE etc.

00900I 10 02 53

5 7 9 13 15 17 19 21 23 25 27 29 31 33 35 37 39 41 43 45 47 49 51 53 55 57 59 61 63 65 67 69 71

CASE TITLE CARD

001000(62HΔΔ Case 23 35 Masses & Prop System Decreased Stiffness between Systems 1 and 3.

OPTION CONTROL CARD

	OP1	OP2	OP3	OP4	OP5	OP6	OP7	OP8	OP9	OP10
00200000										
	9	13	17	21	25	29	33	37	41	45

EDIT CONTROL CARD

00210000					
	9	13	17	21	25 29

GENERAL DATA CARD

	NO. SECTIONS	ω_1(CPS)	ω_2(CPS)	$\Delta\omega$ (CPS)
0030				
	5	9	17	25

SYSTEMS DATA CARDS

	SYSTEMS	RADIUS	INITIAL J
0031			
0031			

TITLE GBRC 1: DATA FORMAT 3 PROGRAMMER DATE

PROBLEM NO. Case 23 PHASE LABEL 640 277 SHEET 24 OF

	n	m	SYSTEM	$K_{n,m}$	$C_{n,m}$	$C_{n,m}/\omega$	$\Delta X_{n,m}$	$(\Delta X/KAG)_{n,m}$	$(I_{\mu z}\Delta X)_{n,m}$	
	5	9	13	17	25	33	41	49	57	65
SPECIAL CONNECTION CARDS										
REAL PART OF SCALING FACTORS										
0051										
0051										
0051										
IMAGINARY PART OF SCALING FACTORS										
0052										
0052										
0052										
PARAMETER VALUES - UNSCALED										
0053	0049	0050	0002	100 000.						
0053	54	55	2	200 000.						
0053										
0053										
0053										
0053										
0053										
0053										
0053										
0053										

TITLE GBRC-1 PROGRAMMER DATE

PROBLEM NO. CASE 24 - Conesco PHASE LABEL 840 277 SHEET OF

35 CONC. Masses + Propulsion Sys. UNIT moment

ABOUT HORIZ. AXIS AT PROPELLER

RUN TITLE CARD

000000(62 HΔΔ SSB(N) 598 Conesco Vertical Bending

DATA CONTROL CARD

NO TYPE NO TYPE etc.

0090	01	10	56	44	02	53	
	5	7	9	11	13	15	17 19 21 23 25 27 29 31 33 35 37 39 41 43 45 47 49 51 53 55 57 59 61 63 65 67 69 71

CASE TITLE CARD

001000(62 HΔΔ 35 SECTIONS + PROP SYSTEM SINE MODE I

OPTION CONTROL CARD

	OP1	OP2	OP3	OP4	OP5	OP6	OP7	OP8	OP9	OP10
00200000										
	9	13	17	21	25	29	33	37	41	45

EDIT CONTROL CARD

00210000					
	9	13	17	21	25 29

GENERAL DATA CARD

	NO. SECTIONS	ω_1 (CPS)	ω_2 (CPS)	$\Delta\omega$ (CPS)
0030				
	5	9	17	25

SYSTEMS DATA CARDS

	SYSTEMS	RADIUS	INITIAL J
0031			
0031			

TITLE GBRC-1 PROGRAMMER DATE

PROBLEM NO. CASE 24 Conesco PHASE LABEL 840 277 SHEET OF

SECTION PARAMETER VALUES - UNSCALED

	SECTION NO (5)	END CONDN. (9)	SYSTEM (13)	MASS (17)	WATER INERTIA (25)	$(\Delta x/EI)_n$ (33)	$(\Delta x)_{n,n+1}$ (41)	$(\Delta x/KAG)_{n,n+1}$ (49)	$(I_{\mu z}\Delta x)_{n,n+1}$ (57)	P_n (65)	(73)
0044	0001										
0044	.										
0044	.										
0044	.										
0044	.										
0044	.		Same As Case 221 and Case 16								
0044	.										
0044	.										
0044	.										
0044	.										
0044	0050										
0043											
0044	0055		003	8220		641.5	55.125		59.0	1.0	
0044	0056		003	27900		75.5				0.0	
0043											
0043											
0043											
0043											
0043											
0043											
0043											
0043											
0043											
0043											

TITLE GBEC-1 PROGRAMMER DATE

PROBLEM NO. CASE 24 Conesco PHASE LABEL SHEET OF

SPECIAL CONNECTIONS - PARAMETER VALUES

	5	9	13	17	25	33	41	49	57	65	73
	n	m	SYSTEM	$K_{n,m}$	$C_{n,m}$	$C_{n,m}/\omega$	$(\Delta x)_{n,m}$	$(\Delta x/KAG)_{n,m}$	$(I_{\mu z}\,\Delta x)_{n,m}$		
0053	0049	0050	0002	232000							
0053											
0053	0054	0055	0002	538000							
0053											
0053											
0053											
0053											
0053											
0053											
0053											
0053											
0053											
0053											
0053											
0053											
0053											
0053											
0053											
0053											
0053											
0053											
0053											
0053											
0053											

TITLE GBRC 1　　PROGRAMMER ______　　DATE ______

PROBLEM NO. CASE 25　Conesco　　PHASE ______ LABEL BNO 277　　SHEET ______ OF ______

35 Conc. masses + Propulsion Sys

Unit Vertical Force at Station 54 (STERN)

RUN TITLE CARD

000000(62HΔΔ　SSB(N) 598　CONESCO Vertical Bending

DATA CONTROL CARD

	NO	TYPE	NO	TYPE etc.
0090	01	10	02	43
	5	7	9	11

CASE TITLE CARD　CASE 26, 35 masses + Prop. System, Unit Vertical Force at Sta 54

001000(62HΔΔ

OPTION CONTROL CARD

	OP1	OP2	OP3	OP4	OP5	OP6	OP7	OP8	OP9	OP10
00200000										
	9	13	17	21	25	29	33	37	41	45

EDIT CONTROL CARD

00210000					
	9	13	17	21	25

GENERAL DATA CARD

	NO. SYSTEMS	ω_1 (CPS)	ω_2 (CPS)	$\Delta\omega$ (CPS)
0030				
	5	9	17	25

SYSTEMS DATA CARDS

	SYSTEMS	RADIUS	INITIAL J
0031			
0031			

TITLE ______ PROGRAMMER ______ DATE ______

PROBLEM NO. CASE 25 Conesco PHASE ______ LABEL 840 277 SHEET ______ OF ______

SECTION PARAMETER VALUES - UNSCALED

	SECTION NO (5)	END CONDN. (9)	SYSTEM (13)	MASS (17)	WATER INERTIA (25)	$(\Delta x/EI)_n$ (33)	$(\Delta x)_{n,n+1}$ (41)	$(\Delta x/KAG)_{n,n+1}$ (49)	$(I_{\mu z}\Delta x)_{n,n+1}$ (57)	P_n (65)
0043	0051		0001	225770	42800	4.773				0
0043	0054	0002	0001	33420	11200	15				1.0
0043										
0043										
0043										
0043					MODIFICATION OF CASE 26.					
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										

TITLE GBRC-1

PROBLEM NO. CASE 26 Conesco

35 Conc. Masses + Propulsion Sys

Unit Vertical Force at Station 51 (STERN)

PROGRAMMER

PHASE

LABEL EMO 277

DATE

SHEET OF

RUN TITLE CARD

000000(62HΔΔ SSB(N) 598 Conesco Vertical Bending

DATA CONTROL CARD

NO TYPE NO TYPE etc.

00000	01	10	56	43

5 7 9 13 15 17 19 21 23 25 27 29 31 33 35 37 39 41 43 45 47 49 51 53 55 57 59 61 63 65 67 69 71

CASE TITLE CARD

001000(62HΔΔ 35 masses + Prop. System Unit Vertical F at Station 51 (STERN)

OPTION CONTROL CARD

	OP	OP2	OP3	OP4	OP5	OP6	OP7	OP8	OP9	OP10
00200000										
	9	13	17	21	25	29	33	37	41	45

EDIT CONTROL CARD

00210000					
	9	13	17	21	25	29

GENERAL DATA CARD

	NO. SYSTEMS	ω_1(CPS)	ω_2(CPS)	$\Delta\omega$ (CPS)
0030				
	5	9	17	25

SYSTEMS DATA CARDS

	SYSTEMS	RADIUS	INITIAL J
0031			
0031			

TITLE GBRC-1 PROGRAMMER ____ DATE ____

PROBLEM NO. CASE 25 - Conosco PHASE ____ LABEL 840 277 SHEET ____ OF ____

SECTION PARAMETER VALUES - UNSCALED

	5 SECTION NO	9 END CONDN.	13 SYSTEM	17 MASS	25 WATER INERTIA	33 $(\Delta x/EI)_n$	41 $(\Delta x)_{n,n+1}$	49 $(\Delta x/KAG)_{n,n+1}$	57 $(I_{\mu z}\Delta x)_{n,n+1}$	65 P_n
0043	0051		0001	225770	42800	4.773				1.0
0043	0055		0003	8220		641.5	55.125		59.0	0.0
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										

Note: All other section parameter values same as case 24, but with card IDENTIFICATION changed back to "0043"

TITLE

PROBLEM NO. Case 27 CONESCO

PROGRAMMER REED

PHASE

LABEL 840 277

DATE 15 Nov 62

SHEET 1 OF 4

35 CONC MASSES + PROPULSION - SPRUNG MASSES RIGID

RUN TITLE CARD

000000 (62 HΔΔ SSBN (598) NOV. 15 1962 Conesco Vertical Bending

DATA CONTROL CARD

NO TYPE NO TYPE etc.

00020	01	10	01	20	01	21	01	30	01	31	03	41	02	42	39	43	03	51	01	52	06	53
	5	7	9	13	15	17	19	21	23	25	27	29	31	33	35	37	39	41	43	45	47	49

CASE TITLE CARD

001000 (62 HΔΔ Case 27 35 Masses + Prop System - Sprung Masses Rigid

OPTION CONTROL CARD

	OP1	OP2	OP3	OP4	OP5	OP6	OP7	OP8	OP9	OP10
00200000	1001									
	9	13	17	21	25	29	33	37	41	45

EDIT CONTROL CARD

00210000	1	31	37	39	
	9	13	17	21	25

GENERAL DATA CARD

	NO. SYSTEMS	ω_1 (CPS)	ω_2 (CPS)	$\Delta\omega$ (CPS)	
					1. cps To 10. cps $\Delta\omega = 0.2$ cps
0030	0037	1.0	~~25~~	~~0.2~~	10. cps To 25. cps $\Delta\omega = 0.5$ cps
	5	9	17	25	

SYSTEMS DATA CARDS

	SYSTEMS	RADIUS	INITIAL J
0031	0001	16	.87
0031			

TITLE ______ PROGRAMMER *REED* DATE *15 NOV 62*

PROBLEM NO. *Case 27* PHASE ______ LABEL. 840 *277* SHEET *2* OF *4*

SECTION DATA CARDS

—	SECTION NO.	END CONDN.	SYSTEM	MASS	WATER INERTIA	$(\Delta x/EI)_n$	$(\Delta x)_{n,n+1}$	$(\Delta x/kAG)_{n,n+1}$	$(I_{\mu z}\Delta x)_{n,n+1}$	P_n	
5	9	13	17		25	33	41	49	57	65	73
REAL PART OF SCALING FACTORS											
00410000			0001	.002591	.002591	0.990 E-12	12	0.990 E-9	2591	1.0	
00410000			0002	1	1	1	1	1		1.0	
00410000			0003	.002591		0.990	1.0		2591	1.0	
IMAGINARY PART OF SCALING FACTORS											
00420000			0001			-0.099 E-12		-0.099 E-9			
00420000			0003			-0.099E-12					
00420000											
PARAMETER VALUES - UNSCALED											
0043	0001	0001	0001	7500	1,350	10	4.5	114			
0043	2		1	69700	47800	1.26	6	53.9			
0043	3		1	157750	115 700	1.39	9	40.8			
0043	4		1	355100	206 000	1.363	9	21.6			
0043	5		1	292 700	367 000	0.317	15	19.3			
0043	6		1	860 475	425 000	0.278	10	12.3			
0043	7		1	425 000	365 000	0.161	7	6.73			
0043	8		1	341 250	355 000	0.095	7	8.35			
0043	9		1	387 000	427 000	0.133	11	12.2			
0043	0010		1	299 450	516 000	0.134	10	10.0			
0043	11		1	316 800	471 000	0.0843	8	6.45			
0043	12		1	582 050	481 000	0.072	9	7.30			
0043	13		1	517 750	563 000	0.0798	10	8.15			
0043	14		1	583 250	607 000	0.0854	10	8.10			
0043	15		1	544 045	585 000	0.0811	9	7.65			

TITLE ______ PROGRAMMER REED DATE 15 NOV 62

PROBLEM NO. Case 27 PHASE ______ LABEL 840 277 SHEET 3 OF 4

SECTION PARAMETER VALUES - UNSCALED

5	9	13	17		25	33	41	49	57	65	73
	SECTION NO	END CONDN.	SYSTEM	MASS	WATER INERTIA	$(\Delta x/EI)_n$	$(\Delta x)_{n,n+1}$	$(\Delta x/KAG)_{n,n+1}$	$(I_{\mu z}\Delta x)_{n,n+1}$	P_n
0043	0016		0001	534 720	560 000	0.0106	4	6.90		
0043	17		1	599 500	591 000	0 0674	10	7.64		
0043	18		1	813 450	745 000	0.0857	14	10.80		
0043	19		1	945 000	930 000	0.1070	16	12.2		
0043	20		1	916 000	868 000	0 1000	12	9.3		
0043	21		1	786 000	772 000	0.0994	13	10.2		
0043	22		1	1040300	953 000	0.1217	18	15.2		
0043	23		1	1531211	1150 000	0.218	22	26.2		
0043	24		1	841 943	824 000	0.216	9	10.6		
0043	25		1	280 600	534 000	0.153	5	6.92		
0043	26		1	483 400	456 000	0.247	16	22.2		
0043	27		1	574 920	623 000	0.353	15	18.6		
0043	28		1	440 420	607 000	0.422	20	32.2		
0043	29		1	384 500	434 000	0.561	11	23.2		
0043	30		1	226 500	223000	0.619	9	22.0		
0043	31		1	116 750	151000	0.632	8	23.9		
0043	32		1	101 050	91800	0.772	5	17.6		
0043	33		1	59 300	68900	1.063	7.42	30.1		
0043	34		1	44 200	48 600	1.794	0.	0.		
0043	36		1	475 720	42 800	4.773	12.	120.		
0043	37	0001	1	23 420	11 200	15.—				
0043	35		0003	13 600		1214.				
0043	38		3	8220		641.5	55.125		59	
0043	39		3	27900		75.5	0.			1.0

TITLE ______ PROGRAMMER REED DATE 15 NOV 62

PROBLEM NO. Case 27 PHASE ______ LABEL 840 277 SHEET 4 OF 4

5 9 13 17 25 33 41 49 57 65 73

SPECIAL CONNECTION CARDS

	n	m	SYSTEM	$K_{n,m}$	$C_{n,m}$	$C_{n,m}/\omega$	$\Delta X_{n,m}$	$(\Delta X/KAG)_{n,m}$	$(I_{\mu z}\Delta X)_{n,m}$
REAL PART OF SCALING FACTORS									
0051			0001	1.0	1.0	1.0	12.0	0.990 E-9	
0051			0002	1.0	1.0	1.0			
0051			0003	1.0	1.0	1.0	1.0		
IMAGINARY PART OF SCALING FACTORS									
0052			0001					-0.099 E-9	
0052									
0052									
PARAMETER VALUES - UNSCALED									
0053	34	35	0002	232 000					
0053	36	38	0002	538 000					
0053	34	36	0001				4.58	28.4	
0053	39		0002		40				
0053	31	35	0002				149.875		
0053	35	38	0003				196.5		
0099									
0053									
0053									
0053									

TITLE ______ PROGRAMMER *REED* DATE *15 Nov. 62*

PROBLEM NO. *Case 30 Conesco* PHASE ______ LABEL 840-377 SHEET *1* OF *7*

Case 18 with Rotary Inertia Added

RUN TITLE CARD

000000(62 HΔΔ *SSBN (598) Nov 15, 1962 CONESCO Vertical Bending*

DATA CONTROL CARD

NO TYPE NO TYPE etc.

0090	21	10	01	20	01	21	01	30	01	31	03	41	01	42	56	43	05	51	01
	5	7	9		13	15	17	19	21	23	25	27	29	31	33	35	37	39	41

52	36	53	
43	45	47	49 51 53 55 57 59 61 63 65 67 69 71

CASE TITLE CARD

001000(62 HΔΔ *Case 30 35 Hull Masses + Prop. System + Sprung Masses Including Rotary Inertia of Hull Masses*

OPTION CONTROL CARD

	OP1	OP2	OP3	OP4	OP5	OP6	OP7	OP8	OP9	OP10
00200000	0001									
	9	13	17	21	25	29	33	37	41	45

EDIT CONTROL CARD

00210000	1	46	54	56	
	9	13	17	21	25 29

GENERAL DATA CARD

NO. SECTIONS	ω_1 (CPS)	ω_2 (CPS)	$\Delta\omega$ (CPS)
0030 0036	1.0	~~25.0~~	~~0.5~~
5	9	17	25

1.0 cps To 10.0 cps $\Delta\omega$ = 0.2 cps
10.0 cps To 25.0 cps $\Delta\omega$ = 0.5 cps

SYSTEMS DATA CARDS

	SYSTEMS	RADIUS	INITIAL J
0031	0001	16	.87
0031			

TITLE ______ PROGRAMMER *REED* DATE *15 NOV 62*

PROBLEM NO. *Case 30 CONESCO* PHASE ______ LABEL 840 *277* SHEET *2* OF *7*

Case 18 with Rotary Inertia Added

SECTION DATA CARDS

5	9	13	17		25	33	41	49	57	65	73
	SECTION NO.	END CONDN.	SYSTEM	MASS	WATER INERTIA	$(\Delta x/EI)_n$	$(\Delta x)_{n,n+1}$	$(\Delta x/kAG)_{n,n+1}$	$(I_{\mu z}\Delta x)_{n,n+1}$	P_n	
REAL PART OF SCALING FACTORS											
00410000			0001	.002591	.002591	0.990E-12	12	0.990E-9	2591	1.0	
00410000			0002	.002591			1.0		2591	1.0	
00410000			0003	.002591		.990E-12	1.0	.990E-9	2591	1.0	
IMAGINARY PART OF SCALING FACTORS											
00420000			0001			-.099E-12		-.099E-9			
00420000											
00420000											
PARAMETER VALUES - UNSCALED											
0043	0001	0001	0001	7500	6350	10	4.5	114	25		
0043	2		1						87.9		
0043	3		1						408		
0043	4		1						1330		
0043	5		1						4450		
0043	6		1						4100		
0043	7		1						2195		
0043	8		1						2665		
0043	9		1						4640		
0043	0010		1						4180		
0043	11		1						6420		
0043	12		1								
0043	15		1						6290		
0043	16		1						9050		
0043	17		1								

SAME AS CASE 17

TITLE ______ PROGRAMMER REED DATE 15 NOV 62

PROBLEM NO. ______ PHASE ______ LABEL 8MC 277 SHEET 3 OF 7

SECTION PARAMETER VALUES - UNSCALED

	5	9	13	17	25	33	41	49	57	65	73
	SECTION NO	END CONDN.	SYSTEM	MASS	WATER INERTIA	$(\Delta x/EI)_n$	$(\Delta x)_{n,n+1}$	$(\Delta x/KAG)_{n,n+1}$	$(I_{\mu z}\Delta x)_{n,n+1}$	P_n	
0043	0020		0001						7930		
0043	21		1						10 000		
0043	22		1						14 120		
0043	23		1						16 000		
0043	24		1						12 380		
0043	25		1						11 290		
0043	26		1								
0043	29		1								
0043	32		1						3060		
0043	33		1						1278		
0043	34		1								
0043	36		1								
0043	38		1								
0043	43		1						1050		
0043	44		1						663		
0043	45		1						390		
0043	46		1						378		
0043	47		1								
0043	49		1								
0043	51		1								
0043	54	0002	1								
0043	13		2								
0043	14		2								
0043											

SAME AS CASE 17

TITLE ______ PROGRAMMER REED DATE 15 NOV 62

PROBLEM NO. ______ PHASE ______ LABEL 840 277 SHEET 4 OF 7

SECTION PARAMETER VALUES - UNSCALED

	SECTION NO	END CONDN.	SYSTEM	MASS	WATER INERTIA	$(\Delta x/EI)_n$	$(\Delta x)_{n,n+1}$	$(\Delta x/KAG)_{n,n+1}$	$(I_{\mu z}\Delta x)_{n,n+1}$	P_n
	5	9	13	17	25	33	41	49	57	65
0043	0018		0002							
0043	19		2							
0043	27		2							
0043	28		2							
0043	30		2							
0043	31		2							
0043	32		2							
0043	37		2							
0043	38		2							
0043	40		2							
0043	41		2							
0043	42		2							
0043	48		2							
0043	52		2							
0043	53		2							
0043	50		3							
0043	55		3						59.0	
0043	56		3							1.0
0043										
0043										
0043										
0043										
0043										
0043										

SAME AS CASE 17

73

TITLE ______ PROGRAMMER REED DATE 15 NOV 62

PROBLEM NO. ______ PHASE ______ LABEL 8HD 277 SHEET 5 OF 7

5 9 13 17 25 33 41 49 57 65 73

SPECIAL CONNECTION CARDS

	n	m	SYSTEM	$K_{n,m}$	$C_{n,m}$	$C_{n,m}/\omega$	$\Delta X_{n,m}$	$(\Delta X/KAG)_{n,m}$	$(I_{\mu z}\Delta x)_{n,m}$	
REAL PART OF SCALING FACTORS										
0051			0001	1.0	1.0	1.0	12.	0.990 E-9	2591	
0051			2	1.0	1.0	1.0		0.	2591	
0051			3	1.0	1.0	1.0	1.	0.990E-9	2591	
IMAGINARY PART OF SCALING FACTORS										
0052			0001					-0.099 E-9		
0052			0003					-.099 E-9		
0052										
PARAMETER VALUES - UNSCALED										
0053	0012	0013	0002							
0053	12	14	2							
0053	17	18	2							
0053	17	19	2							
0053	26	27	2							
0053	26	28	2							
0053	29	30	2							
0053	29	31	2							
0053	34	35	2							
0053	36	37	2							

SAME AS CASE 17

TITLE ____ PROGRAMMER REED DATE 15 NOV 62

PROBLEM NO. ____ PHASE ____ LABEL - 840 277 SHEET 6 OF 7

SPECIAL CONNECTIONS - PARAMETER VALUES

	n	m	SYSTEM	$K_{n,m}$	$C_{n,m}$	$C_{n,m}/\omega$	$(\Delta X)_{n,m}$	$(\Delta X/KAG)_{n,m}$	$(I_{\mu z}\Delta X)_{n,m}$		
	5	9	13	17	25	33	41	49	57	65	73
0053	0038	0039	0002								
0053	38	40	2								
0053	38	41	2								
0053	38	42	2								
0053	47	48	2								
0053	51	52	2								
0053	51	53	2								
0053	12	15	1						7470		
0053	17	20	1						7680		
0053	26	29	1						13200		
0053	29	32	1						12630		
0053	34	36	1						4020		
0053	36	38	1						2540		
0053	38	43	1						2920		
0053	47	49	1						216.6		
0053	49	51	1						117		
0053	51	54	1						141.8		
0053	49	50	2								
0053	54	55	2								
0053	50	55	3								
0053	13		2								
0053	14		2								
0053	52		2								
0053	53		2								

SAME AS CASE 17

TITLE ______ PROGRAMMER REED DATE 15 NOV 62

PROBLEM NO. ______ PHASE ______ LABEL - BHO 277 SHEET 7 OF 7

SPECIAL CONNECTIONS - PARAMETER VALUES

	5 n	9 m	13 SYSTEM	17 $K_{n,m}$	25 $C_{n,m}$	33 $C_{n,m}/\omega$	41 $(\Delta x)_{n,m}$	49 $(\Delta x/KAG)_{n,m}$	57 $(I_{\mu z}\Delta x)_{n,m}$	65	73
0053	0056		0002		SAME AS CASE 17						
0053	46	50	3								
00[illegible]											
0053											
0053											
0053											
0053											
0053											
0053											
0053											
0053											
0053											
0053											
0053											
0053											
0053											
0053											
0053											
0053											
0053											
0053											
0053											
CC53											
0053											

REFERENCES

1. Reed, F. E. and Bradshaw, R. T.
   Ship Hull Vibrations 2 - The Distribution of Exciting Forces Generated by Propellers
   CONESCO Report No. F-101-2 June 1960

2. Reed, F. E. and Bradshaw, R. T.
   Ship Hull Vibrations 3 - Estimation of Amplitude of Vibration of USS GEORGE WASHINGTON SSB(N) 598
   CONESCO Report No. F-105-1 June 1961

3. Gesswein, B. H. and Moses, F. Calculated Modes and Frequencies of Hull Vibration of USS GEORGE WASHINGTON SSB(N) 598
   DTMB Report 1464 Nov. 1960

4. Reed, F. E.
   Ship Hull Vibrations 4 - The Forces and Moments Generated by the Propellers on the USS GEORGE WASHINGTON SSB(N) 598 and the USS ETHAN ALLEN SSB(N) 608 CONESCO Report F-111-1 July 1962

5. General Dynamics Corp. E. B. Div. Plan 98A21-07
   SSB(N) 599 - Weight Distribution - 19 Apr. 1960

6. Bovenzi, Frank G.
   Frequencies and Mode Shapes of SSB(N) 599
   Electric Boat Div., General Dynamics Corp. Report C413-61-001 Jan. 13, 1961

7. "Detailed Weight Summary" SSB(N) 599
   Electric Boat Div., General Dynamics Corp. July 1, 1960

8. Sokolnikoff, I. S. and Sokolnikoff, E. C.,
   Higher Mathematics for Engineers and Physicists
   McGraw Hill, 1934 (pages 407-414)

9. Goodman, Theodore R., and Sargent, Theodore P.
   Effect of Body Perturbations on Added Mass - With Application to Non-Linear Heaving of Ships
   Journ. Ship Research Vol. 4. No. 4 Mar. 1961, p. 22

REFERENCES

10. Lamb, Horace-Hydrodynamics
Dover Publications, N. Y. 1945

11. Macagno, E. O., and Landweber, L.
Irrotational Motion of the Liquid Surrounding a Vibrating Ellipsoid of Revolution,
Journ. Ship Res. June 1958, pp 37-49

12. Greenspan, J. E.
A Shell Type Approach for the Vibration and Acoustic Analysis of Ship and Submarine Hull, J. C. Engineering Research Associates, Tech. Peport No. 1
Contract NOnr - 2862(00)X, May 1960

13. Junger, M. C., and Kleinschmidt, K.
Acoustic Mutual and Self Radiating Impedances for Surface Elements on a Prolate Spheroid and Associated Far Field Sound Pressures, Cambridge Acoustical Associates, Inc., Rep. 6-110-48, Contract NOnr-2739(00).

14. Chertock, G.
Sound Radiation from Prolate Cylinders
Journ. Acoustical Soc. America July 1961 pg. 871

15. Lewis, F. M.
The Inertia of the Water Surrounding a Vibrating Ship, SNAME Trans.
Vol. 37 (1929) pp 1-20

16. Taylor, J. Lockwood
Some Hydrodynamic Inertia Coefficients
Phil. Mag. Series 7 Vol. 9. No. 55 (Jan 1930) pp. 161-183

17. Wendel, Karl
Hydrodynamic Masses and Hydrodynamic Moments of Inertia
DTMB Transl. 260 (July 1956)

18. Weinblum, G.
Über Hydrodynamische Massen, Schiff und Hafen, V. 3 No. 12 (1951)
pp 422-425

REFERENCES

19. Havelock, Sir Thomas H.
Ship Vibrations: The Virtual Inertia of a Spheroid in Shallow Water
Trans. Instn. Nav. Arch. Lond. V. 95 (Jan 1953) pp 1-9

20. Landweber, L. and deMacgno, M. D.
Added Mass of Two-Dimensional Forms Oscillating in a Free Surface
J. Ship Res. V. 1, No. 2 (Nov. 1957) pp 20-31

21. Landweber, L. and Macagno, Matilde
Added Mass of a Three-parameter of Two-dimensional Forms Oscillating in a Free Surface, J. Ship Res. V. 2, No. 4 (Mar. 1959)

22. Kaplan, P.
A Study of the Virtual Mass Associated with the Vertical Vibration of Ships in Water, Davidson Lab. Report No. 734 (Dec. 1959)

23. Landweber, L. and Macagno, M.
Added Mass of a Rigid Prolate Spheroid Oscillating Horizontally in a Free Surface, J. Ship Res. V. 3 No. 4 (1960) pp 30-36

24. McGoldrick, R. T. and Russo, V. L.
Hull Vibration Investigation on SS GOPHER MARINER
Trans. SNAME Vol. 63 1955

25. Kennard, E. H.
Virtual Mass of Ships in Higher Modes of Vibration, Appendix to DTMB Report 762, (June 1951)

26. Leibowitz, R. C. and Kennard, E. H.
Theory of Freely Vibrating Non-uniform Beams, Including Methods of Solution and Application to Ships, DTMB Report 1317, May 1961, pp 235-243

27. Horvay, G., and I. M. Clausen, Membrane and Bending Analysis of Axisymmetrical Shells, ASME J. A. M., V. 22, pp 25-30 (1955)

REFERENCES

28. Schade, H. A., Notes on Primary Strength Calculation. SNAME Trans. V. 65, pp. 154-162 (1959)

29. Schade, H. A., The Effective Breadth of Stiffened Plating Under Bending Loads, SNAME Trans. V. 59, pp 403-420 (1951)

30. Schade, H. A., The Effective Breadth Concept in Ship-Structure Design, SNAME Trans. V. 61, pp 410-424 (1953)

31. Lewis, F. M. and Auslaender, J.
Virtual Inertia of Propellers
Journ. Ship Res. Vol. 3 No. 4 Mar. 1960 p 37

32. Timoshenko, S.
Theory of Plates and Shells
McGraw-Hill (1940)

33. Timoshenko, S. and Goodier, J. N.
Theory of Elasticity
McGraw-Hill (1951)

CONESCO, Inc., Cambridge, Mass., Rep. F-111-2
SHIP HULL VIBRATIONS-5, Analysis of Hull Structures as applied to SSB(N) 598 GEORGE WASHINGTON by F. E. Reed and E. Cuthill, Mar. 1963, XI 270 p. graphs, tables, refs. (Index No. S-F013-11-01, Task 1351; Contract NObs 86111)

A computer program for determining the natural frequency of a ship hull elastically connected to other elastic systems and to sprung masses and containing concentrated and hysteresis damping is developed. This program is applied to the mass-elastic system that represents the SSB(N) 598 GEO. WASHINGTON in detail. The influence upon the hull response to propeller and hull excitation in the stem of variables such as the number of mass points used to represent the hull, the omission of the propulsion sub-system, the stiffness between the propulsion sub-system and the primary hull, the inclusion of the moment of inertia of the hull cross-sections, the treatment of water inertias, the treatment of sprung masses and the amount of hysteresis damping in the hull is shown.

1. Submarines (Nuclear)-Vibration-Mathematical Analysis
2. Propellers (Marine)-Vibration
3. Vibration - Analysis
4. GEORGE WASHINGTON (U. S. fleet ballistic-missile submarine, nuclear, SSB(N) 598)

I Reed, F. E.
II Cuthill, Elizabeth

CONESCO, Inc., Cambridge, Mass., Rep. F-111-2
SHIP HULL VIBRATIONS-5, Analysis of Hull Structures as applied to SSB(N) 598 GEORGE WASHINGTON by F. E. Reed and E. Cuthill, Mar. 1963, XI 270 p. graphs, tables, refs. (Index No. S-F013-11-01, Task 1351; Contract NObs 86111)

A computer program for determining the natural frequency of a ship hull elastically connected to other elastic systems and to sprung masses and containing concentrated and hysteresis damping is developed. This program is applied to the mass-elastic system that represents the SSB(N) 598 GEO. WASHINGTON in detail. The influence upon the hull response to propeller and hull excitation in the stem of variables such as the number of mass points used to represent the hull, the omission of the propulsion sub-system, the stiffness between the propulsion sub-system and the primary hull, the inclusion of the moment of inertia of the hull cross-sections, the treatment of water inertias, the treatment of sprung masses and the amount of hysteresis damping in the hull is shown.

1. Submarines (Nuclear)-Vibration-Mathematical Analysis
2. Propellers (Marine)-Vibration
3. Vibration - Analysis
4. GEORGE WASHINGTON (U. S. fleet ballistic-missile submarine, nuclear, SSB(N) 598)

I Reed, F. E.
II Cuthill, Elizabeth

CONESCO, Inc., Cambridge, Mass., Rep. F-111-2
SHIP HULL VIBRATIONS-5, Analysis of Hull Structures as applied to SSB(N) 598 GEORGE WASHINGTON by F. E. Reed and E. Cuthill, Mar. 1963, XI 270 p. graphs, tables, refs. (Index No. S-F013-11-01, Task 1351; Contract NObs 86111)

A computer program for determining the natural frequency of a ship hull elastically connected to other elastic systems and to sprung masses and containing concentrated and hysteresis damping is developed. This program is applied to the mass-elastic system that represents the SSB(N) 598 GEO. WASHINGTON in detail. The influence upon the hull response to propeller and hull excitation in the stem of variables such as the number of mass points used to represent the hull, the omission of the propulsion sub-system, the stiffness between the propulsion sub-system and the primary hull, the inclusion of the moment of inertia of the hull cross-sections, the treatment of water inertias, the treatment of sprung masses and the amount of hysteresis damping in the hull is shown.

1. Submarines (Nuclear)-Vibration-Mathematical Analysis
2. Propellers (Marine)-Vibration
3. Vibration - Analysis
4. GEORGE WASHINGTON (U. S. fleet ballistic-missile submarine, nuclear, SSB(N) 598)

I Reed, F. E.
II Cuthill, Elizabeth

CONESCO, Inc., Cambridge, Mass., Rep. F-111-2
SHIP HULL VIBRATIONS-5, Analysis of Hull Structures as applied to SSB(N) 598 GEORGE WASHINGTON by F. E. Reed and E. Cuthill, Mar. 1963, XI 270 p. graphs, tables, refs. (Index No. S-F013-11-01, Task 1351; Contract NObs 86111)

A computer program for determining the natural frequency of a ship hull elastically connected to other elastic systems and to sprung masses and containing concentrated and hysteresis damping is developed. This program is applied to the mass-elastic system that represents the SSB(N) 598 GEO. WASHINGTON in detail. The influence upon the hull response to propeller and hull excitation in the stem of variables such as the number of mass points used to represent the hull, the omission of the propulsion sub-system, the stiffness between the propulsion sub-system and the primary hull, the inclusion of the moment of inertia of the hull cross-sections, the treatment of water inertias, the treatment of sprung masses and the amount of hysteresis damping in the hull is shown.

1. Submarines (Nuclear)-Vibration-Mathematical Analysis
2. Propellers (Marine)-Vibration
3. Vibration - Analysis
4. GEORGE WASHINGTON (U. S. fleet ballistic-missile submarine, nuclear, SSB(N) 598)

I Reed, F. E.
II Cuthill, Elizabeth